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METHODS OF BALANCING OF AN AXISYMMETRIC FLEXIBLE ROTOR BY PASSIVE AUTO-BALANCERS

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Застосовано емпіричний критерій настання автобалансування для гнучкого осесиметричного ротора, що балансується n пасивними автобалансирами будь-якого типу. Встановлено, що автобалансування може відбуватися тільки на швидкостях, що перевищують n-ю критичну швидкість обертання ротора. Знайдено діапазони кутових швидкостей обертання ротора, на яких наступатиме автобалансування. Запропоновано способи оптимального балансування гнучкого ротора

Ключові слова: гнучкий ротор, пасивний автобалансир, автобалансування, критерій настання автобалансування, критичні швидкості гнучкого ротора

Применен эмпирический критерий наступления автобалансировки для гибкого осесимметричного ротора, балансируемого n пассивными автобалансирами любого типа. Установлено, что автобалансировка может происходить только на скоростях, превышающих n-ю критическую скорость вращения ротора. Найдены диапазоны угловых скоростей вращения ротора, на которых будет наступать автобалансировка. Предложены способы оптимальной балансировки ротора

Ключевые слова: гибкий ротор, пассивный автобалансир, автобалансировка, критерий наступления автобалансировки, критические скорости гибкого ротора

1. Introduction

Many rotors of aircraft engines, gas turbine engines of power plants, agricultural machines, etc. work at speeds above the first critical one, and therefore behave as flexible [1, 2]. The form and unbalance of the flexible rotor depend on the current speed. In addition, during the operation of such rotors, their unbalance can change due to temperature, wear, dirt sticking, etc. Therefore, it is expedient to constantly balance flexible rotors in motion, in the process of exploitation, by passive auto-balancers [3]. For application of passive auto-balancers, it is necessary to know whether it is possible in principle and on what rotation speeds to balance the flexible rotor installed on the certain supports by them in motion.

Dynamics of rotors without auto-balancers is described by rather difficult differential equations of motion [1–5]. Introduction of auto-balancers (masses movable relative to the rotor) to the system makes the equations even more complicated [3, 6–16]. Therefore, an analytical determination of the conditions for the occurrence of auto-balancing is a complex mathematical problem.

Analytically, the conditions for the occurrence of auto-balancing are determined in [3–16]. At the same time, the most general conditions applicable for auto-balancers of any type and with any number of corrective weights, are received using the empirical criteria [3–5].

Thus, it is actual to find the conditions for the occurrence of auto-balancing in the case of balancing of the flexible massive rotor by any number of auto-balancers of any type.

2. Literature review and problem statement

In detail, the empirical criteria (for the occurrence of auto-balancing or stability of main motions) and the history of their development are described in [4]. In accordance with the criteria, the possibility of the occurrence of auto-balancing is determined by the rotor response to the elementary unbalances applied in the planes of correction (auto-balancers). As a consequence, if within the certain rotor model framework the rotor dynamics had been analytically investigated, then it is possible to analytically determine the conditions for the occurrence of auto-balancing without derivation of the equations of motion of the rotor with auto-balancers.

The efficiency of the empirical criterion for the occurrence of auto-balancing is shown in [4] for a rigid rotor with a fixed point and isotropic elastic support, and [5] for a rigid rotor on two isotropic elastic supports. At the same time, in [5], the results that generalize the results of the papers [6, 7] by extending them for the case of any number of auto-balancers of any type with many corrective weights were obtained.

Let us examine in more detail how deeply the possibility of balancing a flexible rotor by passive auto-balancers is investigated.

In [8], the problem is set to determine the conditions for the occurrence of auto-balancing for the flexible rotor, supported by two spherical bearings, during the rotor balancing by several two-ball balancers. The rotor was modeled as a rotating Euler-Bernoulli beam. To solve the problem, the stability of such motions of the system in which the balls eliminate the rotor deflections in the correction planes was investigated. Because of the complexity of the problem, the case of the rotor balancing by a single two-ball balancer located in the middle of the rotor was investigated. It was found that auto-balancing occurs at speeds above odd critical rotor speeds and below even critical speeds of this rotor with an intermediate support in the auto-balancer plane. Without carrying out the research, the assumption was made that in the case of two auto-balancers, the auto-balancing occurred at speeds above even critical speeds of the rotor and below odd critical speeds of this rotor with intermediate supports in the planes of the auto-balancers.

In [9], the dynamics and stability of the unbalanced flexible shaft-disk system equipped with the auto-balancer with two balls are investigated. It has been determined that the regions of the auto-balancing stability are at supercritical shaft speeds between each flexible mode. These are the narrower regions of rotor speeds at which auto-balancing occurs than the regions found in [8].

In [10], a mathematical model of a massive two-support rotor with one disk mounted on a shaft in any place is constructed. The model takes into account the gyroscopic effect of the disk. In [11] the similar model is constructed in the case when in the disk plane there is a two-ball balancer. It is established that during the passing of the regular critical speed of the rotor, auto-balancing occurs, and with approaching (from below) the next critical speed, it disappears. This agrees with the results of [9].

In [12–14], it is shown that auto-balancers can balance the flexible rotor only partially. Herewith, the auto-balancers eliminate rotor deflections only in their planes. In [12], the flexible rotor was modeled as a massive elastic shaft, on which N two-ball balancers were mounted.

In work [13], the flexible rotor was modelled as several massive disks mounted on a weightless elastic shaft. One of the disks is statically unbalanced. With this or another disk, a two-ball balancer is combined. It was found that when auto-balancing occurs, the accuracy of balancing increases with the approach of the auto-balancer to the unbalanced disk. In [14], unlike [13], a dynamically unbalanced disk and two auto-balancers from different sides of the disk are mounted on a weightless shaft. It was established that the auto-balancers cannot completely balance the disk and eliminate shaft deflections. However, the accuracy of balancing increases with the approach of auto-balancers to the disk.

If in [8–14], the task of decreasing the rotor deflections in the planes of auto-balancers was set, in [15], the task of decreasing the support reactions was set. In [15], the possibility of balancing of the flexible rotor on two supply supports by two ball balancers located near the supports was proved.

It is shown that auto-balancing occurred on above resonance rotor speeds. It is established that auto-balancing can be violated due to a lack of balancing capacity of auto-balancers in the vicinity of the rotor critical speeds. In [16], the transient processes that occur in such balancing of the rotor were evaluated.

Thus, for today it is not investigated how to balance a flexible massive rotor by any number of passive auto-balancers of any type. Ranges of rotor speeds at which the auto-balancing will occur are not found. Below it is investigated using the empirical criterion for the occurrence of auto-balancing.

3. The purpose and problems of the research

The purpose of this work is to receive conditions under which one and more passive auto-balancers of any type will balance a flexible massive axisymmetric rotor.

To achieve this purpose, it is necessary to solve the following research problems:

– to construct the physical and mathematical model of a flexible axisymmetric rotor of constant section, supported by two spherical bearings, with the elementary rotor unbalances applied at future points of suspension of auto-balancers;

– using the empirical criterion for the occurrence of auto-balancing to receive the functionality defining the conditions for the occurrence of auto-balancing;

– to find the conditions for the occurrence of auto-balancing and to generalize them for the case of the rotor of variable section and another type of rotor fixing;

– to find optimum methods of balancing of the flexible rotor by passive auto-balancers.

4. Methods of searching the conditions for the occurrence of auto-balancing

The empirical criterion for the occurrence of auto-balancing is used [13]. The criterion is intended to answer the question – whether it is possible in principle and under what conditions to balance automatically a particular rotor by n passive auto-balancers of any type. According to the criterion, for the occurrence of auto-balancing it is necessary and sufficient that at any elementary unbalances the condition is satisfied:

$$\frac{1}{T}\int_0^T\left(\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t)\right)dt<0, \qquad (1)$$

where t is the time; $\vec{s}_j$ is the elementary rotor unbalance lying in the j^{th} correction plane and applied at the corresponding point j on the longitudinal axis of the rotor $/j=\overline{1,n}/$; $\vec{r}_j$ is the vector of deviation of the point j from its position in the motionless rotor, caused by elementary rotor unbalances $\vec{s}_1,...,\vec{s}_n$; T is the period in case the motion is periodic or another characteristic time interval (time of one or several rotations of the rotor, long interval of time, etc.).

The criterion is applied in the following sequence:

1) the physical-mechanical model of a rotor with the elementary rotor unbalances applied at the future suspension points of auto-balancers, is described;

2) differential equations of motion of the rotor are derived (or equations of the steady-state motion);

3) the steady-state motion of the rotor is searched for;

4) a functional of the criterion for the occurrence of auto-balancing is built;

5) conditions for the occurrence of auto-balancing are determined from the condition of negativity of the functional.

Let us note that, as a rule, the functional of the criterion is a quadratic form of the elementary unbalances. The negative definiteness of this form can be investigated using Sylvester's criterion.

The research is conducted on the example of an axisymmetric rotor of the constant section mounted on two fixed hinge supports. The received results are generalized for rotors with other types of fixing.

5. Results of the research to determine the conditions for the occurrence of auto-balancing for a flexible rotor

5. 1. Description of the physical-mechanical model of the flexible rotor

Fig. 1 shows the scheme of a homogeneous flexible rotor supported by two spherical bearings, length L and linear mass m=ρA, where ρ is the specific density of the rotor material, and A is its cross-sectional area. Without loss of generality, we assume that the rotor bends in one plane. From the left support in the direction of the right support, we draw the axis x. In the rotor bending plane, we draw the axis v perpendicular to the axis x.

At distances x_j from the left support, the independent elementary unbalances s_j, $/j=\overline{1,n}/$, lying in the rotor bending (x, v) plane are applied to the rotor.

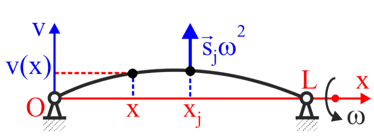

Fig. 1. Motion of the flexible rotor supported by two spherical bearings

5. 2. Differential equation of the steady-state rotor motion

The differential equation of the steady-state rotor motion without taking into account the gyroscopic effect has the form [1]:

$$-\omega^2\rho A v+EJ\frac{\partial^4 v}{\partial x^4}=\omega^2\sum_{j=1}^{n}s_j\delta(x_j-x), \qquad (2)$$

where EJ is the bending rigidity of the rotor; v is the transverse deflection of the rotor in the (x, v) plane at a distance x from the left support; ω is the rotor angular speed; δ is the Dirac's delta function.

The principal functions and natural frequencies of the rotor [1]

$$v_k(x)=\sin\frac{\pi kx}{L},\quad \omega_k^2=\frac{EJ}{\rho A}\cdot\frac{k^4\pi^4}{L^4},\ /k=1,2,.../. \qquad (3)$$

It follows from (3) that the critical speeds of the rotor rapidly increase with respect to k, with each successive speed much greater than the previous ones.

The rotor flexural modes are shown in Fig. 2.

The decomposition of the elementary unbalance s_j over the principal functions has the form

$$s_j\delta(x_j-x)=\frac{2s_j}{L}\sum_{k=1}^{\infty}\sin\frac{\pi kx_j}{L}\sin\frac{\pi kx}{L},\quad /j=\overline{1,n}/. \qquad (4)$$

Then

$$\sum_{j=1}^{n}s_j\delta(x_j-x)=\sum_{j=1}^{n}\left(\frac{2s_j}{L}\sum_{k=1}^{\infty}\sin\frac{\pi kx_j}{L}\sin\frac{\pi kx}{L}\right)=$$

$$=\sum_{k=1}^{\infty}\left[\sin\frac{\pi kx}{L}\left(\sum_{j=1}^{n}\frac{2s_j}{L}\sin\frac{\pi kx_j}{L}\right)\right]=\sum_{k=1}^{\infty}S_k\sin\frac{\pi kx}{L}, \qquad (5)$$

where

$$S_k=\sum_{j=1}^{n}\frac{2s_j}{L}\sin\frac{\pi kx_j}{L},\quad /k=1,2,.../ \qquad (6)$$

is the amplitude of the j^{th} modal unbalance caused by all elementary unbalances.

The rotor modal unbalances corresponding to the rotor flexural modes are shown in Fig. 2.

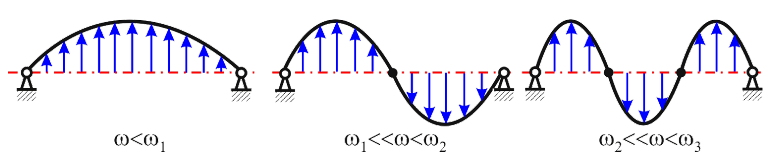

Fig. 2. The flexible rotor supported by two spherical bearings — the rotor flexural modes and modal unbalances, nodal points

The notes for the case if auto-balancers are located in different correction planes:

1. Since the elementary unbalances s_j, $/j=\overline{1,n}/$ are independent, we can choose exactly n independent amplitudes among the amplitudes S_k, /k=1, 2,.../.

2. The determinant of the linear system of equations

$$\sum_{j=1}^{n}\frac{2s_j}{L}\sin\frac{\pi kx_j}{L}=S_k,\quad /k=\overline{1,n}/ \qquad (7)$$

for any n is not equal to zero. Therefore, as independent amplitudes, we can choose the first n ones – $S_1, S_2,..., S_n$.

3. If the elementary unbalances are at the nodes of the rotor flexural k^{th} mode (Fig. 2, k>n≥1), then S_k=0. At the

same time, auto-balancers do not respond to the rotor flexural k^{th} mode and cannot in principle balance the k^{th} modal unbalance.

5. 3. The law of the steady-state motion of the rotor, which corresponds to the applied elementary unbalances

Taking into account (6), the equation of the steady-state motion of the rotor (2) takes the form

$$-\omega^2\rho A v + EJ\frac{\partial^4 v}{\partial x^4} = \omega^2\sum_{k=1}^{\infty} S_k \sin\frac{\pi k x}{L}. \quad (8)$$

The law of the steady-state motion of the rotor

$$v(x) = \omega^2\sum_{k=1}^{\infty}\frac{S_k}{\omega_k^2-\omega^2}\sin\frac{\pi k x}{L}. \quad (9)$$

By a direct substitution, it can be verified that the law (9) satisfies both the equation of the steady-state motion (2) and the zero boundary conditions ($v(0)=v(L)=0$, $v''(0)=v''(L)=0$).

5. 4. Construction of the functional of the criterion for the occurrence of auto-balancing

From (9) we find the displacement of the section j:

$$v(x_j) = \omega^2\sum_{k=1}^{\infty}\frac{S_k}{\omega_k^2-\omega^2}\sin\frac{\pi k x_j}{L}, \quad /j=\overline{1,n}\,/. \quad (10)$$

The integrand of the functional (1) takes the form

$$\sum_{j=1}^{n}\vec{s}_j(t)\cdot\vec{r}_j(t) = \sum_{j=1}^{n} v(x_j)s_j =$$
$$= \sum_{j=1}^{n}\left(\omega^2\sum_{k=1}^{\infty}\frac{S_k}{\omega_k^2-\omega^2}\sin\frac{\pi k x_j}{L}\right)s_j =$$
$$= \omega^2\sum_{k=1}^{\infty}\left[\frac{S_k}{\omega_k^2-\omega^2}\left(\sum_{j=1}^{n} s_j\sin\frac{\pi k x_j}{L}\right)\right] = \frac{L\omega^2}{2}\sum_{k=1}^{\infty}\frac{S_k^2}{\omega_k^2-\omega^2}.$$

Taking into account that this expression does not depend on time, we obtain the following condition for the occurrence of auto-balancing

$$\sum_{k=1}^{\infty}\frac{S_k^2}{\omega_k^2-\omega^2} < 0. \quad (11)$$

Let us transform the sum

$$\sum_{k=1}^{\infty}\frac{S_k^2}{\omega_k^2-\omega^2} = \sum_{k=1}^{\infty}\left[\frac{1}{\omega_k^2-\omega^2}\left(\sum_{j=1}^{n}\frac{2s_j}{L}\sin\frac{\pi k x_j}{L}\right)^2\right] =$$
$$= \frac{4}{L^2}\sum_{k=1}^{\infty}\left[\frac{1}{\omega_k^2-\omega^2}\sum_{j,q=1}^{n} s_j s_q\sin\frac{\pi k x_j}{L}\sin\frac{\pi k x_q}{L}\right] =$$
$$= \frac{4}{L^2}\sum_{j,q=1}^{n}\left[s_j s_q\left(\sum_{k=1}^{\infty}\frac{1}{\omega_k^2-\omega^2}\sin\frac{\pi k x_j}{L}\sin\frac{\pi k x_q}{L}\right)\right].$$

Then the condition for the occurrence of auto-balancing takes also such form

$$\sum_{j,q=1}^{n} a_{jq}s_j s_q < 0, \quad (12)$$

where

$$a_{jq} = \sum_{k=1}^{\infty}\frac{1}{\omega_k^2-\omega^2}\sin\frac{\pi k x_j}{L}\sin\frac{\pi k x_q}{L}, \ (a_{jq}=a_{qj}).$$

This is a quadratic form of the elementary unbalances.

The conditions (11) or (12) can be obtained for a flexible rotor under other its fixing conditions. Indeed, depending on the fixing conditions, the rotor will have its principal functions and natural frequencies $v_k(x)$, ω_k, $/k=1,2,...\,/$. With their use, it will be possible to repeat the above transformations. Therefore, we will further consider the condition (11) or (12) as general for flexible axisymmetric massive rotors.

The consequences of the condition (11):

1. Let n auto-balancers be located in different correction planes. Then, the elementary unbalances form the first n nonzero independent amplitudes S_1, S_2,..., S_n. The auto-balancing occurs immediately when the rotor exceeds the n^{th} critical speed. The necessary condition for the occurrence of auto-balancing is $\omega>\omega_n$.

2. The availability of a nonzero amplitude S_{j+1}, $/j\geq n/$ leads to the disappearance of auto-balancing when the rotor speed approaches (from below) the critical speed ω_{j+1} and the occurrence of auto-balancing at once after exceeding it. In this regard, there is the additional critical speed $\tilde{\omega}_j$: $\omega_j<\tilde{\omega}_j<\omega_{j+1}$, upon passing which the auto-balancing disappears.

3. The availability of a zero amplitude S_{j+1}, $/j\geq n/$ leads to the equality of the additional critical speed $\tilde{\omega}_j$ and the critical speed ω_{j+1}.

5. 5. Optimal methods of balancing of a flexible rotor by passive auto-balancers

1. Let us assume that the flexible rotor works between the n^{th} and $(n+1)^{th}$ critical speeds. The task is to minimize the rotor vibrations and maximize the auto-balancing area.

Let us determine the optimal number of auto-balancers. The auto-balancing will occur if the number of auto-balancers is less than or equal to n. The greater the number of auto-balancers, the greater the number of the modal unbalances the auto-balancers will be able to balance. Therefore, for the best rotor balancing, it is necessary to use n auto-balancers.

Let us define the optimum location of auto-balancers. Between the critical speeds ω_n and ω_{n+1}, there is the additional critical speed $\tilde{\omega}_n$: $\omega_n<\tilde{\omega}_n<\omega_{n+1}$, upon transition through which the auto-balancing disappears. If the auto-balancers are located in the nodes of the rotor flexural $(n+1)^{th}$ mode, the additional critical speed will be equal to the $(n+1)^{th}$ critical speed. At the same time, the auto-balancing will occur at any rotor speed located between the n^{th} and $(n+1)^{th}$ critical speeds.

2. Let us assume that the flexible rotor works between the n^{th} and $(n+1)^{th}$ critical speeds. The task is to minimize the rotor deflections by k auto-balancers, $k\leq n$.

In the case of k automatic balancers, auto-balancing will occur immediately when the rotor speed exceeds the k^{th} critical speed ω_k.

Upon the occurrence of auto-balancing, the rotor will behave as a rotor with intermediate hinge supports in the planes of the auto-balancers. If the intermediate supports (auto-balancers) are placed in the nodes of the rotor flexural $(n+1)^{th}$ mode, the rotor will be the most rigid, and its deflections will be the most limited [1].

3. Let us assume that the flexible rotor works between the n^{th} and $(n+1)^{th}$ critical speeds. The task is to minimize the rotor deflections by k auto-balancers, k≤n, and to obtain auto-balancing on the widest range of the rotor angular speeds.

In this case, it is necessary to place k auto-balancers along the rotor in the nodes of the rotor flexural $(n+1)^{th}$ mode as evenly as possible. Then the auto-balancing will occur between the n^{th} and $(n+1)^{th}$ critical speeds and the rotor will be rather rigid.

6. Discussion of the obtained conditions for the occurrence of auto-balancing

The empirical criterion for the occurrence of auto-balancing is an effective method for determining the conditions under which auto-balancers of any type can balance a certain rotor.

The steady-state motion of a flexible axisymmetric rotor of constant cross-section, supported by two spherical bearings, with the elementary unbalances applied at the future points of the suspension of auto-balancers is described by a linear partial differential equation.

The functional of the criterion for the occurrence of auto-balancing is a quadratic form of the elementary unbalances or amplitudes of the modal unbalances. Its form will not change at other types of fixing of the flexible rotor.

Auto-balancing of the rotor by n passive auto-balancers located in different correction planes is possible only if the rotor speed exceeds the n^{th} critical speed. The number of auto-balancers can be arbitrary. Between the critical rotor speeds, additional critical speeds appear. Auto-balancing occurs whenever the rotor passes a critical speed and disappears whenever the rotor passes an additional critical speed.

If n auto-balancers are located in the n nodes of the rotor flexural $(n+1)^{th}$ mode, the $j\times n^{th}$ additional critical rotor speed matches with the $j(n+1)^{th}$ critical speed, /j=1, 2, 3,.../. When balancing the flexible rotor between the n^{th} and $(n+1)^{th}$ critical speeds, such number and placement of auto-balancers are optimum. Auto-balancers at the same time balance the first n distributed modal unbalances and do not respond to the $(n+1)^{th}$ ones.

The additional speeds are due to the installation of the auto-balancers on the rotor. Upon transition to them, the behavior of auto-balancers changes. At slightly lower rotor speeds, the auto-balancers reduce the rotor unbalance, and at slightly higher ones – increase it.

The received findings are applicable for any number of auto-balancers of any type, and for any type of rotor fixing. These results agree with the results obtained in [8] for the case of one or two two-ball balancers.

The empirical criterion for the occurrence of auto-balancing makes it possible to obtain these conditions in the "zero approximation", since it does not take into account the type and mass of auto-balancers.

In the future, it is planned to obtain, with the help of the empirical criterion of the occurrence of auto-balancing, the conditions for balancing the rotor by any number of passive auto-balancers in the framework of:

– various flat models of the rigid rotor;

– the models of the flexible rotor on pliable supports.

At the same time, a comparison of the results received with the use of the empirical criterion, with the known results received by other methods is planned.

7. Conclusions

The empirical criterion for the occurrence of auto-balancing is an effective method for determining the conditions under which auto-balancers of any type can balance a certain rotor. On the example of the flexible axisymmetric rotor of constant cross-section, supported by two spherical bearings, using this criterion, the following is established.

1. The steady-state motion of a flexible rotor with the elementary unbalances applied at the future points of the suspension of auto-balancers is described by a linear partial differential equation.

2. The functional of the criterion for the occurrence of auto-balancing is a quadratic form of the elementary unbalances or amplitudes of the modal unbalances.

3. Auto-balancing of the rotor by n passive auto-balancers located in different correction planes is possible only if the rotor speed exceeds the n^{th} critical speed. The number of auto-balancers can be arbitrary. Between the critical rotor speeds, additional critical speeds appear. Auto-balancing occurs whenever the rotor passes a critical speed and disappears whenever the rotor passes an additional critical speed.

4. If n auto-balancers are located in the n nodes of the rotor flexural $(n+1)^{th}$ mode, the $j\times n^{th}$ additional critical rotor speed matches with the $j(n+1)^{th}$ critical speed, /j=1, 2, 3,.../. When balancing the flexible rotor between the n^{th} and $(n+1)^{th}$ critical speeds, such number and placement of auto-balancers are optimum. Auto-balancers at the same time balance the first n distributed modal unbalances and do not respond to the $(n+1)^{th}$ ones.

The additional critical speeds are due to the installation of the auto-balancers on the rotor. Upon transition to them, the behavior of auto-balancers changes. At slightly lower rotor speeds, the auto-balancers reduce the rotor unbalance, and at slightly higher ones – increase it.

The received findings are applicable for any number of auto-balancers of any type, and for any type of rotor fixing.

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