1. Introduction

Resistance to motion is one of the most important factors for saving energy in railway transport. Up to 75% of the consumption of energy resources on the traction of trains accounts for overcoming resistance to motion. It is known that rail trucks are directed along a rail track under the action of horizontal forces of the contact between the wheels and the rails, the main ones of which in the process of directing are the forces of friction. Slip friction resistance and resistance in the curve in the accepted classification of motion resistance are in its different components, the first belonging to the basic resistance to motion while the second being in the additional one. The occurrence of this kind of resistance to motion is of a general nature – friction interaction in the contacts between wheels and rails, caused by slipping when the wheelsets are directed by rails. Contact slippage is the main cause of wear of rolling surfaces of wheels and rails and, at the same time, of additional motion resistance associated with directing the wheelsets by rail.

Studies into motion resistance started at the very beginning of development of the railways. The first mention of motion resistance, associated with directing of the rolling stock by a rail track, can be found in experiments on the Morshansko-Syzran railway. Paper [1] noted the impact of “harmful (parasitic) movements of wagons” on the motion resistance, which were lower for passenger trucks compared to the freight wagons.

One of the most important and most resource-intensive challenges facing the railway transport for more than 30 years is considered to be a problem of intensive wear of wheels’ ridges while passing curved sections of the track. The intensity of wear of wheels and rails in the curved sections of the track is considerably greater in comparison with that in the straight sections. This is explained by the fact that directing the rolling stock in curves occurs due to the slip friction forces between the ridges and the rails. The forces of friction are the cause of emergence of additional resistance to motion.

Research into factors affecting motion resistance associated with directing the rolling stock by a rail track are aimed at reducing consumption of electricity and fuel needed for the traction of trains. Given the importance of energy saving issues and the amount of actual energy consumption by railroads, the present article is relevant.

2. Literature review and problem statement

The problem of motion resistance is closely linked to the energy efficiency of rail transport. In article [2], it is
in particular noted that effective reduction of resistance to motion of trains is essential to save electricity on railroad transport. Papers [3]–[8] investigated effect of the design of wheelsets on the motion resistance. Traditional design of wheelsets includes wheels’ ridges, a rigid coupling of wheels in a wheelset, and conical profiles of the wheels’ rolling surfaces [3]. These design features of the wheelsets have for almost 200 years enabled reliable routing of trucks along the rail track. When passing the curves, design of the wheelsets contributes to a reduction in the directing efforts between ridges of the wheels along the rails [4]. However, in the curves of small and medium radius, the lack of taper of the rolling surfaces causes slippage of the wheels in the longitudinal direction. The forces of longitudinal slippage lead to an increase in the skew of a wheelset axis in the curve and an increase in the wheel’s angle of attack on the rail head, which enhances transverse reactions of rail and motion resistance [5]. A confirmation of this phenomena is characteristic creak and rattle when the carriage moves along the curve, which is noted in paper [6]. When moving along straight sections of the track, there occur intense auto-oscillations of the wheelsets’ waggling with a periodic contact between ridges of the wheels and the rails. Paper [7] examines kinematical characteristics of meandering movement and, in particular, slippage in the contacts between wheels and rails. Authors of article [8] analyze transverse forces of contact interaction between wheels and rails during oscillations at high speed. The authors concluded that the transverse forces acting on rails could reach significant values. This may lead to the intensive wear of ridges, and to an increase in the motion resistance.

Paper [9] addresses improvement of the classification of the motion resistance components. It is proposed to isolate, as a separate component, the motion resistance related to directing the trucks by a rail track. The issue of directing the wheelsets by rails is considered in study [10]. Authors propose to approach this process as a control process. This makes it possible to employ the elements of the theory of manageability of transportation vehicles applied in the automobile transport, aviation etc.

The differential slippage and the rolling resistance related to it are tackled in article [11]. The analysis of this type of rolling resistance was performed using the kinematics of rolling bearings as an example. That is why quantitative results of the study cannot be applied to the contact between a wheel and a rail.

Almost all known studies into motion resistance were limited to an analysis of the so-called speed characteristics, that is, a dependence of resistance on speed. Research results could not be used to improve design of the rolling stock in order to reduce motion resistance. Thus, for example, article [12] reports methods for improving the weight-strength properties of rolling stock by optimizing the moments of resistance of bearing systems. However, the functionality of such an optimization does not determine the impact of changes in the designs of the examined object on motion resistance.

In paper [13], authors explored an issue of the influence of friction in the elements of joints and nodes of wagons on the indicators of dynamics. At the same time, the effect of friction in the contacts between wheels and rails on the indicators of motion resistance is not addressed.

Paper [14] presented the concept of promising rolling stock of the railroad. In this case, the issues of design optimization in terms of minimizing the motion resistance are not considered.

Authors of study [15], based on computer simulation, analyze the share of additional motion resistance along the curved track section for a two-axle wagon bogie. The possibility of its reduction by 15–40% is noted. At the same time, the authors indicate that this component of the resistance to motion is not the key one. It is concluded in the paper that an actual reduction in the overall motion resistance of a wagon cannot exceed 3.3%. The shortcoming of the work is the authors’ consideration of the contact between a wheel and a rail as a single-point contact. In this case, they did not take into account spatial distribution of slippage at a ridge contact.

A role of the ridge contact for the creation of additional motion resistance was discussed in study [16] in the analysis of kinematics of a two-point contact between a wheel and a rail. It is shown that the share of additional motion resistance related to differential slippage in a ridge contact is significant. This can significantly increase additional motion resistance associated with directing the wheelsets in the curve.

The disadvantages of traditional design of wheelsets, those particularly associated with the motion resistance, gave rise to projects of improved wheelsets. The projects can be conditionally divided into two groups. The first group contains various suggestions in order to control alignment of wheelsets in curves [17–19]. One of the modern developments of rolling stock that implies controlled directing of trucks along a rail track is the three-axle locomotive truck HTC-R2 Trucks, designed by the company Electro-Motive Diesel (USA) [20]. The letter R in the indexing of the truck denotes a modification with the radial alignment of wheelsets in a curve. However, the complexity of control mechanism and high cost hamper wide implementation of trucks with radial alignment of the wheelsets.

The second group of improvements in wheelsets include designs with independent rotation of wheels. Article [21] analyzes improvement in directing the truck by a track based on the prospective designs of wheelsets. In particular, the need is noted for a detailed description of the geometry of wheels and rails and the kinematics of slippage in contacts when modeling motion resistance. Paper [22] reports results of experimental research into trucks with independently rotating wheels. It is noted that along with advantages of the new wheelsets, certain drawbacks were revealed related to the loss of self-control. The only manufacturer that utilizes wheelsets with separate rotation of wheels is the Spanish company Talgo. Due to the advanced design of the chassis, trains made by Talgo are distinguished by the improved comfort for passengers and minimum wear of wheels and rails [23]. An analysis of the scientific publications related to directing the trucks by a rail track [1–23] shows that the object of present research is at the junction of two areas of science about wheel/rail adhesion. The first area is the motion resistance, the second is the horizontal dynamics. Neither the first nor the second area has sufficient evidence about the resistance in question. Much of the known research into resistance to motion [1–10, 15, 18–24] were carried out with the aim of obtaining formulae for practical application in traction calculations. In this case, the issues of reducing a component of motion resistance related to directing the trucks by a rail track were not dealt with. The main proof is the fact that the accepted classification misses such a type of
motion resistance and its components. The impact of truck and track parameters on the components of motion resistance was not examined either.

3. The aim and objectives of the study

The studies conducted were aimed at defining the nature of occurrence of resistance to motion of railway vehicles, related to directing the wheelsets by a rail track and determining the prospects for its reduction.

To achieve the set aim, the following tasks have been solved:

– to construct a mathematical model for calculating motion resistance related to directing the wheelsets by a rail track;
– to determine effect of certain parameters of the track and the undercarriage on motion resistance;
– to determine the prospects for reducing motion resistance related to directing the wheelsets by a rail track.

4. Object and methods of research into resistance to motion of railway trucks, related to directing the wheelsets by a rail track

Modeling of additional resistance to motion when directing the trucks by a rail track in curve was performed using the four-axle freight semi-wagon, model 12-9046, with the two-axle trucks 18–100, as an example.

To study motion resistance, related to directing the wheelsets by a rail track, we employed elements from the theory of closed power circuits and the principle of kinetostatics, known as the D’Alembert’s principle.

5. Results of research into motion resistance, related to directing the wheelsets by a rail track

5. 1. The nature of motion resistance, related to directing the wheelsets by a rail track

The nature of resistance under examination is associated with parasitic slippage in the closed power circuits formed in the system that directs wheelsets by a rail track. The study is built based on an energy hypothesis on that the mechanical energy of overcoming the friction of parasitic slippage is the energy of motion resistance.

It is proposed to term the component of motion resistance related to directing the wheelsets by a rail track the kinematic resistance to motion. According to the accepted classification, the kinematic resistance to motion possesses attributes of both primary and additional resistance. Therefore, conditionally, during motion along the straight track sections, it should be considered as part of the primary; when passing the curves – as part of the additional motion resistance.

In the system of directing the undercarriages by a rail track, several closed power circuits can be differentiated.

At a two-contact contact between the wheel and the rail, there forms a closed power circuit with two nodal points in the main and ridge contacts. In this circuit, there occurs differential slippage, which may be causing an additional kinematic resistance to motion due to an increase in rolling resistance.

Fig. 1 shows as example a schematic of the possible distribution of normal reactions \( (N_1, N_2) \) and adhesion forces \( (S_1, S_2) \) in a two-contact ridge contact between a wheel and a rail. \( F_t \) is the external longitudinal reaction on the wheel from the truck’s side. The reaction \( F_t \) is the force of motion resistance, which needs to be overcome for a wheel to roll.

\[
\begin{align*}
\sum M &= S_1 \cdot R_1 - S_2 \cdot R_2 = 0; \\
\sum F &= S_2 - S_1 + F_t = 0.
\end{align*}
\]

From equations (1), it is possible to derive the magnitude of kinetic resistance to motion \( W_k \)

\[
W_k = F_t = S_1 \left(1 - R_1 / R_2\right).
\]

Based on (2), it is possible to draw an important conclusion: the force of motion resistance \( W_k = F_t \) cannot be equal to zero in the presence of a ridge contact between the wheel and the rail. Under different conditions, the difference between radii of the main and ridge contacts reaches 15–30 mm. That is why the kinematic motion resistance can amount to 6% of adhesion force \( S_1 \).

Wheelsets together with a rail track also create closed power circuits. The absence of longitudinal slippage in the contacts between wheels and rails is possible only in the ideal case. Ideal is the case when a single wheelset slowly rolls without a ridge touch with rails. Under actual conditions of motion of wheelset as part of the track, there always occurs slippage in the contacts between wheelsets and rails. This slippage is parasitic and creates an additional kinematic motion resistance, both in straight and curved sections of track.

Fig. 2 shows a simplified schematic of adhesion forces \( S_1, S_2 \) and journal reactions \( F_{\theta 1}, F_{\theta 2} \) acting on the wheelset.

A wheelset is set with lateral shift \( \Delta y \) relative to the track axis and rolls by force along the rails in a straight line. In this case, wagging is constrained by journal reactions \( F_{\theta 1}, F_{\theta 2} \).
From Fig. 2, it is possible to build the following equilibrium equations

\[
\begin{align*}
(S_k + s_{k2}) - (F_{b1} + F_{b2}) &= 0; \\
S_{k1} - S_{k2} - R_1 &= 0; \\
F_{b1} - F_{b2} - S_{k1} + S_{k2} &= 0.
\end{align*}
\tag{3}
\]

It is possible to obtain values for the kinematic motion resistance from equations (3),

\[
W = S_{k1} - S_{k2} = (1 - R_1 / R_2).
\tag{4}
\]

Formulae (2) and (4) disclose the nature of occurrence of the kinematic motion resistance. The kinematic resistance to motion emerges in the process of directing the wheelsets by a rail track at the expense of parasitic slipping in the nodes of closed power circuits. Such nodes are the contacts between wheels and rails.

5.2. Mathematical model of the process of directing the undercarriage by a track in curve

5.2.1. Estimated scheme for fitting the truck curve to the curved track section

We consider steady motion of a two-axle semi-wagon’s truck along a circular curve in the absence of external influence except for a rail track. Profiles of the wheels are new, in line with GOST 10791.

Fig. 3 shows the estimated scheme for fitting the truck at free alignment in a track.

In Fig. 3, the following notations are used:
- \( \delta \) – full clearance of the wheelset in a rail track;
- \( \delta_{jk} \) – clearances in the ridge contacts of respective wheels (\( j = 1, 2 \)) of the corresponding wheelsets (\( k = 1, 2 \));
- \( \alpha \) – angle between a lateral axis of symmetry of the truck and a radius of the curve that passes through a truck’s pivot;
- \( \psi_k \) – angles of attack of respective wheelsets on rails (\( k = 1, 2 \));
- \( 2C \) – base of the truck;
- \( \rho, \rho_1, \rho_2 \) – radii of lines of symmetry of the curve and of the inner and outer rails;
- \( G \) – point of rotation of the truck relative to a vertical axis – the truck’s pivot;
- \( V \) – the truck’s motion direction, perpendicular to the radial line that passes through the truck’s pivot;
- \( OXY \) – absolute coordinate system;
- \( O_{kX_kY_k} \) – coordinate systems related to the corresponding wheelsets. Directions of the \( y_k \) axes coincide with the radial straight lines, passing through the centers of the respective wheelsets.

The first wheelset is guiding and it moves with the ridge pressed to the outer rails, while the second one is in the free position. The actual position of the second wheelset is determined in the process of calculation.

Fig. 4 shows a schematic of the contact forces for the wheels of wheelsets: \( a \) – vertical cross sections in the contact planes; \( b \) – horizontal projections of the contact forces.
The following notations are used:

- $K_{ij}$ – notations of the respective contacts between wheels and rails; $i$ – the number of contact by type (main contacts – $i$=1, ridge contacts – $i$=2); $j$ – notations of contacts by the wheel’s number (left wheels – $j$=1, right wheels $j$=2); $k$ – notations of contacts by the wheelset’s number (first wheelset – $k$=1, second wheelset – $k$=2);
- $N_{ijk}$ – normal loads in contacts;
- $P_{ijk}$ – vertical components of normal loads in contacts;
- $H_{ijk}$ – horizontal lateral components of normal loads in contacts in the $O_{ij}X_{ij}Y_{ij}$ coordinate systems;
- $S_{ijk}$, $S_{ijk1}$, $S_{ijk2}$ – adhesion forces and their longitudinal and lateral components in the corresponding contacts in the $O_{ij}X_{ij}Y_{ij}$ coordinate systems;
- $F_{x}$ – longitudinal external force acting on the truck from the side of the body and models the tractive force needed to overcome additional resistance to motion in a curve. Force $F_{x}$ is applied to the truck’s pivot and is perpendicular to the radius of the curve that passes through the pivot;
- $F_{y}$ – lateral force acting on the truck from the side of the body and it models the unsuppressed centripetal force of inertia. The force $F_{y}$ is applied to the truck’s pivot and is directed along the radius of the curve that passes through the pivot.

In order to study the influence of the truck’s and track’s parameters on the motion resistance related to directing the truck by a rail track, we employed the principle of quasi-static dynamics. The undercarriage is considered during motion along a circular curve under the action of contact track forces (including the forces of motion resistance), tractive force from the locomotive, and the unsuppressed inertia force due to circular motion.

The main vectors of external force impact on the wheelsets, particularly, the main vector of forces ($\vec{F}$) and the main vector of horizontal force moments ($\vec{M}$), are equal to zero

$$\vec{F} = \vec{F}_{x} + \vec{F}_{y} + \vec{F}_{z} = 0; \quad \vec{M} = 0,$$

where $\vec{F}_{x}$ is the main vector of horizontal components of forces in the contact between wheels and rails.

Based on Fig. 3, 4, it is possible to record a series of interrelations.

$$\delta_{11} + \delta_{12} = \delta_{21} + \delta_{22} = \delta; \quad (6)$$

$$\alpha = a \cos(\gamma) - \sin(\gamma) + \delta = \frac{\rho_{2} - \delta_{22}}{2c} \sqrt{1 - d^2}; \quad (7)$$

$$\psi_{1} = \frac{\pi}{2} \sin \left( \frac{\rho_{2} - \delta_{22}}{2c} \sqrt{1 - d^2} \right); \quad (8)$$

$$\psi_{2} = \frac{\pi}{2} \sin \left( \frac{\rho_{2}}{2c} \sqrt{1 - d^2} \right). \quad (9)$$

In formulae (7)–(9):

$$d = \frac{2\rho_{2}(\rho_{2} - \delta_{22}) - 2d^2 + \delta_{22}^2}{2\rho_{2}(\rho_{2} - \delta_{22})}. \quad (10)$$

The adhesion forces in contacts $K_{gh}$ were calculated by the procedure outlined in [24].

The main vector of horizontal components of forces in the contacts between wheels and rails

$$\vec{F}_{x} = \sum_{i,j,k=1}^{2} (H_{ijk} + S_{ijk1} + S_{ijk2}); \quad (11)$$

The main vector of moments of horizontal forces relative to the vertical axis passing through the truck’s pivot (point $G$):

$$\vec{M}_{x} = \sum_{i,j,k=1}^{2} (H_{ijk}G_{ijk} + S_{ijk1}G_{ijk1} + S_{ijk2}G_{ijk2}), \quad (12)$$

where $G_{ijk}$, $G_{ijk1}$, $G_{ijk2}$ are the shoulders of respective forces to calculate the moments relative to point $G$.

Based on (6)–(12), it is possible to construct a system of equilibrium equations (5).

5.3. Results of calculation of the motion resistance, related to directing the wheelsets by a rail track.

As a result of solving the system of equations (5), we obtained dependences of additional motion resistance related to directing the wagon by a rail track in the curve. We accepted the following input parameters for calculation: motion speed, radius of the curve, elevation of the outer rail, clearance of the wheelset in a track, loading of the wagon. The initial data for calculations were the dependences of motion resistance on the specified parameters. Fig. 5 shows estimated dependences of specific resistance to motion in curve $\omega$, on motion velocity $V$, the radius of curve $\rho$, and the elevation of outer rail $h$.

Fig. 6 shows in the form of lines of the level the estimated dependences of specific resistance to motion in curve $\omega$, on the truck’s base $2C$ and the radius of curve $\rho$ for the fixed values of motion velocity $V=30$ m/s, relative elevation of outer rail $h=120$ mm and two variants of values for a wheelset clearance in a rail track – $\delta=20$ mm and 40 mm.
One of the reasons for increased motion resistance of undercarriages on the base of the truck and the clearance of wheelsets in a rail track is the nature of motion resistance related to directing by a track, which is caused by the parasitic cutting of ridges and lateral wear of the rail heads, which are the result of action of forces of motion resistance. The kinematic motion resistance is based on the analysis of the resistance to motion of undercarriages on the base of the truck and the clearance of wheelsets in a rail track. The results obtained could substantiate choosing the rational parameters of trucks during their design. The results might also be used when selecting permissible parameters for the lateral wear of rails.

6. Discussion of results of research into resistance to motion of railroad undercarriages

For many decades, at the design stage of new types of rolling stock, characteristics of resistance to motion have not been examined. Improvement of running gear parts of locomotives and wagons has often led to the deterioration of characteristics of motion resistance. An indirect confirmation of this is the data on an increase in the wear of wheel rolling surfaces during operation of the new types of multi-axle locomotives. Of special importance is the intensive cutting of ridges and lateral wear of the rail heads, which are the result of action of forces of motion resistance.

The nature of motion resistance related to directing the undercarriages by a track. One of the reasons for increased wear of wheels and rails is the lack of detailed studies into the nature of motion resistance, related to directing the undercarriages by a track, or the kinematic motion resistance. The source of the kinematic motion resistance is closed power circuits in the system of directing the rail undercarriages. The kinematic motion resistance is caused by the parasitic slippage in the nodes of closed power circuits. According to the theory of closed power circuits, contacts between wheels and rails are the resolving nodal points of the friction type.

The kinematic motion resistance has two components. The first is the differential motion resistance associated with the existence of a two-point ridge contact under certain modes of motion. Formula (2) proves the existence of this kind of resistance and explains its nature. The second component is the circular resistance connected to the design feature of the wheelset, particularly, the inability of relative rotation of the wheels around the axle. The nature of the circular part of kinematic resistance is explained by Fig. 2 and formula (4).

Study of the influence of parameters of the track and the chassis of the undercarriage on the kinematic motion resistance. A method of mathematical modeling of the process of directing an undercarriage in the curve is based on the principles of quasi-static dynamics. A steady motion of the undercarriage along a circular curve is modeled, at constant velocity under the action of contact track forces, tractive force from the locomotive, and the unsuppressed force of inertia. The system of contact forces is considered in detail separately. The modeling of adhesion forces was performed in line with the procedure outlined in [14]. The truck is freely aligned in a track. Its actual position is determined by the clearances of wheelsets in a track. The values of clearances are determined in the process of solving a system of equilibrium equations. The kinematic motion resistance is defined as the force that must be applied to the truck’s pivot in order to balance all other forces.

A mathematical model and the procedure for estimating resistance to motion make it possible to obtain estimated dependences of motion resistance on the following parameters:

- motion velocity;
- track’s parameters: elevation of the outer rail, radius of the track, deviation in the track’s width that affects the clearance of wheelsets in a track;
- truck’s parameters: base of the truck, diameter and profile of wheels, violation in the geometry of alignment of wheelsets in the frame of the truck.

Results of the calculations confirmed the existence of a vividly expressed minimum of the dependence of motion resistance on velocity (Fig. 5). However, this minimum does not match the equilibrium speed in a curve. Velocity of the minimum is, on average, 15–20 % lower than the estimated equilibrium speed.

We obtained another important result: the lowest motion resistance is demonstrated by trucks with the base from 2.5 to 6.0 m (Fig. 6). Thus, the parameters of the truck’s base of 18–100 are not the best in terms of resistance to motion.

It is an interesting conclusion on that an increase in the wheelsets’ clearance in a track from 20 to 40 mm enhances motion resistance in the curve by the magnitude of up to 25 % (Fig. 6).

Clarifying the nature of motion resistance, related to directing the rolling stock by a rail track, opens up certain prospects in terms of reducing the kinematic motion resistance through design parameters of the undercarriages and the track. The kinematic motion resistance could become an additional criterion when choosing optimal characteristics for the mechanical part of the undercarriages. The same applies to permissible deviations in the parameters of the rolling stock and the track.

As evidenced by the review of publications [1–24], the vast majority of studies into resistance to motion of the rolling stock were experimental. The purpose of the experiments was to obtain formulae to calculate the traction. In their essence they were passive, stating the patterns related to motion resistance. Our results are active as they confirm the possibility to influence the motion resistance of railway undercarriages through design parameters of the chassis and the track.

The results obtained show that the component of motion resistance, related to directing by a track, acquires a substantial magnitude only in curves with a radius of less than 350 m. Given this, the research results could yield maximum effect only at the railways with the presence of curves with a small radius. The findings might also prove useful in the development and modernization of the rolling stock of urban rail transport.

The present study is an attempt to confirm the potential of reducing motion resistance based on the analysis of influence exerted on it by the design parameters of the undercarriages. Further research may cover at least three
areas. The first is associated with the choice of optimal design parameters of the undercarriages based on traditional approaches. The second is research into prospective chassis designs, based on the so-called controlled motion of the wheelsets. The third is to study the influence of permissible deviations in parameters of the rolling stock and the track on motion resistance. These include, first of all, skewes and attacks of wheelsets in the trucks, deviation in the wheelsets’ wheel diameters, etc.

The first two of the indicated areas are predetermined by the need for changes, sometimes essential, in the designs of wheelsets and trucks. This can turn out to be quite problematic given the economic efficiency of modernization. The third approach appears to be more promising as it is confined to technological requirements for maintaining the rolling stock.

### 7. Conclusions

1. Based on the theory of closed power circuits and the principle of kinetostatics, we performed modeling of directing the undercarriage by a rail track. The research was conducted using the four-axle freight semi-wagon, model 12-9046, with two-axle trucks 18–100, as an example. It was established that because of the parasitic slippage in the nodes of closed power circuits in the system of directing the wheelsets by a rail track, there occurs additional motion resistance. In this case, the contact between wheels and rails are the friction resolving nodal points of the circuits. Given the nature of this resistance, we proposed to term it the kinematic resistance to motion.

2. The mathematical model considered a spatial distribution of reactions in the contacts between wheels and rails. Special attention was paid to model accurately the forces of adhesion in the main and ridge contacts. It is suggested that the kinematic motion resistance be defined as a longitudinal force, applied to the track’s pivot in order to balance all external forces.

3. We obtained estimated dependences of specific kinematic resistance to motion on the motion velocity, base of the truck, radius of the curve, clearances of the wheelset in a rail track, and elevation of the outer rail.

### References


