Запропоновано метод пошуку оптимального управління технологічними процесами, заснований на аналізі рішення системи диференціальних рівнянь (СДР), яка є математичною моделлю керованого процесу. Показано, що отримані при цьому рішення узгоджуються з принципом максимуму Понтрягіна для задачі про швидкодію, але відкриваються додаткові можливості в управлінні кінцевим станом. Запропоновано спосіб мультиальтернативного параметричного опису кінцевого стану

Ключові слова: оптимальне керування технологічними процесами, принцип максимуму Понтрягіна, швидкодія, лінія кінцевого стану, мультіальтернатівний опис кінцевого стану, рідж-аналіз

Предложен метод поиска оптимального управления технологическими процессами, основанный на анализе решения системы дифференциальных уравнений (СДУ), являющейся математической моделью управляемого процесса. Показано, что получаемые при этом решения согласуются с принципом максимума Понтрягина для задачи о быстродействии, но открываются дополнительные возможности в управлении конечным состоянием. Предложен способ мультиальтернативного параметрического описания конечного состояния

Ключевые слова: оптимальное управление технологическими процессами, принцип максимума Понтрягина, быстродействие, линия конечного состояния, мультиальтернативное описание конечного состояния, ридж-анализ

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SYNTHESIS OF OPTIMAL CONTROL OF TECHNOLOGICAL PROCESSES BASED ON A MULTIALTERNATIVE PARAMETRIC DESCRIPTION OF THE FINAL STATE

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1. Introduction

The search for optimal control of specific technical applications requires an informed choice of the methods that would be most suitable for these objects. Dealing with management of industrial objects and corresponding technological processes, it is necessary to take into account that such a choice of optimal control should help cope with a number of uncertainties. It is clear that such uncertainties occur due to the conditions of control objects' operation or the specificity of their operation. However, making a choice in favor of a method, one more circumstance should be taken into account – whether a particular problem is considered from the standpoint of engineering or from the position of the mathematical theory of control. The presence of an obvious, sometimes "competing", dualism in this issue requires additional research on a number of fundamental aspects for successful determination of optimal management.

In this connection, it is necessary to note such key areas as the selection of the criterion of management optimality [1], the structure of the mathematical description of the control object [2–4], the substantiation of the effectiveness of the individual operation estimation for the implementation of the selected management [5–7], and the uncertainty in the estimation of state variables [8], which generates the need for their fuzzy description [9–11]. All these aspects are more related to applied issues in solving problems of control optimization, developed mostly just in the framework of engineering for specific technical problems. The main objects of research in the field of mathematical control theory are, directly, methods of searching for optimal control, in particular for improving the evidence base about necessary and sufficient conditions for the existence of optimal solutions. In this regard, one interesting fact cannot be overlooked. Despite the active development of new directions in the solution of optimization tasks in the field of management, for example, the use of genetic algorithms or artificial neural networks, it is the classical methods of searching for optimal control that are very much in demand for research. Among them, in particular, is Pontryagin's maximum principle, which makes it possible to solve a wide class of problems for objects described by systems of linear or nonlinear differential equations, to search for optimal control of processes with distributed parameters and discrete processes. This is confirmed by studies devoted to the development of the maximum principle for a number of applications: the search for optimal control of technological processes [12–14], transport in the fields of its production and operation [15–18], the design of structures in power engineering [19, 20], and the field of economics [21].

All the aforementioned arguments substantiate the relevance of research subjects devoted to improving the methods of searching for optimal control, both in the applied aspect and in terms of developing the mathematical theory of control.

2. Literature review and problem statement

In study [1], it is argued that so far there are no valid rules for selecting a universal criterion for optimality of management, and the common practice is to set independently some indicator of the role of the efficiency criterion. At the same time, the search for optimal control makes it necessary to take into account the fact that the achievement of the strategic goal of control is ensured by the formation of a sequence of technological operations within a time interval under study. The benchmark for choosing the most profitable technological process is the technological operation with its own, local, maximum efficiency indicator, which can be used as a single criterion for optimal control. Thus, the choice of a criterion for the optimality of control implies the necessity of using particular models for describing the control object. This circumstance is also mentioned in [2–4], which also specify, in particular, that mathematical models used in control systems of complex energy-technological complexes have a number of specific features the essence of which is the reflection of "local" principles of construction in their structure. This "locality" is understood in the sense of constructing a number of particular models and revealing the main relationships between them instead of building a common model. Justification of the expediency of such an approach is found by the authors in that any model is one-sided, and in a complex process comprehensive modeling is achieved by constructing a set of one-sided but interrelated models. In this case, the development of the control object model is considered as a multi-stage conditional-extreme goal-oriented process, which can be formulated as the task of optimizing the functional opt F (X, Y, Z, Q, B, S, and T) under a variety of conditions imposed by the specific operation of the control object. In this functional, X is the input vector, Y is the output space, Q is the space of constraints on the processes of machine study, B is the constraints on the design object for specific requirements imposed by operating conditions and the constraints imposed on the sensitivity requirements of measuring equipment systems, reliability, and energy intensity, S is the reaction space of the design process to environmental influences, T is time, and F is the conditional operator.

It is clear that the structure of such a functional reflects the interrelations, processes, characteristics and parameters of energy-efficient and managerial nature. Along with developing the criterion of effectiveness of resource use as a component of the criterion of management optimality, it allows identifying the control object and determining the most effective target operation. The development of such views can be found in [5–7], where it is shown how classes of simple target operations can be distinguished for managed systems, and their effectiveness is estimated from the point of view of forming a general criterion of management optimality.

In the case of uncertainty, the description of state variables requires their mathematical formalization. As its variants, the methods of the theory of Pawlak's approximation spaces [8] or fuzzy regression analysis are possible [9–11]. The first approach allows, on the basis of the final expert assessments on the state of the system and objective or subjective assessments of its parameters, to build forecasting algorithms based on logical inference procedures. The second approach is based on the use of fuzzy regression analysis, including the use of artificial orthogonalization of the results of a passive experiment, to construct a mathematical description of the relationship between output and input variables that are fuzzy numbers [9–11].

The described studies reveal a number of principal aspects relating to the contradictory nature of the requirements imposed on the quality of management criteria and the objective complexity of an adequate mathematical description. This particularly applies to industrial facilities operating in conditions of significant uncertainty. Despite the fact that these studies do not contain complete algorithms for synthesizing optimal control for specific applications, their results are very useful. First of all, this is connected with the approaches substantiated in them to the formation of principles for considering real technological objects for which the problems of searching for optimal control are being solved.

Of particular interest are studies devoted to the mathematical description of optimal control problems for those industrial facilities that can function in several technological regimes. Obviously, this interest is justified by the ubiquity of exploiting such specific technological objects, and the requirements for finding optimal control programs for them involve solving the problem of speed. Traditionally, this is done by using the Pontryagin maximum principle, which is actively developed in studies devoted to the mathematical theory of control. Among the key aspects of researching the maximum principle within the framework of developing this theory, the directions that can be singled out are dedicated to exploring transversality conditions [22–24], optimal control of a determinate system with discrete time and impulse control under mixed constraints [25, 26], and necessary optimality conditions for control problems on an infinite interval [27, 28].

In particular, a brief geometric analysis of the Pontryagin maximum principle is given in [22], and key concepts such as separating hyperplanes as well as boundary conditions and minimizers (normal and abnormal minimizers) are analyzed. It is shown that they all have natural contact-geometric interpretations that make it possible to obtain a simple derivation of the transversality conditions for an optimal control in the final state. The Pontryagin maximum principles for the optimal Mayer control problem for a system described by a functional-differential equation are established in [23]. The singularities of these solutions are related to the fact that the control functions are piecewise-continuous, and the state functions are piecewise-continuously differentiable. In this case, the Michel method is used for controllable systems described by ordinary differential equations, but taking into account the properties of the resolvent of a linear functional-differential equation.

In [24], the maximum principle for control problems with a bounded horizon is presented and discussed, and the value formed in the final state of the system is chosen as the objective function. In this sense, the criterion for the quality of management coincides with the criterion chosen in [22], but there is no emphasis on geometric interpretation. Instead, the necessary conditions for optimality in the form of Hamiltonian inclusions are presented for the solution of the problem of control of the final state, but with the new transversality condition taken into account. A distinctive feature of the search for such control is that the requirements to the state trajectory are advanced by their asymptotic stability, and the equilibrium point is limited to a given closed set. It is shown that the obtained conditions can also be extended to impulse control problems the dynamics of which are described by differential equations obtained on the basis of measurements. In this case, an important role is assigned to the concept of a solution based on the concept of equilibrium introduced for this class of problems.

The use of the Pontryagin maximum principle for solving the problem of optimal control of a continuous deterministic system and the discrete maximum principle for solving the optimal control problem for a determinate system with discrete time is considered in [25]. Noting the significant shortcoming of the maximum principle associated with the need to transform the problem of optimal control into a two-point boundary problem, an alternative approach is presented. It is based on a stochastic modification of the maximum principle for both continuous and discrete-time systems. Using the proposed method, optimal control strategies are derived, the results of which are consistent with the results obtained with the help of stochastic dynamic programming. It should be noted that the authors use the concept of cautious control strategy by a linear stochastic system based on the principle of a stochastic minimum. In this case, the replacement of all random variables by their mathematical expectations, leading to an equivalent strategy, provides the possibility of obtaining the same results as when using dynamic programming. Obviously, the expansion of the class of control objects under consideration and consideration of additional constraints imposed on them would give the opportunity to expand the solutions obtained in the field of specific technological applications. The emphasis on this side is made in [26], in which, in particular, necessary conditions are derived in the form of the Pontryagin maximum principle for impulse control problems with mixed constraints. A new mathematical concept of impulse control is introduced as a requirement for the consistency of the impulse structure. In addition, this management concept allows taking into account engineering needs when considering the usual control influence in the development of a momentum. It should be noted that the assumption of regularity, under which the maximum principle is proved, is weaker than in known studies. However, the proof is based on Ekeland's variational principle and the discontinuous change in the Lebesgue time variable [26].

In [27], necessary optimality conditions are studied for control problems on an infinite interval with a quality functional containing a discount factor not necessarily exponential. The discussed assumptions guarantee the necessity of both a Michel condition for the maximized Hamiltonian and the Cauchy type formula for the conjugate variable. It is shown that due to this approach the maximum principle is supplemented to a complete system of relations.

In [28], it is discussed that the problem of the fact that the spatial model described by a parabolic-type system for searching for optimal control with an infinite horizon, although solved by means of dynamic programming, cannot be solved using the Pontryagin maximum principle. More precisely, Pontryagin's conditions, although necessary, do not allow determining the unique solution of the optimal control problem. The authors of this study show that such a conclusion is not entirely justified and should be revised. In particular, if a Michel transversality condition is introduced and the fact that the conjugate variable should be non-negative is taken into account, the maximum principle is able to give a unique solution for the mathematical model in question.

The aforementioned results of [22–28] allow stating that there are several promising directions of research devoted to the maximum principle. However, developing the mathematical apparatus of the maximum principle, they do not emphasize the applied directions of its use in the field of controlling technological processes. Here it should be noted that in the synthesis of optimal control using the maximum principle, it is necessary to consider whether the technological processes are periodic or continuous. The nature of technological processes forms different limitations in solving the problem of finding optimal control. For example, in [29] it is shown that with optimal control of batch devices in the case when its performance limits the efficiency of the entire production, it is necessary to minimize the cycle time. In the absence of such a restriction, it is sufficient to maximize the specific yield of the finished products. It is noted that the application of the maximum principle for the solution of these problems in the conditions of interphase transitions requires the control of the constants of a kinetic equation describing in these conditions the dynamic model of apparatuses of periodic action. A brief description of the operation principle of the process algorithm is given for optimal non-stationary values of these constants called form factors. The latter, as noted in this paper, depend on the state variables characterizing the initial conditions of the interphase transition process. However, the mathematical implementation of the optimal control search algorithm is not given.

In [30], one of the main problems of optimal control of the process of non-stationary thermoelectric cooling is formulated to consist in determining the optimum dependence of the supply current on time, which ensures a minimum of the cooling temperature. To solve this problem, a method for discretizing the mathematical model of the process of non-stationary thermoelectric cooling along a coordinate is proposed, which makes it possible to apply the Pontryagin maximum principle to calculate optimal control functions. Nevertheless, the emphasis in the study is on the construction of the model, but in relation to the search for optimal control it is said only that the problem is solved numerically, by using an iterative algorithm implemented in the Matlab environment.

An alternative method and algorithm for controlling the catalytic reforming process by optimal distribution of the values of input process variables is given in [31]. To solve the optimization problem, a method is used based on the Hook-Jeeves non-linear optimization method supplemented by a procedure for controlling the boundaries of variable parameters. As settings for the input of the optimization algorithm, vectors that describe the upper and lower boundaries of the optimized variables are transmitted. To minimize the error in calculating optimal modes before the initial data input, the optimization algorithm initiates the refinement of the tuning coefficients that take into account the activity of the catalyst in the mathematical model. After this, the temperature of the reaction mixture at the inlet begins to be calculated cyclically for each reactor, providing the maximum increment of aromatic hydrocarbons at the outlet of the reactor, taking into account the adopted limitations. However, comparison with classical algorithms for searching for optimal control of similar processes is not given in the study.

The problem of synthesizing optimal control of continuous technological processes is investigated in [32, 33], in which the management difficulties of real industrial objects are noted to occur due to the lack of the possibility of obtaining objective reliable information about the process in real time. The authors specify that this is exactly the situation that takes place in the conditions of actual operation of industrial facilities. It should also be noted that the absence of well-founded arguments as to the appropriateness of choosing a criterion of management quality that is most relevant for these conditions and production requirements complicates the process of searching for optimal control even at the stage of its mathematical construction [34]. It is clear that in this context the possibility of measuring or some other adequate estimation of the state variables is of particular importance. This circumstance is noted in [35], and the idea developing it is described in [36]. In particular, it is shown that in most practical cases operational management of technological processes is carried out on the basis of using the results of monitoring the production situation. The control is realized by the collection and primary processing of data, including the results of the use of measuring tools and complexes and laboratory analyses of intermediate and marketable products. The results of analyses obtained by means of factory laboratories as a rule do not possess the necessary level of completeness and efficiency, and practical experience with them shows that their reliability in some cases is unsatisfactory. Hence, there arises the problem of increasing the completeness, efficiency and reliability of information provision for technological personnel by creating and implementing a virtual monitoring system. The main idea of it is to obtain new knowledge about the current state of the technological process and the dynamics of its evolution through deep mathematical processing of operational and retrospective data obtained by existing control and measuring equipment.

Despite the obvious correct identification of such a problem and recommendations as to what directions of solutions can be used, the mathematical apparatus and algorithms for its implementation for the synthesis of optimal control of continuous technological processes are given only conceptually in [35]. The author of the study only describes the conditional scheme of the interaction of virtual analyzers with a typical process control system, the scheme for formalizing a single object of the diagram of the structural model, vector diagrams of the main tasks of virtual analysis, the functional matrix of virtual analyzers, and the DSS matrix of decision support systems, as well as a general representation of the mathematical tools of virtual analyzers.

Systematization of the results of the above-mentioned studies makes it possible to assume that the existing approaches to solving the problem of synthesizing optimal control are based on the choice of a particular method. In other words, it is assumed that the method is already known, and the rationale for its expediency is not discussed. Moreover, this conclusion is valid for applied technical and technological problems and for research problems of the mathematical apparatus of the control theory.

In the considered variants, the emphasis is made only on one of the classical methods of searching for optimal control – the Pontryagin maximum principle. Such a choice is justified by two circumstances. First, the method itself from the point of view of the mathematical apparatus has by no means exhausted itself as an object of special investigation. Obviously, the results obtained as a result of such studies can find applied continuations. Secondly, for tasks with speed, this method is unique, and its choice is unambiguously justified in the case of synthesis of optimal program control of periodic technological processes. Moreover, overcoming the objective difficulties associated with measuring or adequately describing the state variables in real time can help expand the possibilities to applying them in the management of continuous technological processes. Finally, it should be noted that even with an alternative method for solving problems of the same class for which the maximum principle is adapted, it is possible to use the results obtained with the use of the maximum principle as a test of their performance.

Among the limitations in the described studies, including those devoted to the maximum principle as a mathematical apparatus, it is necessary to point out the lack of attention to the description of the final state of the system for problems of different classes and its influence on the effectiveness of the obtained solutions. At the same time, efficiency should be understood as the possibility of choosing the optimal strategy for finding the optimal solution in the case of several alternatives in the description of the final state.

3. The aim and objectives of the study

The aim of the study is to investigate the possibility of synthesizing the optimal control for the speed and final state of technological processes in the case of alternatives in describing the final state of the system. This will make it possible to choose the best management option in the given conditions on the basis of operational information on the actual current state of the technological system as a control object.

To achieve this aim, it is necessary to solve the following tasks:

– to check the possibility of finding the optimal control, relying only on the analysis of the system of differential equations describing the mathematical model of the control object;

– to develop an algorithm for describing the final state of the system of a controlled process.

4. Investigation of the solutions of a system of differential equations describing the mathematical model of a control object in the search for optimal control

Taking into account the fact that the choice of the method for finding optimal control is based on the specifics of a particular task and the requirements for the quality of management, it is important to find the answer to the question whether an optimal solution can be obtained in an easy way in order to achieve a given management purpose. In particular, is it possible, by analyzing the solutions of the system of stochastic differential equations (SDEs), a mathematical model of the control object, to find the speed-optimal control over the system that transmits it for a minimum time from a predetermined initial to a given finite state? Traditionally, such a problem has been solved with the help of the Pontryagin maximum principle. To answer the question of the possibility of obtaining a simpler solution, it is advisable to consider a simple system of the type

$$\dot{x}_1(t) = x_2(t), \dot{x}_2(t) = u(t), t \geq 0, |u(t)| \leq 1, \quad (1)$$

where $x_1(t)$ is the coordinate of the material point, $x_2(t)$ is the velocity of the displacement of the material point, and $u(t)$ is the control.

The SDE of form (1) is a mathematical model of the control object for both the initial conditions $x_1(0)=x(0)$, $x_2(0)=\dot{x}(0)$ and the final state, $x_1(T)=x_2(T)=0$, $T\to\min$, where T is the moment of the end of the motion, and the problem of finding optimal control is known as the problem of damping the material point [37]. Its solution using the Pontryagin maximum principle is shown in Fig. 1.

A change in the description of the final state leads to analogous conclusions regarding the obtaining of the solution; for the example of describing the final state in the form of $x_1=\xi x_2$ and the general form of the SDE (2) for $a_0=1$, $a_1=1$, $b=1$, and $\xi=1$, the solution has the form presented in Fig. 2 [38, 39].

$$\dot{x}_1(t)=a_0+a_1x_2(t),\dot{x}_2(t)=bu(t),t\geq 0,|u(t)|\leq 1. \qquad (2)$$

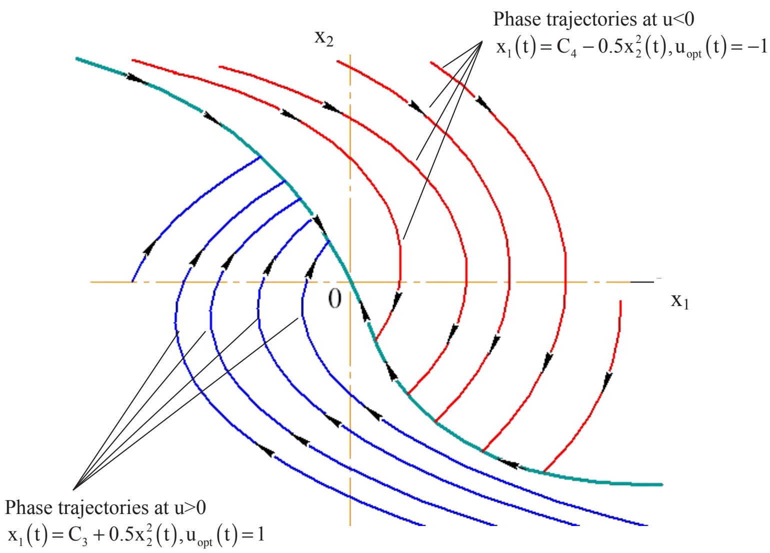

Fig. 1. Solving the problem of damping a material point using the Pontryagin maximum principle: $u_{opt}(t)=\text{sign}(-C_1t+C_2)$, $u_{opt}(t)$ is the optimal control; C1, C2, C3, and C4 are integration constants

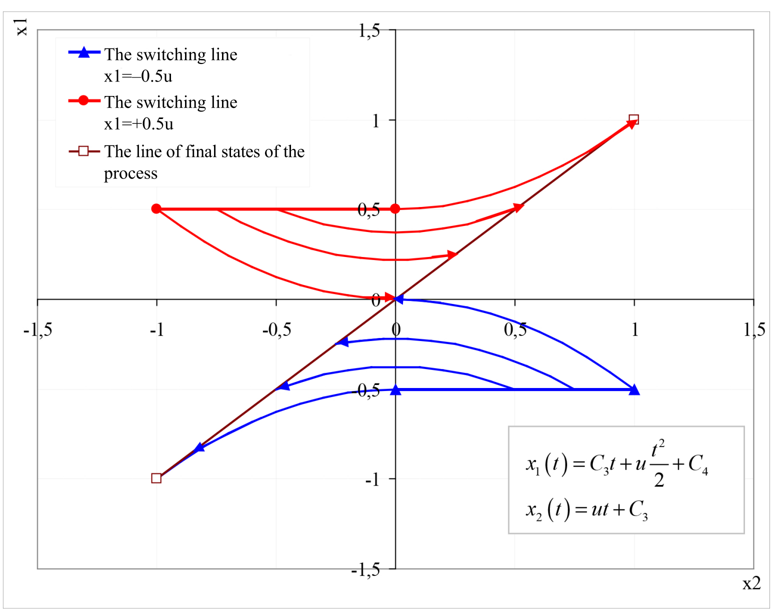

Fig. 2. Solving the problem of finding the optimal control with a final state of the form $x_1=x_2$ with the use of the Pontryagin maximum principle: $u_{opt}(t)=\text{sign}(-C_1t+C_2)$

The universality of the mathematical description of form (2) for a number of applied problems in the field of workpiece production for machine building and metallurgy is described, for example, in [40]. It is shown that the results analogous to the Pontryagin maximum principle can be obtained directly from the SDE of form (1) when performing a series of procedures. However, first, a number of comments should be made regarding the terminology used below. With respect to the parameter u(t) in descriptions (1) and (2), the term "control" is often used. In the context of the problem under consideration, it cannot be considered successful because in its content "management" presupposes an action. The parameter u(t) within the SDE is a physical factor having its own dimension. This may be the force F or the moment of forces M acting on the mechanical system, the pressure of oil in the hydraulic system or compressed air in the pneumatic system p, a pressure drop in the pneumatic system Δp, voltage U or amperage I, etc. Therefore, it makes sense to use the term "control parameter" instead of the term "control". The connection between these two concepts can be expressed as follows. Each range of the control parameter corresponds to a certain stage of the control switch. In this case, the choice of a certain stage at a given time corresponds to the control leading to a change in the physical magnitude of the control action. The latter, in turn, changes the phase trajectory of the system. Taking this into account, the problem of finding the optimal control can be solved as follows. By assigning fixed values $u=(u_1, u_2,\dots u_n)$ to the values of the control parameter and by solving the SDEs, the time required to reach the given final state is determined for a fixed value of the control parameter. Based on the obtained set of numerical solutions, the dependence t=f(u) is constructed, on the basis of which the minimization problem is solved. The desired parameter t for each fixed u can be found by solving SDE (2) on the basis of a system of equations of the following form:

$$\begin{cases} x_1=(a_0+a_1x_2(0))t+a_1bu\dfrac{t^2}{2}+x_1(0), \\ x_2=but+x_2(0), \\ x_1-\xi x_2=0. \end{cases} \qquad (3)$$

The third equation of system (3) is an analytical description of the final state that can have any form, depending on the specific nature of the problem.

In this case, the time t that is necessary to transfer the system from a given initial state to a given final state described by the third equation of system (3) can be determined by one of the alternative equations:

$$t=\frac{x_1-x_1(0)}{a_0+a_1x_2(0)}, \qquad (4)$$

$$t=\frac{x_2-x_2(0)}{u}. \qquad (5)$$

It should be noted that at u=0 it is necessary to apply equation (4), and in all other cases the applied equation should be (5).

When a_0=1, a_1=1, b=1, and ξ=1, SDE (2) takes the form of SDE (1) and, without taking into account the description of the final state, it has a simple and understandable physical meaning, which is displacement of the material point by the force F acting as the controlling parameter. In fact, it concerns the basic equation of dynamics. Setting the values of F in the range from 0N to 10N with a step of 2N and assigning them the values $u=(u_0,..., u_5)$ when solving system (3) make it possible to obtain the optimal solution for $t=t_{min}$ at $F=F_{max}=u_5$. This conclusion does not contradict the physical meaning of the task, and it fully corresponds to the Pontryagin maximum principle, which is easy to verify with the use of its geometric interpretation. Fig. 3 shows an example of the dependence $t=\varphi(u)$ for an arbitrary initial state at $x_1(0)$=0.45 m, $x_2(0)$=0.55 m/s, from which it can be seen that the time of reaching the final state with an increasing value of the control parameter falls, reaching its minimum value at its boundary. The same results are essentially obtained for any initial state of the system.

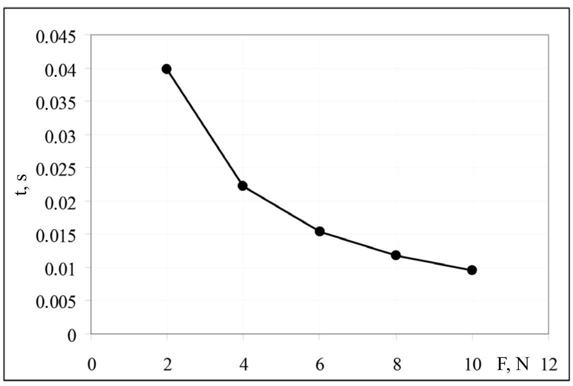

Fig. 3. Dependences $t=\varphi(u)$ for the initial state when $x_1(0)$=0.45 m and $x_2(0)$=0.55 m/s

This means that by solving together the system of equations (3) for each of the fixed values of the control parameter, using (4) or (5) and then minimizing the function $t=\varphi(u)$, we can obtain the optimal solution in terms of speed without resorting directly to the Pontryagin maximum principle. The thus obtained results are completely consistent with it. It is obvious that the determining point for choosing the optimal control is the position of the point characterizing the actual position of the initial state with respect to the final state line. If it is a straight line, its description can be reduced to the normal form in the standard way, using the normalizing factor. In this case, the sign of the deviation of the corresponding point of the phase space from the line of the final state will determine the required control sign. For the space (N×2), this can be expressed in the following form:

$$\delta_i(x_{1i},x_{2i})<0 \rightarrow u<0,\quad \delta_i(x_{1i},x_{2i})>0 \rightarrow u>0, \qquad (6)$$

where $d_i(x_{1i}, x_{2i})$ is the deviation of the point describing the initial state from the line of the final state.

The choice of the optimal control in this case is reduced to calculating the value $d_i(x_{1i}, x_{2i})$.

However, one question remains unresolved. Namely, the achievement of the final state as an end in itself is not always good for managing technological processes. For such processes, the speed requirement must be met only when the duration of the technological operation as part of the cycle is a priority. This requirement is directly related to such a criterion of quality management as performance. In this case, the moment when the final state is reached, for example, as the time necessary for obtaining a technological product in a given volume, must correspond to the moment of its delivery, in general, to the process line. However, reaching the final state, the object continues its motion along the phase trajectories, leaving the final state line. To illustrate this, it is sufficient to analyze the behavior of the object on the basis of solving the system of equations (3). As an example of such an illustration, Fig. 4, 5 show the behavior of the object with negative control (u<0, Fig. 4) and positive control (u>0, Fig. 5).

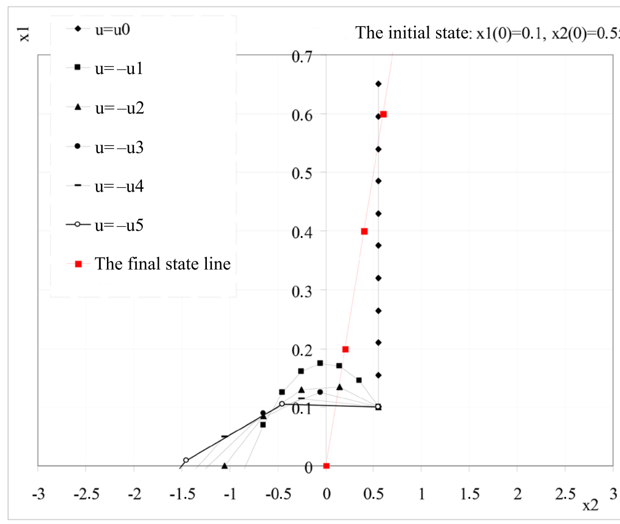

Fig. 4. Behavior of the object under negative control (u<0)

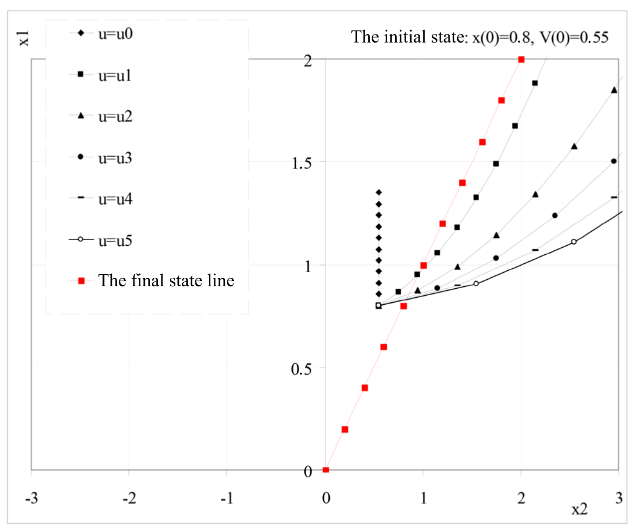

Fig. 5. Behavior of the object under positive control (u>0)

Proceeding from this, it is necessary to expand the criterion of the quality of control by adding the requirement of reaching the final state in minimum time to the requirement of keeping the object in the given area of the final state for the required time. This requirement has a simple technological meaning, which is as follows. If a delay is required with the delivery of the technological product to the line, then the object must remain in the final state for a predetermined period of time. Meanwhile, one more situation is possible: at the moment of the product delivery to the line, its quality does not correspond to the preset one, and bringing it to specified quality indicators requires a certain time. If the volume of the product at the same time corresponds to the required quantity, the time for bringing the product in compliance with the specified parameters implies holding the object in the final state area during this time period. It is possible to suggest the following solution. Let the area of the final state have lower and upper admissible boundaries while the line of the final state is described by the equation $x_1=x_2$. Then the equation of the lower admissible area has the form $x_1=-\delta+x_2$, whereas the equation of the upper admissible area has the form $x_1=\delta+x_2$. Here δ is the permissible interval for varying the variable x_1.

The phase trajectory of the system when the SDE of form (1) is selected as a mathematical model of the chosen control crosses it at two points. Accordingly, the time when the object reaches the lower limit of the admissible area is a solution of the system of equations

$$\begin{cases} x_1 = x_2(0)t + u\dfrac{t^2}{2} + x_1(0), \\ x_2 = ut + x_2(0), \\ x_1 = -\delta + x_2, \end{cases} \tag{7}$$

and the time the object reaches the upper boundary of the admissible area is a solution of the system of equations

$$\begin{cases} x_1 = x_2(0)t + u\dfrac{t^2}{2} + x_1(0), \\ x_2 = ut + x_2(0), \\ x_1 = \delta + x_2. \end{cases} \tag{8}$$

Therefore, there are at least two options for synthesizing control. The initial data for both options is the actual initial state in which the process system or technological process is located. Depending on this, a control sign is selected in accordance with (6).

Option number 1

As soon as the object reaches the upper permissible area of the final state, the control action (or control signal) of the corresponding sign is produced. This time t_{s1}, therefore, corresponds to the first switching, and the coordinates of the object at the given moment of time correspond to a new initial state. In other words, if the previous initial state had the form $x_1(0), x_2(0))=(x_1(i-1), x_2(i-1))$, then the new initial state into which the object falls when the upper boundary of the admissible area of the final state is reached at time t_{s1} has the form $(x_1(0), x_2(0))=(x_1(i), x_2(i))$. Thus, with respect to the initial state, it is possible to refer to the concepts of "previous state" and "subsequent state". As a result, the phase trajectory changes, and the object moves to the lower boundary of the admissible area. As soon as the object reaches it, the control action (or the control signal) of the corresponding sign is produced again. The switching time moment corresponds to t_{s2}, whereas the intersection point of the phase trajectory and the lower boundary of the admissible area forms a new initial state $(x_1(0), x_2(0))=(x_1(i+1), x_2(i+1))$, and so on as long as the system can be maintained within the specified range $\pm\delta$ for a given time. The latter corresponds to the requirements of the technological process for the volume and quality of the technological product.

The described principle of constructing control is shown in Fig. 6, and the results of applying the SDE of form (1) are shown in Fig. 7.

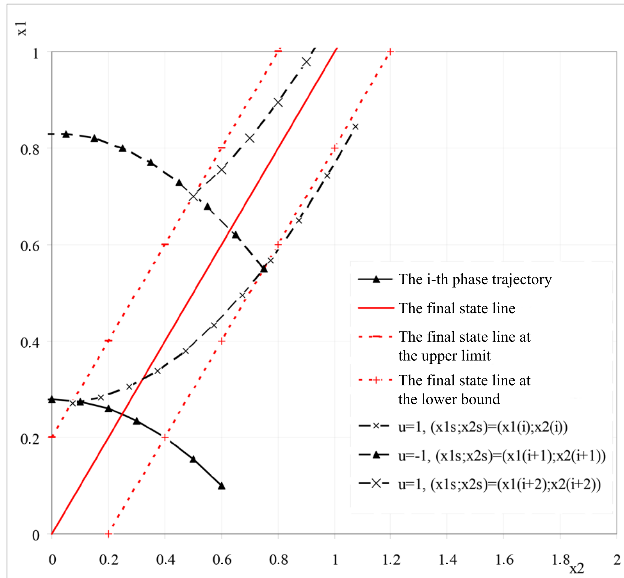

Fig. 6. The principle of synthesis of control for option No. 1

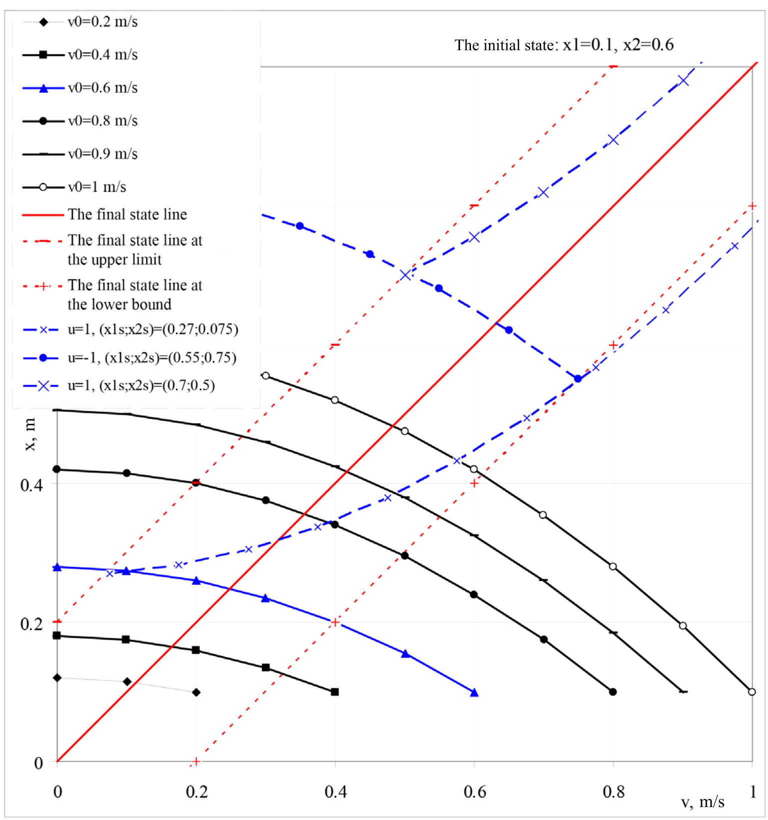

Fig. 7. Control construction for the SDE of form (1), describing the displacement of a material point

Option number 2

As soon as the object reaches the lower permissible area of the final state, the control action (or control signal) is removed. This is equivalent to setting the control parameter to zero (u=0). In accordance with (1), an object having a nonzero initial velocity continues to move uniformly. This time t_{s1}, therefore, corresponds to a switch-off, and the coordinates of the object at the given time correspond to a new initial state. As soon as the object moving uniformly reaches the upper limit of the permissible area of the final state, the control action (or control signal) of the corresponding sign is produced, and the phase trajectory begins to change. The time of the new switch-off corresponding to t_{s2} occurs when the phase trajectory crosses the lower boundary of the admissible area. A new initial state is formed at (x_1(i+1), x_2(i+1)), and so on as long as the system can be maintained within the specified range ±δ for a given time.

The described principle of constructing control is shown in Fig. 8.

The results of the control synthesis for option No. 1 are shown in Fig. 9, and for option No. 2 they are shown in Fig. 10.

Based on the obtained results of the synthesis of control for the two options, the latter can be compared and a choice can be made in favor of the one that provides the best indicators for stabilizing the object. The latter can be estimated, for example, on the basis of the time of the object (technological system) stay within the permissible area of the final state.

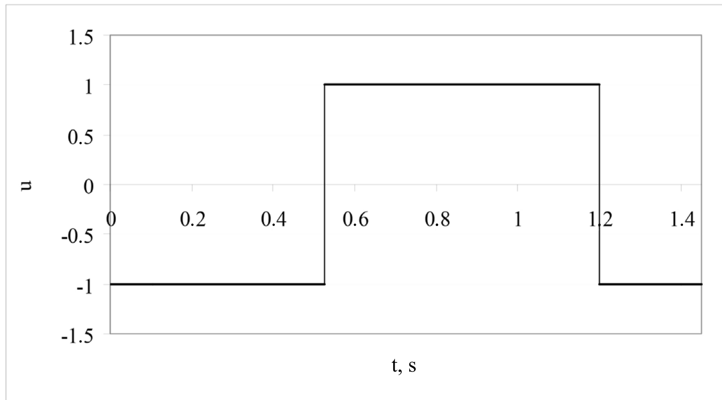

Fig. 9. Results of the synthesis of control for option No. 1

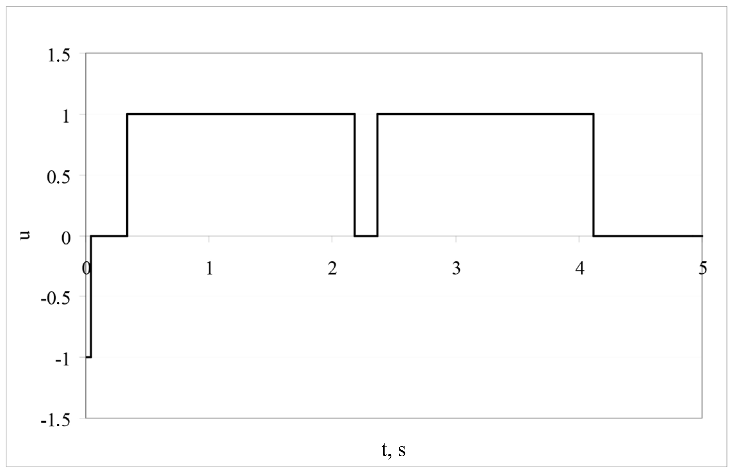

Fig. 10. Results of the synthesis of control for option No. 2

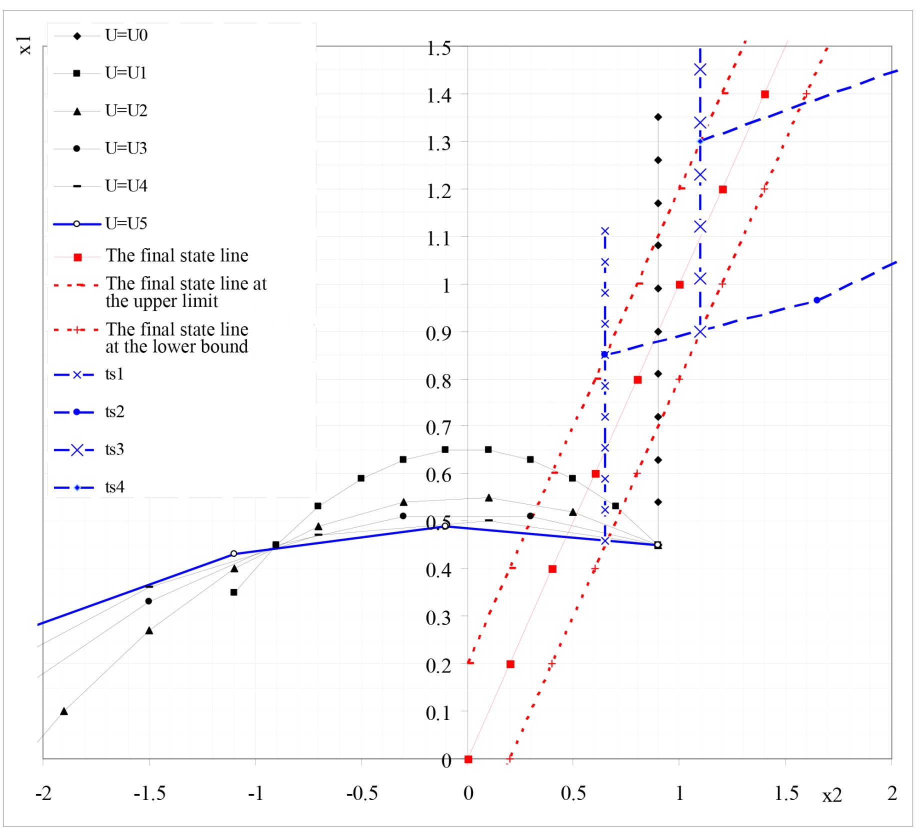

Fig. 8. The principle of synthesis of control for option No. 2

5. Synthesis of a multialternative description of the final state of a system in the task of searching for optimal control

The choice of the final state is uniquely dependent on the task being solved and, in relation to the management of technological processes, the requirements imposed in specific conditions must be taken into account. The presence of a qualitative mathematical model of the control object is decisive; therefore, special attention is paid to problems of mathematical modeling for specific technological problems. A number of such applied studies can be found in the following:

– works [41, 42], devoted to modeling in the search for optimal control of drilling processes;

– works [43–45], devoted to modeling and managing in technologies of growing crystals;

– works [46–48], devoted to the modeling and identification of controlled objects using the analysis of interval sets and the study of the convergence of the solution of an extreme problem under constraints.

All of them are united by the general idea that the modeling and management of real objects involve the overcoming of objective difficulties caused by a fuzzy description and the need to take into account the significant, often multilevel, uncertainty. Some generalizations of such problems can be found in [49], and general principles for their solution are described in [50].

In the event that an adequate mathematical model of the technological process is unknown, it can be obtained in at least two ways. The first of them is based on the implementation of an active experiment as a result of which the values of the coefficients of the regression equation describing the influence of input variables on output variables are estimated. The subsequent carrying out of the procedures of experimental optimization makes it possible to obtain a description of the stationary area on the basis of calculating the corresponding coefficients of the regression equation a_i [51]:

$$a_i = c_1 \sum_{j=1}^{N} x^j y^j, \quad i = 1,\ldots,n, \tag{11}$$

$$a_i = c_2[(x_{i-n}^j)^2 - \beta] y^j, \quad i = n+1,\ldots,2n, \tag{12}$$

$$a_i = c_3 \sum_{j=1}^{N} x_\mu^j x_\lambda^j y^j, \quad i = 1,\ldots,n, \mu \neq \lambda, \quad i = 2n+1,\ldots,k, \tag{13}$$

$$a_0 = \frac{1}{N}\sum_{j=1}^{N} y^j - \beta \sum_{j=1}^{N} a_{n+i}. \tag{14}$$

In formulae (11) through (14), c_1, c_2, and c_3 are the coefficients for linear, quadratic and paired relationships, respectively; μ and λ are the indices in the description of estimates of the coefficients for pair interactions; n is the number of linear terms of the equations; N is the number of experiments; B is a parameter calculated depending on the number of points in the core of the compositional plan 2^{n-p}, the arm of the "star" points α and the number of points in the plan according to the formula

$$\beta = \frac{\sum_{j=1}^{N} (x_i^j)^2}{N} = \frac{2^{n-p} + \alpha}{N}. \tag{15}$$

The second way is based on processing the experimental data of the passive experiment by the least squares method (LSM), and these data should be obtained directly from the current production. In this case, the mathematical model is a regression equation, and the least-squares functional has the form

$$J = (FA - Y)^T (FA - Y), \tag{16}$$

whereas the minimizing (16) vector of estimating the coefficients is calculated by the formula

$$A = \left(F^T F\right)^{-1} F^T Y. \tag{17}$$

Here F is the matrix of the experimental design, A is the matrix of the coefficients of the regression equation, and Y is the matrix of the values of the output variables, which is a column vector of the values of the resulting parameter in the i-th experiment.

Examples of obtaining such descriptions on the basis of industrial data can be found in [52–54].

Thus, if an adequate mathematical model of the technological process is obtained by any of the above procedures, then the description of the final state can be a solution to the optimization problem. The latter is formulated as follows: it is necessary to find such values of input variables that provide a maximum or a minimum of the value of a given output variable. For this purpose, it is advisable to use a special procedure for studying the response surface. It is a matter of a "ridge" analysis. In this case, it is proposed to use as a final state a parametric description of the form

$$\begin{cases} x^*(\lambda) = (\lambda I - A)^{-1} a, \\ r(\lambda) = \sqrt{x^{*'} x^*}, \\ y^*(\lambda) = a_0 + 2a' x^* + x^{*'} A x^*, \end{cases} \tag{9}$$

where a_0, a, and A are estimates of the coefficients in the regression equation and $x_i^* = \frac{a_i}{2\lambda}$ means suboptimal values of the input variables in the search for optimal control of technological processes, i. e. state variables; $r = \sqrt{r^2}$, $r^2 = \sum_{i=1}^{n} \left(\frac{a_i}{2\lambda}\right)^2$ are constraints in the factor space, and

$$y^* = a_0 + \sum_{i=1}^{n} \frac{a_i^2}{2\lambda}$$

stands for suboptimal values of the output variable.

Taking into account the features of the ridge analysis, connected with obtaining a set of suboptimal values of the output variable, several alternative variants of describing the final state are formed. Each of them for the factor space (N×2) is a curve of the form $x_1=f(x_2)$ describing the set of suboptimal values of the input variables that provide the given values of the output variable. In other words, each of the descriptions of the final state, in essence, forms the requirements for the quality of the technological product. The multialternative final state thus formed requires the solution of the problem of choosing the one of which the optimum control should be sought.

An illustration of the solution of the optimization problem in the parametric form (9) and the derivation on the basis of the multialternative description of the final state are shown in Fig. 11–13. Fig. 13 also shows the principle of choosing the final state with respect to which optimal control should be sought.

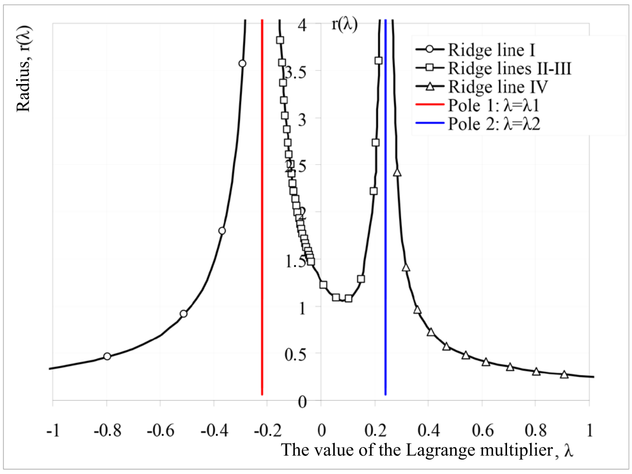

Fig. 11. Parametric description of constraints

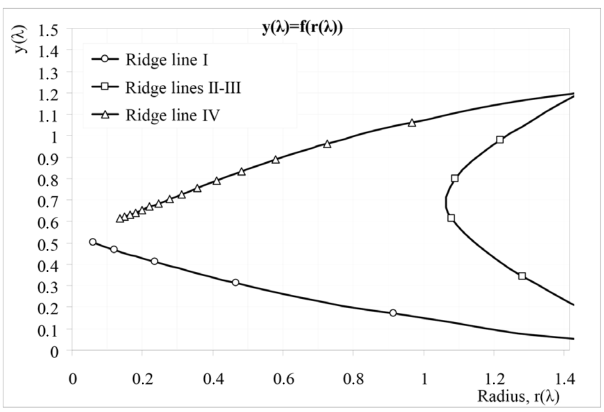

Fig. 12. Lines of ridges as curves describing a set of suboptimal meanings

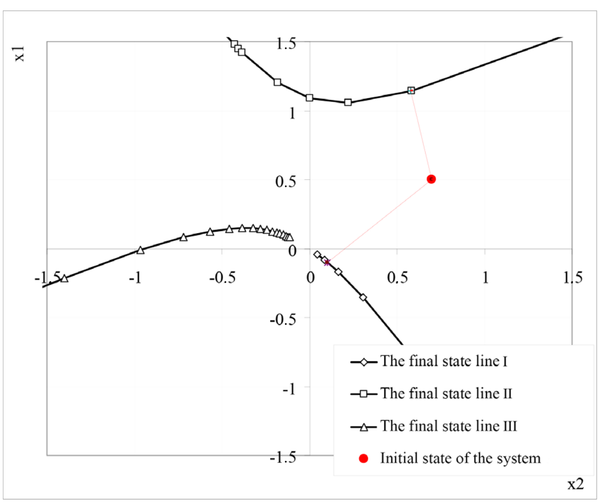

Fig. 13. The principle of a multialternative description of the final state and the principle of choosing the final state with respect to which it is expedient to search for optimal control

Obviously, the search for optimal control should be conducted relative to the nearest final state line. Moreover, if the line is straight, the distance can be determined in the known way

$$d_{ij} = \sqrt{(x_{1j} - x_{10})^2 + (x_{2j} - x_{20})^2}, \qquad (10)$$

where x_{1j} and x_{2j} are the coordinates of the points on the line of the final state, while x_{10} and x_{20} are the coordinates of the point describing the initial state.

6. Discussion of the results of studying the optimal control synthesis method based on the analysis of the mathematical model of the control object and the multialternative description of the final state

The proposed method of searching for optimal control of technological processes, based on an analysis of the solution of the SDEs, has the advantage of being quite simple. The solutions obtained with its application are completely consistent with the results obtained using the Pontryagin maximum principle. It should be noted that its distinctive feature is associated with the possibility of solving the problem of stabilizing the control object – the technological process – in the sense of retaining its parameters within a given area for the necessary time. The point is that if the moment of the time that is the minimum for the transition from the actual initial state to the final one corresponds to the moment of the time of issuing the technological product, then the speed is the priority. Meanwhile, the obtained solutions for finding the optimal control in terms of the speed of operation are the desired results.

If the parameters of the technological process must correspond to the values set for a certain time interval, for example, before the technological product is dispensed, the control that maximizes the time of their values staying within the given area will be considered optimal. Here, it is also possible to talk about minimizing the new time to reach a given area if the phase trajectory goes beyond it. In this case, at least two alternative control implementations are essentially possible, differing in the principle of selecting control switching times. Obviously, the determining factor for choosing the optimal control in this case is the initial state of the system, described by the position of the phase space point characterizing the actual initial state relative to the final state line.

In this regard, it should be noted that when solving real technological problems in the conditions of commercial production, there is often an uncertainty in the estimation of the initial state of the system on which the position of the phase trajectory depends under the chosen control. Obviously, while speaking about the search for optimal control, we are concerned with the sign of control rather than with the optimal values of the control parameter (it corresponds to its boundary value $|u(t)|=1$). Its choice depends on the position of the initial state of the system relative to the line of the final state. Moreover, often the initial state of a technological system as a control object can only be determined only with a time shift associated with the necessary laboratory testing of the technological product. This means that the choice of control must depend on taking into account the existence of such a temporary drift, and at the time of the control action the system has already passed from its initial (previous) state to the subsequent one. The evaluation of the latter is a separate, independent task, but without its solution, the choice of the optimal management at the given time is dubious.

The proposed procedure for estimating the initial state with respect to the final state, based on the reduction of the straight line describing the final state, to the normal form is simple enough and makes it possible to uniquely determine the sign of control. If the line of the final state is not described by the equation of a straight line, other variants of estimation are possible, but the essence remains the same – it is necessary to estimate the position of the point of the phase space with respect to the final state.

From all this it follows that the procedure for synthesizing the optimal control of the technological process must be preceded by the procedure for obtaining a description of the final state. If a parametric description of form (9) is applied, multialternativeness arises in the choice of the final state. As follows from the proposed selection principle (Fig. 13), the most appropriate choice is the line of the final state to which the initial state is closer.

It should be noted that the proposed method of searching for optimal control, based on the analysis of the solutions of the SDEs as a mathematical model of the management object, does not take into account such an important criterion of management quality as minimizing energy costs. This is its obvious shortcoming and possible direction of development. The proposed principle of choosing the final state in the conditions of its multialternativeness, although it seems to be effective, is still not without its obvious shortcoming. It is associated with the requirement to represent the mathematical model of the technological process in the form of a regression equation. It is clear that in the presence of a small sample of experimental data, as well as with uncertainty in the description of input variables, to obtain an adequate mathematical model for the subsequent parametric multialternative description is a difficult task. Obviously, these shortcomings form potentially interesting directions for the development of the proposed method.

7. Conclusion

1. The proposed method of searching for the optimal speed and final state of control of technological processes, based on an analysis of the solution of the SDEs, makes it possible, along with its simplicity, to obtain solutions completely consistent with the results obtained using the Pontryagin maximum principle. In doing so, it opens up additional opportunities in solving the task of retaining the parameters of the technological process within a given area. It has been shown that for this there are at least two alternative control implementations differing in the principle of selecting switching times of the control. The determining factor for choosing the optimal control in this case is the initial state of the system, described by the position of the phase space point characterizing the actual initial state relative to the final state line. In this case, the concept of "initial state" is replaced by the equivalent concept of "previous state", which is more suitable for describing the technological process in the context of searching for optimal control. It is proposed, in the case that the final state is described by the equation of a straight line, to render it in a normal form. As a result, the corresponding deviation of the point of the preceding state from the straight line that uniquely determines the control sign can be calculated, taking into account the sign.

2. It is proposed to obtain a multialternative parametric description of the final state for the search for optimal control of the technological process using a ridge analysis. It has been shown that each of the alternatives is a set of suboptimal values of the output variable, which provides optimal values of the output variable in the chosen senses. The latter are formed by the requirements for the quality of the finished technological product. It has been shown that in the synthesis of optimal control in the case of multialternative descriptions of the final state, it is most expedient to choose the one that is the closest to the point in the phase space, describing the previous state of the technological system.

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