be interconnected, which requires that company managers pay maximum attention to the organization and operation of the infrastructure.

The functionality of IT-service involves a large number of functions of the IT as a service department. At the stage of IT-service planning, functionality is aligned to the strategy, standards and plans within the framework of strategic functions of the IT-service. Finally, at the stage of IT-service operation, the functionality is provided by data and equipment management, systems software and support of end-users [2, 3]. The respective functions of IT-service make it possible to take into account the costs, related to the IT-service support, as well as compliance with conditions, agreements between a customer and the IT-service. Service time, availability, reliability, and efficiency of the IT-service are defined in the course of negotiating an agreement with requirements to the IT-service with a customer and are subsequently controlled by the monitoring functions. These parameters are provided by the functions of end-user support and data and equipment management, as well as systems software. Data on the efficiency of operations, significant for end-user, can be retrieved based on the statistics about using the applied systems. Given this, one can see that the characteristics of IT-service directly depend on the functional requirements from end-users.
2. Literature review and problem statement

When designing a new device or IT-service, it is necessary at the early stage to clearly identify target groups of the users and their requirements. Paper [4] focuses on the importance of correct and full account of the users’ and organizational requirements for any new device or IT-service. It is noted that development of a new multi-user IT-service is an expensive and risky process. That is why it is important to design a resulting reliable IT-service, which definitively meets the requirements of users and implements the entire set of functions required by the users.

It is expedient to divide end-user requirements to IT-services, which form a part of the information system, into levels [5, 6]. The levels of requirements relate, on the one hand, to the level of abstraction of information system, on the other hand, to the level of enterprise management. One of the relevant problems arising in the process of working out the requirements is the inability to implement a clear transition between different levels of requirements. Paper [7] proposed a graphical tool for refining the requirements, which guides software architects during the transition from the user’s requirements to the system’s requirements (the architectural level). However, the visual models, consequently obtained, cannot be represented in the form of software implementation.

Article [8] considers an application of service-oriented architecture for the information systems at large organizations. The authors note that when designing applications, it is necessary to use the service principles of abstraction and autonomy; however, the article does not specify how these principles can be actually employed at the stage of collecting requirements. The paper explores the ways SOA, NFV, and SDN can complement each other in the implementation of various functions of the information system, through specifying the composition of IT-services, IT-services selection, IT-services provision and the deployment tasks.

Software architecture is a bridge between requirements and software realization of the information system [10]. Specification of software architecture represents a high-level model and defines the structure of an information system by determining architectural components and relationships between them for the selection of the best way to meet all key requirements, including requirements to the reliability of the solution being developed. Software architects when compiling the requirements typically talk to customers and analyze existing architectural templates in order to understand which components they should apply, how the selected components behave, the pattern by which they must be linked and which problems might arise at the simultaneous application of various components. That is why the interrelation between the requirements of end users and the architecture of information system has recently been in the focus of attention, especially regarding the tasks to provide reliability and quality of the developed system.

Paper [11] proposes a new approach to the reliability of information systems based instead of the probability theory on the non-equilibrium theory of processes. It is assumed that defects in the software implementation of IT-services occur as a result of incoming and outgoing defect flows. Emphasis is on the fact that most existing models for the evaluation of reliability of information systems with a service-oriented architecture follow from the dynamics of software systems. To create an information system, a sequence of transformations is performed, starting with requirements and ending with the implementation. However, requirements are mainly represented in the form of a text, rather than a model that can be easily read by computers [12, 13]. That is why an automatic transformation from the requirements to the analytical models is extremely difficult. In this connection, a solution was proposed to the problem on the formation of a formalized description of end-user requirements, using a theoretical-categorical apparatus.

The analysis performed revealed that the description of relationship between the IT-services of an information system and the set of functional end-user requirements is an extremely challenging problem and it defies implementation employing classical mathematical methods.

3. Research goal and objectives

The goal of present study is to develop a mathematical model for determining the alignment of IT-services of the information system with the functional end-user requirements. This makes it possible to describe IT-services and end-user requirements and to apply them in order to formalize relations between them.

To accomplish the set goal, the following tasks should be solved:

- to represent IT-services of the information system in the form of a category and to prove the possibility of existence of such a representation;
- to represent functional requirements of end-users in the form of category and to prove the possibility of existence of such a representation;
- to determine the alignment of the presented categories of IT-services with the functional requirements of end-users.

4. Determining a category of IT-services

Development of a mathematical model for determining the alignment of IT-services of the information system with the functional requirements of end-users will make it possible to describe an interrelation between the information system, which is represented by a set of IT-services, and the functional requirements of end users. Describing such a model, which takes into account all components of IT-services of the information system, by classical mathematical methods is not possible, due to the weak formalization of connections between components, which is why we refer to a theoretical-categorical apparatus [13, 14].

A category implies projecting objects onto a certain totality of “related” objects and makes it possible to determine the inner structure of the examined objects by the attributes of representations. In a general form, a category can be represented as a totality of unambiguous mathematical structures and representations between these structures (morphisms), in which a number of additional conditions (functionality between objects) are satisfied [13].

While applying categories, there are different ways to describe objects, the results of which do not depend on the description means; the objects remain integral and the representations, displayed by morphisms, allow comparing (matching) objects to find similar elements in them. A category is considered defined if a class of objects, represented by
the structured sets, is assigned, as well as a set of morphisms among them.

In a general form, a model of category $L$ is mapped as objects (tasks, functions) $\mathcal{O} \mathcal{B} \mathcal{L}$ and morphisms $\mathcal{M} \mathcal{O} \mathcal{R} \mathcal{L}$, describing connections between the related objects in the following form:

$$L = \{ \mathcal{O} \mathcal{B} \mathcal{L}, \mathcal{M} \mathcal{O} \mathcal{R} \mathcal{L} \}.$$  

(1)

To obtain a model of IT-services of the information system in accordance with its composition, we will define categories of the IT-services whose objects are the sets of their characteristics, required to describe IT-services of the information system [14].

Let us consider the process of obtaining such categories. To determine the categories of services $L'$, we will assign their sets of objects and corresponding morphisms. The objects of services ($t$) are: $A'$ – a set of enterprises' services, $B'$ – a set of services of the enterprises' subdivisions, $C'$ – a set of services, required by particular employees of an enterprise.

Based on such structural allocation of services within an enterprise, a set of services of subdivisions $B'$ is distributed among a set of services of enterprise $A'$ in such a way that a certain set of services $B'_a$ from set $B'$ belongs to each set of services $A'$. Distribution of the set of services $B'$ among the set of services $A'$ is matched with such representation $F$ so that any set of enterprise's services $a', a \in A'$ is matched with at least a particular set of services $b'$. Then the distribution of the set of services $B'$ among a set of services $A'$ will be represented in the following form:

$$F^{A'}_{B'}: A' \rightarrow 2^{B'_a},$$

(2)

where $2^{B'_a}$ is the set of all subsets of services $B'$ (Boolean $B'$), included in the set of services $A'$, then:

$$F^{A'}_{B'}(a') = B'_a, \quad B'_a \in B',$$

(3)

where $B'_a$ is the set of services of the subdivision, included in the set of services of a particular enterprise $y a', a \in A'$. In this case, the following condition should be satisfied:

$$B' = \bigcup_{a \in A'} B'_a.$$  

(4)

This means that a set of services of subdivisions $B'$ must belong to the set of services of enterprise $a', a \in A'$, or else it has no a prototype at all. Therefore, the condition must be satisfied:

$$b' \in B': |F^{-1}\{b'\}| \leq 1,$$

(5)

where $F^{-1}$ is the inversion $F$. By analogy, a certain set of employees’ services $C'$ corresponds to a set of services of subdivisions $B'$. Then the distribution of the set of services $C'$ among set $B'$ will be presented by analogy in the form of the following representation:

$$F^{B'}_{C'}: B' \rightarrow 2^{C'_a},$$

(6)

where $2^{C'_a}$ is the set of all subsets of services $C'$ (Boolean $C'$), included in the set of services $B'$, then:

$$F^{B'}_{C'}(b') = C'_a, \quad C'_a \in C',$$

(7)

where $C'_a$ is the set of services of subdivisions, included in a set of services of a particular enterprise $y b', b' \in B'$. In this case, the condition must be satisfied:

$$C' = \bigcup_{b \in B'} C'_b.$$  

(8)

This allocation means that the set of services $C'$ must belong to the set of services of subdivisions $B'$, $b' \in B'$. This distribution $F$ is an injective mapping of $C'$ onto $B'$, if every set of services $c'$ is the pattern of only one set of services of a particular subdivision $b', b' \in B'$, or else has no a prototype at all, that is:

$$c' \in C': |F^{-1}\{c'\}| \leq 1.$$  

(9)

Because a set of services of employees $C'$, in turn, is included in a set of services of enterprise $A'$, the distribution $C'$ in $A'$ will be represented as follows:

$$F^{A'}_{A'} = F^{A'}_{B'} \times F^{B'}_{C'},$$

(10)

where $\times$ indicates distribution composition. Thus, the distinguished structured sets $C$, $B'$, $A'$ and representations $F^{A'}_{B'}$, $F^{B'}_{C'}$ are the elements of a mathematical model of the structure of enterprise's services in the form of:

$$M' = \{ A', B', C', F^{A'}_{B'}, F^{B'}_{C'} \}.$$  

(11)

To obtain the categories of services, we will determine the following morphisms:

$$\phi_{A':} A' \rightarrow A', \quad \phi_{B':} B' \rightarrow B', \quad \phi_{C':} C' \rightarrow C'.$$

(12)

Structured sets of services $C$, $B'$, $A'$, forming a region of services $D'$, is a constituent element of the generalizing set of enterprise's services $D'$ with structured sets $C''$, $B''$, $A''$ provided $D' \subset D''$. By the definition of a category, such morphisms should be functional, which is confirmed by the corresponding commutative diagrams (Fig. 1).

Description of commutative diagrams is represented by the respective identities:

$$F^{A'}_{B'} \circ \phi_{A'} = \phi_{B'} \circ F^{A'}_{B'}, \quad F^{B'}_{C'} \circ \phi_{B'} = \phi_{C'} \circ F^{B'}_{C'}.$$  

(13)

In this case, internal relationships between structured sets $C$, $B'$, $A'$ in $D'$ are also preserved in $D''$ due to their functional purpose, and, therefore, the condition of morphism definition is satisfied.

Using the introduced structured sets of enterprises' services ($A'$, $B'$, $C'$) and the corresponding morphisms ($\phi_{A'}$, $\phi_{B'}$, $\phi_{C'}$), a categorical model of enterprise's services will be represented in the following form:

$$L = \{ A', B', C', \phi_{A'}, \phi_{B'}, \phi_{C'} \}.$$  

(14)

Thus, in order to establish a relation between the IT-services and the end-user requirements, it is necessary by analogy to represent the category that describes the end-user functional requirements.
5. Determining an alignment of the IT-services with the end-user requirements

The sets of services of enterprises and their subdivisions are connected with the necessity to implement their correspondent functional requirements \( z \). To define a category of the functional requirement, we will introduce structured sets: \( A^z \) is the set of requirements for an enterprise, \( B^z \) is the set of requirements of subdivisions, \( C^z \) is the set of personal requirements from employees.

By analogy, using the introduced structured sets, we will define their relationship in the form of representation:

\[
\begin{align*}
F^S_x & \colon A^z \to 2^{B^z}, \\
F^S_y & \colon A^z \to B^z,
\end{align*}
\]

(15)

where \( 2^{B^z} \) is the set of all subsets of functional requirements for an enterprise, distributed for the whole set of functional requirements of subdivisions. In this case, the following conditions must be satisfied:

\[
F^S_x(a^z) = B^z, \quad B^z \subseteq B^z.
\]

(16)

Such representation indicates belonging of the functional requirements of subdivisions \( B^z \) to the set of functional requirements of a particular enterprise \( a^z \), which is an injection. Thus:

\[
b^z \in B^z : |F^{-1}(b^z)| \leq 1.
\]

(17)

By analogy, we will establish the relationship between functional requirements of subdivisions \( B^z \) and those directly of employees \( C^z \) by the following representation:

\[
\begin{align*}
F^S_z & \colon B^z \to 2^{C^z}, \\
F^S_z & \colon b^z \to C^z.
\end{align*}
\]

(18)

When similar conditions are met:

\[
F^S_z(b^z) = C^z, \quad C^z \subseteq C^z,
\]

(19)

\[
C^z = \bigcup_{a^z \in A^z} C^z,
\]

(20)

\[
b^z \in B^z : |F^{-1}(b^z)| \leq 1.
\]

(21)

Using the introduced structured sets and obtained representations, we will derive a mathematical model of the structure of the functional end-user requirements in the form:

\[
M^S = \{A^z, B^z, C^z, F^S_x, F^S_y, F^S_z \}.
\]

(22)

To obtain the category of functional requirements of an enterprise, by analogy, we will introduce the following morphisms:

\[
\phi_x : A^z \to A^z,
\]

\[
\phi_y : B^z \to B^z,
\]

\[
\phi_z : C^z \to C^z.
\]

(23)

By definition, such morphisms should be functional, which was proved by the corresponding commutative diagrams (Fig. 2).

Description of commutative diagrams is represented by the corresponding identities:

\[
F^S_x \circ \phi_y = \phi_x \circ F^S_y,
\]

\[
F^S_z \circ \phi_z = \phi_z \circ F^S_z.
\]

(24)

By analogy, internal relationships between the structured sets of functional requirements of an enterprise are maintained in a generalized set of functional requirements; therefore, the condition of morphism definition is satisfied.

By the definition of a category, structured sets \((A^z; B^z; C^z)\) and morphisms \((\phi_x; \phi_y; \phi_z)\) form a category (model) of functional requirements of an enterprise in the following form:

\[
L^S = \{A^z, B^z, C^z, \phi_x, \phi_y, \phi_z \}.
\]

(25)

Description of the relationship between a set of enterprise’s services and a set of functional requirements is considered to be extremely difficult, however, by employing the notion of a functor, this problem can be resolved. By definition, a functor describes relationships between the structured sets and the morphisms of categories on condition of the functionality of their objects and morphisms. Functionality of relationship between categories of services and functional requirements is explained by the correspondence of selection of services for resolving a specific list of functional tasks. That is why such a relationship between the introduced categories will be described by the appropriate functor:

\[
\Phi^S_L : L \to L^S.
\]

(26)
The functor is represented by the mapping of category $L'$ onto category $L^2$, consistent with the structure of these categories. Objects of category $ObL^2$ are mapped onto objects of category $ObL^3$, while the morphisms of category $MorL^2$ are mapped onto the morphisms of category $MorL^3$. Consequently, their functionalities $(ObL^2\to ObL^3, MorL^2\to MorL^3)$ provided the two conditions are satisfied.

1. For each object:

$$T \in ObL, \Phi(t) = 1_{\Phi(t)} \tag{27}$$

2. For any morphisms:

$$\Phi(\varphi) : A' \to A', \Phi(\varphi) : B' \to B', (MorL^2). \tag{28}$$

$$\Phi(\Phi(\varphi) \times \Phi(\varphi)). \tag{29}$$

Justification of formalization of the relationship between categories $L'$ and $L^2$ by functor (26) is determined by the functionality of morphisms of these categories. The presented model makes it possible to consider IT-services and functional requirements as categories, which allows the use of such representation of these objects in order to formalize relations between them.

Such model makes it possible to coordinate the availability of IT-services, supported by the information system, at the structural level, in accordance with specific end-user requirements. The given model implements at the verbal level coherence of the infrastructure of an enterprise management and the infrastructure of the information system, which is impossible to execute by using classical methods. Implementation of such coordination is carried out using the introduced morphisms, which can be realized through visual models in the form of UML diagrams. In turn, by using a CASE-tool of the IBM Rational Software Architect, visual UML diagrams can be converted into a code in the JAVA programming language.

6. Discussion of results of the study on defining the alignment of IT-services with the end-user requirements

Due to the complexity of the examined objects (IT-services and end-user requirements), the process of implementation of a mathematical model for determining their alignment is a difficult task, which is why the most appropriate description of such a system is the mathematical models, given in the form of categories. Application of the category theory when studying complex information systems makes it possible to examine the internal structure of an information system. The theory of categories examines, instead of separate sets, all equally structured states, which can be divided into separate classes of states, by identifying them with the content concepts of the examined system. This means that a totality of all similarly structured sets makes up a class of objects of the category, a class of states of the system. Categories imply projecting their objects onto a certain totality of their “related” objects and, by the properties of projections, determination of the internal structure of the examined objects.

Application of the theory of categories for solving the problem on determining the alignment of IT-services of the information system of an enterprise with the end-user requirements allowed us to devise a model of the structure of IT-services and a model of the structure of the end-user requirements. The categories of IT-services and end-user requirements are described in the form of structured sets and morphisms. The relationship between the introduced categories is described by the appropriate functor. Functionality of relationship between the category of IT-services and the category of end-user requirements is explained by the correspondence of the choice of IT-services to the implementation of a specific list of functional requirements.

The shortcomings of the designed model might include a verbal description of the considered categories, which involves the necessity of developing tools in order to establish a relationship between the obtained categories.

Because the present model does not take into account the relationship between parameters that characterize an IT-service, the disadvantages of the developed model might also include the absence of a description of interactions between the sets, which directly describe an IT-service.

In the future, a software implementation of the developed model is planned.

7. Conclusions

1. A model of the structure of IT-services of the information system was developed. We identified the structured sets and representations, which form a mathematical model of the structure of IT-services of an enterprise information system. IT-services of the information system of an enterprise are represented in the form of categories. To define a category of the IT-services of an information system, the sets of structured objects and related morphisms were assigned. The objects of IT-services include: a set of an enterprise’s services, a set of services of enterprise subdivisions, a set of services required by particular employees of the enterprise. Functionality of the morphisms, describing relationships among the objects in this category, is proved by commutative diagrams.

2. A model of the structure of functional requirements of end users of the information system was developed. Structured sets and representations, which form a mathematical model of a structure of end-user requirements were defined. The requirements are represented in the form of a category. A category of requirements of end-users of the system in the form of a set of objects and related morphisms was determined. The objects of end-user requirements include: a set of an enterprise requirements, a set of requirements of subdivisions, a set of personal requirements from particular employees of the enterprise. Functionality of the morphisms, describing relationships among objects in this category, is proved by commutative diagrams.

3. A categorical model for determining the alignment of IT-services of the information system to the end-user requirements was developed. Relationships between categories of IT-services category and end-user requirements were defined in the form of a functor. Justification of formalization of the relationship between categories by a functor is determined by the functionality of morphisms of the given categories. Such model makes it possible to establish relationships between requirements of end-users of the infrastructure of the control object and the corresponding IT-services, which should be supported by the information system. This problem cannot be solved in practice by classical methods because of the impossibility to describe dependences between a set of end-user requirements and a set of IT-services at the mathematical level. In addition, selection of the appropriate IT-services of information system for a particular user is also a problem.
References


