Досліджено двочастотні режими руху одномасної вібромашини з прямолінійним поступальним рухом платформи і віброзбудником у вигляді кульового, роликового або маятникового автобалансира. Методом малого параметра наближено знайдені частоти, на яких можуть застрявати вантажі і відповідні закони двочастотних вібрацій платформи. Оцінені величини неврахованих в законах складових

Ключові слова: інерційний віброзбудник, двочастотні вібрації, резонансна вібромашина, автобалансир, одномасна вібромашина, ефект Зомерфельда

Исследованы двухчастотные режимы движения одномассной вибромашины с прямолинейным поступательным движением платформы и вибровозбудителем в виде шарового, роликового или маятникового автобалансира. Методом малого параметра приближенно найдены частоты, на которых могут застревать грузы и соответствующие законы двухчастотных вибраций платформы. Оценены величины неучтенных в законах составляющих

Ключевые слова: инерционный вибровозбудитель, двухчастотные вибрации, резонансная вибромашина, автобалансир, одномассная вибромашина, эффект Зоммерфельда

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SEARCH FOR TWO-FREQUENCY MOTION MODES OF SINGLE-MASS VIBRATORY MACHINE WITH VIBRATION EXCITER IN THE FORM OF PASSIVE AUTO-BALANCER

V. Yatsun
PhD, Associate Professor
Department of Road Cars and Building**
E-mail: yvkr@i.ua

G. Filimonikhin
Doctor of Technical Sciences,
Professor, Head of Department*
E-mail: filimonikhin@ukr.net

K. Dumenko
Doctor of Technical Sciences, Associate Professor
Department of Operation and Repair of Machines**
E-mail: dumenkokm@gmail.com

A. Nevdakha
PhD*
E-mail: aunevdaha@ukr.net

*Department of Machine Parts and Applied Mechanics**
**Central Ukrainian National Technical University
Universytetskyi ave., 8, Kropivnitskiy, Ukraine, 25006

1. Introduction

The most promising among vibratory machines, such as screeners, vibratory tables, conveyers and mills, etc., are the multi-frequency, resonance and multi-frequency-resonance machines.

Multi-frequency vibratory machines have better performance [1], resonance vibratory machines are the most energy efficient [2], and multi-frequency-resonance vibratory machines combine advantages of both multi-frequency and resonance vibratory machines [3].

The simplest way to excite resonance dual-frequency vibrations employs a ball, a roller, or a pendulum auto-balancer as a vibration exciter [4].

Up to now, the applicability of the new method for exciting dual-frequency vibrations in single-mass vibratory machines with translational rectilinear motion of the vibratory platform has not been theoretically studied.

2. Literature review and problem statement

In [4], it was proposed to excite dual-frequency resonance vibrations of the platform by a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer. To do this, a special mode of motion of pendulum [5], balls or rollers [6] is used. The mode occurs at small forces of resistance to loads' motion relative to the auto-balancer body. Under this mode, loads get together, cannot catch up with the shaft, onto which the auto-balancer is mounted, and get stuck at resonance frequency of the platform oscillations. Because loads get stuck, slow resonance oscillations of the platform are excited. That is why the new method is based on the Sommerfeld effect [7]. It is also proposed to put the unbalanced mass on the auto-balancer body. The unbalanced mass rotates synchronously with the rotor (auto-balancer body). In this way, rapid oscillations of the platform are excited. Parameters of dual-frequency vibrations alter by changing the rotor speed, the unbalanced mass, and the total mass of loads.

A vibration exciter in the form of a passive auto-balancer is supposed to be applicable for single-, two-, and three-mass vibratory machines with different kinematics of the platform motion.

The feasibility of the new method was examined for a screener with rectilinear translational motion of the box using 3D modeling [8] and field experiment [9]. It is relevant to explore analytically the workability of the new method for a single-mass vibratory machine.

In [10], authors developed generalized models of single-, two-, and three-mass vibratory machines with translational motion of vibratory platforms and a vibration exciter in the form of a ball, a roller or a pendulum auto-balancer. Differential equations for the motion of vibratory machines were derived.

Analytically, the effect of getting stuck was examined:

[11] – for a pendulum, mounted onto the shaft of a low-power electric motor installed on a vibratory platform;

[12] – for a pendulum, mounted onto the shaft of an electric motor whose nominal rotation speed is slightly larger than the resonance frequency of a vibratory platform;

[13] – for the wind wheel with an unbalanced mass, mounted on the vibratory platform;

[14] – for two pendulums, mounted onto two shafts of two low-power electric motors, installed on a vibratory platform;

[15] – for two balls within a spatial model of static balancing of the rotor by a ball auto-balancer;

[16] – for two balls within a flat model of the rotor with a two-ball auto-balancer;

[17] – for pendulums in the auto-balancer, mounted on the flexible rotor for its balancing in one plane of correction;

[18] – for pendulums in two auto-balancers, mounted on the rotor for its dynamic balancing (in two planes of correction).

Stability of motions, corresponding to the effect of getting stuck, was investigated: in [11–16], using a method of synchronization of dynamical systems [11]; in [17, 18], applying a method of separation of motions.

In the course of studies, carried out in [4–18], it was found that balls, pendulums, etc. get stuck on one of the natural frequencies of a rotor or a vibratory platform.

The method of synchronization of dynamic systems and the method of separation of motions are the methods of a small parameter (perturbation). The results obtained using them can become inapplicable if there is violation of the ratios of smallness between parameters for which they were obtained. Typically, these methods simultaneously solve the following tasks: a search for synchronous motions; studying their stability.

3. The aim and objectives of the study

The purpose of present research is to explore dual-frequency modes of motion of a vibratory platform of a single-mass vibratory machine with translational rectilinear motion of the vibratory platform, excited by a passive auto-balancer.

To achieve this purpose, the following problems must be solved:

– under condition of loads getting stuck in the auto-balancer, to find an approximate solution to differential equations of the motion of a vibratory machine and to estimate the magnitudes of unconsidered (rejected) components of the solution;

– to find frequencies at which loads get stuck depending on the rotor speed.

4. Research methods

We use differential equations of motion for a single-mass vibratory machine with translational rectilinear motion of the vibratory platform and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer [10].

To search for the approximate solution to the system of differential equations and frequencies of the loads getting stuck, we employ methods of perturbations and the elements of a theory of nonlinear oscillations [19].

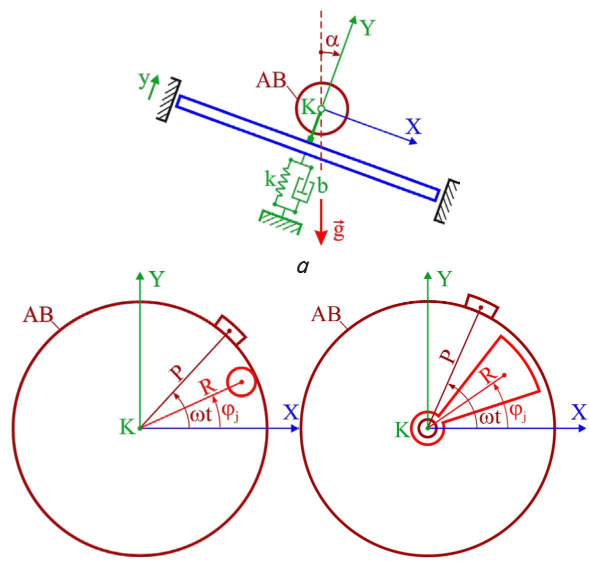
Synchronous modes of motion are sought for at different ratios between parameters for the cases that are relevant for practice:

– when forces of external and internal resistance are small;

– mass of the loads is much smaller than the platform mass.

4. 1. Description of a generalized model of the vibratory machine [10]

The vibratory machine includes (Fig. 1) a platform, which has mass M and a vibration exciter in the form of a ball, a roller (Fig. 1, *b*) or a pendulum (Fig. 1, *c*) auto-balancer. The platform can move only in a rectilinear translational way executed with two fixed guides. Direction of the platform motion forms angle α with a vertical. The platform is based on an elastic-viscous support with a rigidity coefficient k and coefficient of viscosity b. Position of the platform is determined by coordinate y, equal to zero in the position of the static equilibrium of the platform.



b *c*

Fig. 1. Model of a single-mass vibratory machine with the rectilinear translational motion of vibratory platform: *a* – motion kinematics of platform; *b* – motion kinematics of unbalanced mass, a ball or a roller; *c* – motion kinematics of unbalanced mass and pendulum

An auto-balancer body revolves around a shaft – point K with constant angular velocity ω.

The point unbalanced mass μ is rigidly attached to the auto-balancer body. It is located at distance P from point K. Two mutually perpendicular axes X, Y originate at point K and form the right coordinate system. Axis X is parallel to the platform, and axis Y is parallel to the direction of the platform motion. The position of the unbalanced mass relative to the body is determined by angle ωt, where t is the time. The angle is measured from axis X to the line segment that originates at point K and ends at the unbalanced mass.

The auto-balancer consists of N identical loads. The mass of one load is m. The center of load's masses can move around a circle with radius R, centered at point K (Fig. 1, *b, c*). Position of load number j relative to the body is determined by angle φ_j, $/ j=\overline{1,N} /$. The angle is measured from axis X to the line segment that originates at point K and ends at the center of masses of load number j. The motion of the load relative to the auto-balancer body is prevented by the force of viscous resistance that has module

$$F_j = b_W v_j^{(r)} = b_W R\,|\,\phi_j' - \omega\,|, \quad /\,j=\overline{1,N}\,/,$$

where b_W is the coefficient of force of viscous resistance, $v_j^{(r)} = R\,|\,\phi_j' - \omega\,|$ is the module of velocity of the center of mases of load number j relative to the auto-balancer body, with a bar by the magnitude denoting a derivative from time t.

4. 2. Differential equations of motion of a single-mass vibratory machine [10]

Differential equations of the platform motion

$$M_\Sigma y'' + by' + ky + S_y'' = S_d \omega^2 \sin \omega t, \tag{1}$$

where $M_\Sigma = M + Nm + \mu$ is the mass of the whole system,

$$S_x = mR\sum_{j=1}^{N}\cos\phi_j, \quad S_y = mR\sum_{j=1}^{N}\sin\phi_j, \quad S_d = \mu P \tag{2}$$

respectively, projections of the total unbalance from loads on axes X, Y and the unbalance from the unbalanced mass.

We note that this is a linear differential equation with constant coefficients relative to unknown y and S_y.

Differential equations for the load motion

$$\begin{aligned}&\kappa m R^2 \phi_j'' + b_W R^2(\phi_j' - \omega) + \\ &+ mgR\cos(\phi_j - \alpha) + mRy''\cos\phi_j = 0, \quad /\,j=\overline{1,N}\,/,\end{aligned} \tag{3}$$

where for a ball, a roller, and a pendulum, respectively,

$$\kappa = \frac{7}{5}, \quad \kappa = \frac{3}{2}, \quad \kappa = 1 + J_C/(mR^2) \tag{4}$$

and J_C is the principal central axial moment of inertia of a pendulum. We note that for a mathematical pendulum $J_C = 0$, $\kappa = 1$.

We note that these are non-linear equations.

We note that the form of differential equations of the motion of system (1) and (3) does not depend on the auto-balancer type.

In the research that follows, the impact of gravity forces is not taken into consideration.

5. Research results

5. 1. Reducing motion equations to dimensionless form

We shall introduce dimensionless variables and time

$$v = y/\tilde{y}, \quad s_x = S_x/\tilde{s}, \quad s_y = S_y/\tilde{s}, \quad \tau = \tilde{\omega} t, \tag{5}$$

where $\tilde{y}$, $\tilde{s}$, $\tilde{\omega}$ are the characteristic scales that will be chosen later.

Then,

$$\frac{d\,\cdot}{dt} = \frac{d\,\cdot}{d\tau}\frac{d\tau}{dt} = \tilde{\omega}\frac{d\,\cdot}{d\tau}, \quad \frac{d^2\,\cdot}{dt^2} = \tilde{\omega}^2\frac{d^2\,\cdot}{d\tau^2}, \tag{6}$$

and equations of motion (1) and (3) will take the form

$$M_\Sigma \tilde{\omega}^2 \tilde{y}\ddot{v} + b\tilde{\omega}\tilde{y}\dot{v} + k\tilde{y}v + \tilde{\omega}^2\tilde{s}\ddot{s}_y = S_d\omega^2 \sin\frac{\omega}{\tilde{\omega}}\tau,$$

$$\kappa m R^2 \tilde{\omega}^2 \ddot{\phi}_j + b_W R^2(\tilde{\omega}\dot{\phi}_j - \omega) + \tilde{\omega}^2 \tilde{y}\ddot{v} mR\cos\phi_j = 0, \tag{7}$$

where the point above the magnitude denotes a derivative from τ.

We shall divide the first equation in (7) by $M_\Sigma \tilde{\omega}^2 \tilde{y}$, and the second – by $\kappa m R^2 \tilde{\omega}^2$, and obtain

$$\ddot{v} + \frac{b}{M_\Sigma\tilde{\omega}}\dot{v} + \frac{k}{M_\Sigma\tilde{\omega}^2}v + \frac{\tilde{s}}{M_\Sigma\tilde{y}}\ddot{s}_y = \frac{S_d}{M_\Sigma\tilde{y}}\cdot\frac{\omega^2}{\tilde{\omega}^2}\sin\frac{\omega}{\tilde{\omega}}\tau,$$

$$\ddot{\phi}_j + \frac{b_W}{\kappa m\tilde{\omega}}\left(\dot{\phi}_j - \frac{\omega}{\tilde{\omega}}\right) + \frac{\tilde{y}}{\kappa R}\ddot{v}\cos\phi_j = 0. \tag{8}$$

We shall introduce new dimensionless parameters and characteristic scale:

$$2h = \frac{b}{M_\Sigma\tilde{\omega}}, \quad n = \frac{\omega}{\tilde{\omega}}, \quad \tilde{y} = \frac{\tilde{s}}{M_\Sigma}, \quad \varepsilon = \frac{\tilde{y}}{\kappa R} = \frac{\tilde{s}}{\kappa R M_\Sigma},$$

$$\varepsilon\beta = \frac{b_W}{\kappa m\tilde{\omega}}, \quad \beta = \frac{b_W}{\varepsilon\kappa m\tilde{\omega}} = \frac{b_W R M_\Sigma}{\tilde{s} m\tilde{\omega}},$$

$$\tilde{\omega} = \sqrt{\frac{k}{M_\Sigma}}, \quad \delta = \frac{S_d}{\tilde{s}}. \tag{9}$$

Then equations (8) will take the form:

$$\ddot{v} + 2h\dot{v} + v + \ddot{s}_y = \delta n^2 \sin n\tau,$$

$$\ddot{\phi}_j + \varepsilon\beta(\dot{\phi}_j - n) + \varepsilon\ddot{v}\cos\phi_j = 0, \quad /\,j=\overline{1,N}\,/. \tag{10}$$

Let

$$\tilde{s} = NmR. \tag{11}$$

Then

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\phi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\phi_j, \tag{12}$$

$$\tilde{y} = \frac{NmR}{M_\Sigma}, \quad \varepsilon = \frac{Nm}{\kappa M_\Sigma},$$

$$\beta = \frac{b_W M_\Sigma}{Nm^2\tilde{\omega}}, \quad \delta = \frac{S_d}{NmR} = \frac{\mu P}{NmR}. \tag{13}$$

In this case, the form of equations (10) will be the same.

5. 2. Transformation of equations of the loads motion

We add the equations of loads motion from (10), we shall obtain

$$\sum\nolimits_{j=1}^{N}\ddot{\phi}_j + \varepsilon\beta\sum\nolimits_{j=1}^{N}(\dot{\phi}_j - n) + \varepsilon\ddot{v}\sum\nolimits_{j=1}^{N}\cos\phi_j = 0. \tag{14}$$

We shall introduce an average angle for consideration:

$$\phi = \frac{1}{N}\sum_{j=1}^{N}\phi_j. \tag{15}$$

Then, considering (12), (15) equation (14) will take the form

$$\ddot{\phi}+\varepsilon\beta(\dot{\phi}-n)+\varepsilon\ddot{v}s_x=0. \tag{16}$$

We shall introduce the system of equations

$$\ddot{v}+2h\dot{v}+v+\ddot{s}_y=\delta n^2\sin n\tau,$$

$$\ddot{\phi}+\varepsilon\beta(\dot{\phi}-n)+\varepsilon\ddot{v}s_x=0. \tag{17}$$

It is meant to search for dual-frequency modes of the platform motion.

5. 3. Dual-frequency motion mode in zero approximation

At $\varepsilon=0$, system (10) takes the form:

$$\ddot{v}+2h\dot{v}+v+\ddot{s}_y=\delta n^2\sin n\tau,\ \ \ddot{\phi}_j=0,\ \ /\,j=\overline{1,N}\,/. \tag{18}$$

From the last N equations, we obtain

$$\phi_j^{(0)}=\Omega_j\tau+\psi_j,\ \ \Omega_j,\psi_j-\text{const},\ \ /\,j=\overline{1,N}\,/. \tag{19}$$

Then

$$\phi^{(0)}=\tau\frac{1}{N}\sum_{j=1}^{N}\Omega_j+\frac{1}{N}\sum_{j=1}^{N}\psi_j=\Omega\tau+\psi. \tag{20}$$

Hence, we obtain

$$\Omega=\frac{1}{N}\sum_{j=1}^{N}\Omega_j,\ \ \ \psi=\frac{1}{N}\sum_{j=1}^{N}\psi_j. \tag{21}$$

Since the balls or rollers are on the same run track, then:

$$\Omega_j=\Omega,\ \ /\,j=\overline{1,N}/;$$

$$s_x=\frac{1}{N}\sum_{j=1}^{N}\cos\phi_j=\frac{1}{N}\sum_{j=1}^{N}\cos(\Omega\tau+\psi_j)=$$
$$=\frac{1}{N}\sum_{j=1}^{N}(\cos\Omega\tau\cos\psi_j-\sin\Omega\tau\sin\psi_j)=$$
$$=\frac{\cos\Omega\tau}{N}\sum_{j=1}^{N}\cos\psi_j-\frac{\sin\Omega\tau}{N}\sum_{j=1}^{N}\sin\psi_j;$$

$$s_y=\frac{1}{N}\sum_{j=1}^{N}\sin\phi_j=\frac{1}{N}\sum_{j=1}^{N}\sin(\Omega\tau+\psi_j)=$$
$$=\frac{1}{N}\sum_{j=1}^{N}(\sin\Omega\tau\cos\psi_j+\cos\Omega\tau\sin\psi_j)=$$
$$=\frac{\sin\Omega\tau}{N}\sum_{j=1}^{N}\cos\psi_j+\frac{\cos\Omega\tau}{N}\sum_{j=1}^{N}\sin\psi_j. \tag{22}$$

We shall demand that:

$$s_x=A\cos(\Omega\tau+\gamma_0)=A(\cos\Omega\tau\cos\gamma_0-\sin\Omega\tau\sin\gamma_0);$$

$$s_y=A\sin(\Omega\tau+\gamma_0)=A(\sin\Omega\tau\cos\gamma_0+\cos\Omega\tau\sin\gamma_0). \tag{23}$$

Then

$$A\cos\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\cos\psi_j,$$

$$A\sin\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\sin\psi_j,$$

$$A^2=\frac{1}{N^2}\left[\left(\sum_{j=1}^{N}\cos\psi_j\right)^2+\left(\sum_{j=1}^{N}\sin\psi_j\right)^2\right],$$

$$\operatorname{tg}\gamma_0=\sum_{j=1}^{N}\sin\psi_j\Big/\sum_{j=1}^{N}\cos\psi_j. \tag{24}$$

From (22), we find

$$\ddot{s}_y=-A\Omega^2\sin(\Omega\tau+\gamma).$$

Then the first equation in system (10) takes the form

$$\ddot{v}+2h\dot{v}+v=A\Omega^2\sin(\Omega\tau+\gamma_0)+\delta n^2\sin n\tau. \tag{25}$$

A partial solution to this equation, corresponding to the steady-state mode of motion, takes the form

$$v_0=\frac{A\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau+\gamma_0)-2h\Omega\cos(\Omega\tau+\gamma_0)\right]+$$
$$+\frac{\delta n^2}{(1-n^2)^2+4h^2n^2}\left[(1-n^2)\sin(n\tau)-2hn\cos(n\tau)\right]. \tag{26}$$

This is a dual-frequency mode of the platform motion, found at zero approximation ($\varepsilon=0$). In it, the value of constant parameter Ω is undefined.

5. 4. Dual-frequency motion mode in the first approximation

We search for the steady-state motion of system (17) in the first approximation. We assume that Ω=const, and γ is a slowly changing periodic function. Then

$$\phi=\Omega\tau+\gamma,\ \ \dot{\phi}=\Omega+\dot{\gamma},\ \ \ddot{\phi}=\ddot{\gamma},$$

$$\dot{s}_y=A(\Omega+\dot{\gamma})\cos(\Omega\tau+\gamma),$$

$$\ddot{s}_y=A\ddot{\gamma}\cos(\Omega\tau+\gamma)-A(\Omega+\dot{\gamma})^2\sin(\Omega\tau+\gamma) \tag{27}$$

and system of equations (17) takes the form

$$\ddot{v}+2h\dot{v}+v=A(\Omega+\dot{\gamma})^2\sin(\Omega\tau+\gamma)-$$
$$-A\ddot{\gamma}\cos(\Omega\tau+\gamma)+\delta n^2\sin(n\tau),$$

$$\ddot{\gamma}+\varepsilon\beta(\Omega+\dot{\gamma}-n)+\varepsilon\ddot{v}\cos(\Omega\tau+\gamma)=0. \tag{28}$$

We search for the first approximation of solution to system (28) in the form (truncated series)

$$v=v_0+\varepsilon v_1,\ \ \gamma=\gamma_0+\varepsilon\gamma_1. \tag{29}$$

Substituting (29) in (28), we shall obtain

$$\ddot{v}_0+\varepsilon\ddot{v}_1+2h(\dot{v}_0+\varepsilon\dot{v}_1)+v_0+\varepsilon v_1=$$
$$=A(\Omega+\varepsilon\dot{\gamma}_1)^2\sin(\Omega\tau+\gamma_0+\varepsilon\gamma_1)-$$
$$-A\varepsilon\ddot{\gamma}_1\cos(\Omega\tau+\gamma_0+\varepsilon\gamma_1)+\delta n^2\sin(n\tau),$$

$$\varepsilon\ddot{\gamma}_1+\varepsilon\beta(\Omega+\varepsilon\dot{\gamma}_1-n)+$$
$$+\varepsilon(\ddot{v}_0+\varepsilon\ddot{v}_1)\cos(\Omega\tau+\gamma_0+\varepsilon\gamma_1)=0. \tag{30}$$

Expand into series of sin and cos:

$$\sin(\Omega\tau+\gamma_0+\varepsilon\gamma_1)=$$
$$=\sin(\Omega\tau+\gamma_0)+\varepsilon\gamma_1\cos(\Omega\tau+\gamma_0),$$

$$\cos(\Omega\tau+\gamma_0+\varepsilon\gamma_1)=$$
$$=\cos(\Omega\tau+\gamma_0)-\varepsilon\gamma_1\sin(\Omega\tau+\gamma_0). \quad (31)$$

Substituting (31) in (30), we shall obtain

$$\ddot{v}_0+\varepsilon\ddot{v}_1+2h(\dot{v}_0+\varepsilon\dot{v}_1)+v_0+\varepsilon v_1=$$
$$=A(\Omega+\varepsilon\dot{\gamma}_1)^2[\sin(\Omega\tau+\gamma_0)+\varepsilon\gamma_1\cos(\Omega\tau+\gamma_0)]-$$
$$-A\varepsilon\ddot{\gamma}_1[\cos(\Omega\tau+\gamma_0)-\varepsilon\gamma_1\sin(\Omega\tau+\gamma_0)]+\delta n^2\sin(n\tau),$$

$$\varepsilon\ddot{\gamma}_1+\varepsilon\beta\left(\Omega+\varepsilon\dot{\gamma}_1-n\right)+$$
$$+\varepsilon(\ddot{v}_0+\varepsilon\ddot{v}_1)[\cos(\Omega\tau+\gamma_0)-\varepsilon\gamma_1\sin(\Omega\tau+\gamma_0)]=0. \quad (32)$$

Collecting in (32) components at the same powers ε and equating them to zero, we shall obtain:

$$\varepsilon^0:\ \ \ddot{v}_0+2h\dot{v}_0+v_0=$$
$$=A\Omega^2\sin(\Omega\tau+\gamma_0)+\delta n^2\sin(n\tau),$$

$$\varepsilon^1:\ \ \ddot{\gamma}_1+\beta\left(\Omega-n\right)+\ddot{v}_0\cos(\Omega\tau+\gamma_0)=0,$$

$$\varepsilon^1:\ \ \ddot{v}_1+2h\dot{v}_1+v_1=$$
$$=2A\Omega\dot{\gamma}_1\sin(\Omega\tau+\gamma_0)+A\Omega^2\gamma_1\cos(\Omega\tau+\gamma_0)-$$
$$-A\ddot{\gamma}_1\cos(\Omega\tau+\gamma_0)=$$
$$=-A\frac{d^2}{d\tau^2}[\gamma_1\cos(\Omega\tau+\gamma_0)]. \quad (33)$$

In zero approximation, v_0 is determined from the first equation in system (33). It was found previously and takes the form (26). We shall find the second derivative

$$\ddot{v}_0=-\frac{A\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau+\gamma_0)-2h\Omega\cos(\Omega\tau+\gamma_0)\right]-$$
$$-\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\left[(1-n^2)\sin(n\tau)-2hn\cos(n\tau)\right]. \quad (34)$$

Substituting it in the second equation in (33), we shall obtain

$$\ddot{\gamma}_1+\beta\left(\Omega-n\right)-\frac{A\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times[(1-\Omega^2)\sin(\Omega\tau+\gamma_0)\cos(\Omega\tau+\gamma_0)-$$
$$-2h\Omega\cos^2(\Omega\tau+\gamma_0)]-\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\times$$
$$\times[(1-n^2)\sin n\tau\cos(\Omega\tau+\gamma_0)-2hn\cos n\tau\cos(\Omega\tau+\gamma_0)]=0.$$

We shall transform this equation:

$$\ddot{\gamma}_1=-\beta\left(\Omega-n\right)+\frac{1}{2}\cdot\frac{A\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left\langle(1-\Omega^2)\sin[2(\Omega t+\gamma_0)]-2h\Omega\{1+\cos[2(\Omega t+\gamma_0)]\}\right\rangle+$$
$$+\frac{1}{2}\cdot\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\left\langle(1-n^2)\{\sin[(n+\Omega)\tau+\gamma_0]+\right.$$
$$+\sin[(n-\Omega)\tau-\gamma_0]\}-2hn\{\cos[(n+\Omega)\tau+\gamma_0]+$$
$$\left.+\cos[(n-\Omega)\tau-\gamma_0]\}\right\rangle=0. \quad (35)$$

A condition for the existence of periodic solution:

$$-\beta\left(n-\Omega\right)+\frac{Ah\Omega^5}{(1-\Omega^2)^2+4h^2\Omega^2}=0.$$

This condition is equivalent to the following

$$P(\Omega)=\chi\Omega^5-\left(n-\Omega\right)[(1-\Omega^2)^2+4h^2\Omega^2]=$$
$$=a_0\Omega^5+a_1\Omega^4+a_2\Omega^3+a_3\Omega^2+a_4\Omega+a_5=0, \quad (36)$$

where

$$\chi=Ah/\beta,\ a_0=1+\chi,\ a_1=-n,$$
$$a_2=-2(1-2h^2),\ a_3=2n(1-2h^2),\ a_4=1,\ a_5=-n. \quad (37)$$

From (36), frequencies of the loads that get stuck are determined. This condition will be explored below.

When condition (36) is satisfied, we obtain from (35) the following equation in order to search for γ_1:

$$\ddot{\gamma}_1=\frac{1}{2}\cdot\frac{A\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\{(1-\Omega^2)\sin[2(\Omega t+\gamma_0)]-$$
$$-2h\Omega\cos[2(\Omega t+\gamma_0)]\}+\frac{1}{2}\cdot\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\times$$
$$\times\left\langle(1-n^2)\{\sin[(n+\Omega)\tau+\gamma_0]+\sin[(n-\Omega)\tau-\gamma_0]\}-\right.$$
$$\left.-2hn\{\cos[(n+\Omega)\tau+\gamma_0]+\cos[(n-\Omega)\tau-\gamma_0]\}\right\rangle=0.$$

Hence, we shall find a periodic component

$$\gamma_1=-\frac{1}{8}\cdot\frac{A\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\{(1-\Omega^2)\sin[2(\Omega t+\gamma_0)]-2h\Omega\cos[2(\Omega t+\gamma_0)]\}-$$
$$-\frac{1}{2}\cdot\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\times$$
$$\times\left\langle(1-n^2)\left\{\frac{\sin[(n+\Omega)\tau+\gamma_0]}{(n+\Omega)^2}+\frac{\sin[(n-\Omega)\tau-\gamma_0]}{(n-\Omega)^2}\right\}-\right.$$
$$\left.-2hn\left\{\frac{\cos[(n+\Omega)\tau+\gamma_0]}{(n+\Omega)^2}+\frac{\cos[(n-\Omega)\tau-\gamma_0]}{(n-\Omega)^2}\right\}\right\rangle.$$

Or, after transformations

$$\gamma_1=-\frac{1}{8}\cdot\frac{A\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\{(1-\Omega^2)\sin[2(\Omega t+\gamma_0)]-2h\Omega\cos[2(\Omega t+\gamma_0)]\}-$$
$$-\frac{1}{2}\cdot\frac{\delta n^4}{(1-n^2)^2+4h^2n^2}\times$$
$$\times\left\langle\frac{1}{(n+\Omega)^2}\{(1-n^2)\sin[(n+\Omega)\tau+\gamma_0]-2hn\sin[(n+\Omega)\tau+\gamma_0]\}+\right.$$
$$\left.+\frac{1}{(n-\Omega)^2}\{(1-n^2)\sin[(n-\Omega)\tau-\gamma_0]-2hn\sin[(n-\Omega)\tau-\gamma_0]\}\right\rangle. \quad (38)$$

From the last equation in (33), it is possible to find v_1. Correction for v_0 will be of order ε. For actual vibratory machines, ε<50 and that is why correction will not exceed 2 % of the found dual-frequency mode of motion. That is why this correction is not determined below.

Estimation of the magnitudes of discarded (unconsidered) components shows that, despite a strong asymmetry of supports, the platform performs almost perfect dual-frequency oscillations.

5. 5. Search for the frequencies at which loads can get stuck

We shall find approximately the roots of polynomial (36).

We note that

$P(0)=-n<0$, $P(n)=\chi n^5>0$, $P(1)=\chi-4h^2(n-1)>0$

and

$\forall\Omega\leq 0$ $P(\Omega)<0$, $\forall\Omega\geq n$ $P(\Omega)>0$.

Hence, it follows that:

all real roots of polynomial (36) are located in the open interval $(0, n)$;

– $\forall n>0$ there exists at least one real positive root $\Omega_1\in(0, n)$ is the frequency at which loads get stuck.

It follows from the Descartes theorem (Descartes rules of signs) that polynomial (36) may have:

– $\forall h<1/\sqrt{2}$ – 1 or 3 real roots;

– $\forall h>1/\sqrt{2}$ – 1, 3 or 5 real roots.

Let us find approximately the roots of polynomial (36) depending on rotor speed n. To do this, we shall expand the roots by powers of small parameters and leave only positive real roots.

1. In the case of small rotor speeds $n<<1$ (n~ε), polynomial (36) has the only real root, close to n:

$$\Omega_1\approx n(1-\chi n^4). \tag{39}$$

2. In the case when the rotor rotates rapidly $n>>1$ (n~$1/\varepsilon$), polynomial (36) has the only real root, close to n:

$$\Omega_1\approx\frac{n}{1+\chi}-\frac{2\chi(1-2h^2)}{n}. \tag{40}$$

3. Consider the case when the rotor speeds are equivalent to 1 (n~1). In this case, we additionally assume the smallness of parameters h, χ (smallness of the force of viscous friction in supports, smallness of the ratio of loads mass to the mass of the system, smallness of the force of viscous resistance to loads motion, etc.).

3. 1. In the case when χ~ε, h~ε and n~1 (angular velocity of rotor rotation is equivalent to unity), polynomial (36) has three real roots

$$\Omega_{1/2}\approx 1\mp\frac{1}{2}\sqrt{\frac{\chi}{n-1}+\frac{\chi(4n-3)}{8(n-1)^2}},$$

$$\Omega_3\approx n-\frac{\chi n^5}{(n^2-1)^2}. \tag{41}$$

It is evident that real $\Omega_{1/2}$ exist only at the above-resonance of the rotor speeds (n>1) and expansions are applicable at some distance from resonance frequency $|n-1|\sim 1$.

Given the results of point 1 and point 2, it is necessary to determine at which characteristic rotor speed the two frequencies of the loads getting stuck occur and at which speed the two frequencies disappear.

3. 2. In the case when $|n-1|\sim\sqrt[3]{\varepsilon},\ \chi,\ h\sim\varepsilon$, the rotor rotates at about-resonance velocity, polynomial (36) has one or three real roots, equivalent to 1:

$$\Omega_i\approx 1+\sqrt[3]{\varepsilon}\tilde{\Omega}_i,\ \ /i=1,2,3/,$$

$$\Omega_i\approx 1+\sqrt[3]{\varepsilon}\tilde{\Omega}_i,\ \ /i=1,2,3/,\ \ n=1+\sqrt[3]{\varepsilon}\nu, \tag{42}$$

where ν is the parameter, equivalent to 1, and $\tilde{\Omega}$ are the roots of cubic equation.

$$f(\tilde{\Omega})=c_0\tilde{\Omega}^3+c_1\tilde{\Omega}^2+c_2\tilde{\Omega}+c_3=0,$$

$$c_0=1,\ \ c_1=-\nu,\ \ c_2=0,\ \ c_3=\chi/4. \tag{43}$$

Cubic equation (43) will have three real roots when the following condition is satisfied [6, 19]

$$\Delta=-c_1^2c_2^2+4c_1^3c_3+27c_0^2c_3^2-18c_0c_1c_2c_3+4c_0c_2^3<0.$$

Substituting coefficients from (43) in it, we shall obtain

$$\Delta=-\frac{\chi}{16}(16\nu^3-27\chi)<0.$$

This condition will be satisfied if

$$\nu>\frac{3}{4}\sqrt[3]{4\chi},\ \left(n>1+\frac{3}{4}\sqrt[3]{4\chi}\right).$$

Let us introduce for consideration the first characteristic rotor speed

$$\tilde{n}_1\approx 1+\frac{3}{4}\sqrt[3]{4\chi}. \tag{44}$$

Then if $n<\tilde{n}_1$, the platform has one dual-frequency mode of motion, and if $n>\tilde{n}_1$, it has three modes of motion.

If $n=\tilde{n}_1$, the roots of cubic equation (43) are determined by equalities

$$\tilde{\Omega}_1=-\frac{1}{4}\sqrt[3]{4\chi},\ \ \tilde{\Omega}_{2,3}=\frac{1}{2}\sqrt[3]{4\chi}.$$

Then, taking (42) into consideration, frequencies of the loads getting stuck are approximately determined from equalities

$$\Omega_1\approx 1-\frac{1}{4}\sqrt[3]{4\chi},\ \ \Omega_{2,3}\approx 1+\frac{1}{2}\sqrt[3]{4\chi}. \tag{45}$$

Given the form of cubic equation (43), we conclude that for ν~1 it always has a negative root and when ν increases, two more positive roots appear. That is why at any ν~1, the platform always has one dual-frequency motion mode with frequency of the loads getting stuck less than 1. With increasing angular velocity, there appear two more dual-frequency motion modes with frequencies of the loads getting stuck larger than 1.

We shall replace parameter ν and variable $\tilde{\Omega}$ in (43):

$$\nu=\frac{3}{4}\sqrt[3]{4\chi}(1+w),\ \ \tilde{\Omega}=\frac{z}{2}\sqrt[3]{4\chi}. \tag{46}$$

Then cubic equation in (45), with an accuracy to a constant multiplier, takes the form

$$\tilde{f}(z)=2z^3-3z^2(1+w)+1=0. \tag{47}$$

Its roots have the following expansion for w:

$$z_1 \approx -\frac{1}{2}(1-w/3), \quad z_{2/3} = 1 \mp \sqrt{w} + \frac{2}{3}w.$$

Finally, in the parametric form, we find approximately the following frequencies of the loads getting stuck

$$\Omega_1 \approx 1 - \frac{1}{4}\sqrt[3]{4\chi}(1-w/3),$$

$$\Omega_{2/3} \approx 1 + \frac{1}{2}\sqrt[3]{4\chi}(1 \mp \sqrt{w} + 2w/3),$$

$$n = 1 + \frac{3}{4}\sqrt[3]{4\chi}(1+w), \qquad (48)$$

where w is the parameter. For this purpose, to obtain solution in the obvious form, it is necessary to substitute in (48)

$$w = \frac{4}{3}\frac{(n-1)}{\sqrt[3]{4\chi}} - 1. \qquad (49)$$

3. 3. We shall find characteristic velocity above which frequencies of the loads getting stuck will be above-resonance (exceeding 1).

Condition for the existence of below-resonance frequency of the loads getting stuck is $P(1)>0$. We shall find from it

$$\chi > 4h^2(n-1), \quad \frac{\chi}{4h^2} > n-1, \quad n < 1 + \frac{\chi}{4h^2}.$$

Let us introduce for consideration the second characteristic rotor speed

$$\tilde{n}_2 = 1 + \frac{\chi}{4h^2} = 1 + \frac{A}{4\beta h}. \qquad (50)$$

At $n < \tilde{n}_2$, at least one root of polynomial (36) will be less than 1 and loads will get stuck at the below-resonance rotor speed.

Let us consider limit case $n \approx \tilde{n}_2$. We introduce a new variable and a parameter.

$$\Omega = 1 + w, \quad n = \tilde{n}_2 + \nu. \qquad (51)$$

Then polynomial (36) will be transformed to the form

$$P(w) = (1+\chi)w^5 - \left[\frac{\chi}{4h^2}(1-20h^2) + \nu - 4\right]w^4 -$$
$$-\left[\frac{\chi}{h^2}(1-10h^2) - 4(1+h^2-\nu)\right]w^3 -$$
$$-\left[\frac{\chi}{h^2}(1-9h^2) + 4\nu(1+h^2) - 8h^2\right]w^2 +$$
$$+[3\chi + 4h^2(1-2\nu)]w - 4\nu h^2 = 0. \qquad (52)$$

From (52), for small ν, we find

$$w \approx 4\nu h^2 / [3\chi + 4h^2(1-2\nu)]. \qquad (53)$$

When speed n overpasses value $\tilde{n}_2$ (ν overpasses 0), parameter w overpasses 0 and root Ω overpasses 1. This case is not special because roots do not become multiple or complex.

Two characteristic speeds $\tilde{n}_1$, $\tilde{n}_2$ will exist under condition that $\tilde{n}_1 < \tilde{n}_2$. Using (44) and (50), we find the condition under which it is possible:

$$1 + \frac{3}{4}\sqrt[3]{4\chi} < \frac{\chi}{4h^2} + 1, \quad 0 < \frac{\chi}{4h^2} - \frac{3}{4}\sqrt[3]{4\chi},$$

$$\frac{\chi}{h^2\sqrt[3]{4\chi}} > 3, \quad \frac{\chi^3}{h^6\chi} > 4\cdot 27, \quad \frac{\chi^2}{h^6} > 4\cdot 27,$$

$$\frac{\chi}{h^3} > 6\sqrt{3} \quad \left(\frac{A}{\beta h^2} > 6\sqrt{3}\right). \qquad (54)$$

Considering the ratios of smallness between parameters, we conclude that this condition is satisfied if the loads create imbalance equivalent to 1 (A~1) and the forces of external and internal resistance are small (β, h<<1). These conditions are met in nearly all important practical cases. That is why, within a certain range of angular rotational speed of the rotor, the platform theoretically may have three dual-frequency modes of motion.

3. 4. In the case when the rotor rotates rapidly n~$1/\varepsilon^2$, and the forces of resistance to loads motion are small h~ε, polynomial (36) has one or three real roots:

$$\Omega_{1/2} \approx 1 \mp \sqrt{\frac{\chi - 4h^2 n}{4n}}, \quad \Omega_3 \approx \frac{n}{1+\chi} - \frac{2\chi}{n}, \qquad (55)$$

in this case, $\Omega_{1,2}$ are real when the following condition is satisfied

$$n < \tilde{n}_3, \ \tilde{n}_3 \approx \frac{\chi}{4h^2}. \qquad (56)$$

It is evident that $\tilde{n}_3 >> 1$, and $\tilde{n}_3 \approx \tilde{n}_2$.

To refine critical speed $\tilde{n}_3$, at exceeding of which the two first frequencies of the loads getting stuck $\Omega_{1,2}$ cease to exist, we shall search for it and the related critical root in the form

$$\tilde{n}_3 = \frac{\chi}{4h^2} + r_0 + r_1 h + \cdots,$$

$$\tilde{\Omega}_{1,2} = 1 + \omega_{3/2}(h)^{3/2} + \omega_2 h^2 + \cdots, \qquad (57)$$

where r_i, ω_i are the expansion coefficients. As a result, we obtain critical speed and critical roots, corresponding to it, in the form of

$$\tilde{n}_3 \approx \frac{\chi}{4h^2} + 1 + \frac{9}{16}\chi + \frac{3}{2}\left(1 + \frac{27}{32}\chi\right)h^2, \quad \Omega_{1,2} \approx 1 + \frac{3}{2}h^2. \qquad (58)$$

In the vicinity of critical speed

$$n \approx \frac{\chi}{4h^2} + 1 + \frac{9}{16}\chi + \sigma h^2,$$

$$\Omega_{1/2} \approx 1 + \frac{3}{2}h^2 \mp h^3\sqrt{\frac{96 + 81\chi - 64\sigma}{16\chi^2}}, \qquad (59)$$

where σ is the real parameter, equivalent to 1.

One can see that critical roots are somewhat larger than 1 and disappear at speed $\tilde{n}_3$, slightly exceeding $\tilde{n}_2$

4. In the case when χ<<1 (χ~ε, ε<<1) (the load's mass is much smaller than the rotor's mass and the forces of internal

resistance are finite), polynomial (36) has the only real root close to n:

$$\Omega_1 \approx n - \frac{\chi n^5}{(n^2-1)^2 + 4h^2n^2}. \qquad (60)$$

5. In the case when $\chi>>1$ (χ~$1/\varepsilon$, $\varepsilon<<1$) (small forces of external resistance β~ε), polynomial (36) has the only real root (at n~1) smaller than 1:

$$\Omega_1 \approx \sqrt[5]{n/\chi}. \qquad (61)$$

Results of the research conducted are given in Table 1.

Table 1

Frequencies of the loads getting stuck (Ω_i) for different rotor speeds (n)

No. of entry	Smallness ratios of parameters	Frequencies of the loads getting stuck – expansion of roots of polynomial (36)
1	n~ε	$\Omega_1 \approx n(1-\chi n^4)$
2	n~$1/\varepsilon$	$\Omega_1 \approx \frac{n}{1+\chi} - \frac{2\chi(1-2h^2)}{n}$
3. 1	$\lvert n-1\rvert$~1, χ~ε, h~ε	$\Omega_{1/2} \approx 1 \mp \frac{1}{2}\sqrt{\frac{\chi}{n-1}} + \frac{\chi(4n-3)}{8(n-1)^2}$, $\Omega_3 \approx n - \frac{\chi n^5}{(n^2-1)^2}$
3. 2, 3. 3	$n \approx \tilde{n}_1$: $(n-1) \sim \sqrt[3]{\varepsilon},\ \chi, h \sim \varepsilon$	$\Omega_1 \approx 1 - \frac{1}{4}\sqrt[3]{4\chi}(1-w/3)$, $\Omega_{2/3} \approx 1 + \frac{1}{2}\sqrt[3]{4\chi}(1 \mp \sqrt{w} + 2w/3)$, $w = \frac{4}{3}\frac{(n-1)}{\sqrt[3]{4\chi}} - 1$
3. 4	$n \approx \tilde{n}_2$: n~$1/\varepsilon^2$, h~ε	$n \approx \frac{\chi}{4h^2} + 1 + \frac{9}{16}\chi + \sigma h^2$, $\Omega_{1/2} \approx 1 + \frac{3}{2}h^2 \mp h^3\sqrt{\frac{96+81\chi-64\sigma}{16\chi^2}}$, $\Omega_3 \approx \frac{n}{1+\chi} - \frac{2\chi}{n}$
4	$\chi \sim \varepsilon,\ \varepsilon << 1$	$\Omega_1 \approx n - \frac{\chi n^5}{(n^2-1)^2 + 4h^2n^2}$
5	$\chi \sim 1/\varepsilon,\ \varepsilon << 1$	$\Omega_1 \approx \sqrt[5]{n/\chi}$

Table 1 might be used for approximate calculation of frequencies of the loads getting stuck depending on the ratios of smallness between parameters of the system.

6. Discussion of results of research into dual-frequency modes of motion of single-mass vibratory machines

Conducted theoretical study proves that a single-mass vibratory machine with rectilinear translational motion of the platform and a vibration exciter in the form of a passive auto-balancer has the steady-state motion modes, close to dual-frequency modes. At these motions, loads in the auto-balancer create constant imbalance, cannot catch up with the rotor and get stuck at a certain frequency. These loads operate as the first vibration exciter, exciting vibrations with frequency of the loads getting stuck. The second vibration exciter is formed by unbalanced mass on the auto-balancer body. The mass rotates at rotor rotation frequency and excites more rapid vibrations.

Despite a strong asymmetry of supports, the auto-balancer excites almost perfect dual-frequency vibrations. Deviations from the dual-frequency law are proportional to the ratio of loads' mass to the mass of the entire machine. That is why for real machines, they do not exceed 2 %.

In important, from the practical point of view, cases, in particular, when h, $\chi<<1$ (small forces of external and internal resistance, the loads' mass is much smaller than the platform's mass, etc.), there are three characteristic rotor speeds $\tilde{n}_1,\ \tilde{n}_2,\ \tilde{n}_3$. In this case, $1 < \tilde{n}_1 << \tilde{n}_2 < \tilde{n}_3 << n$ and:

– at the rotor speeds smaller than $\tilde{n}_1$ $(0 < n < \tilde{n}_1)$, there exists a single frequency of the loads getting stuck Ω_1, in this case $0<\Omega_1<1$;

– at the above-resonance speeds, exceeding $\tilde{n}_1$, but smaller than $\tilde{n}_2$ $(\tilde{n}_1 < n < \tilde{n}_2)$, there exist such three frequencies of the loads getting stuck $\Omega_{1,2,3}$, that $0<\Omega_1<1<\Omega_2<\Omega_3<n$;

– at the above-resonance speeds, exceeding $\tilde{n}_2$, but smaller than $\tilde{n}_3$ $(\tilde{n}_2 < n < \tilde{n}_3)$, there exist such three frequencies of the loads getting stuck $\Omega_{1,2,3}$, that $1<\Omega_1<\Omega_2<<\Omega_3<n$;

– at the above-resonance speeds, exceeding $\tilde{n}_3$ $(n > \tilde{n}_3)$, there is such single frequency of the loads getting stuck Ω_1, that $1<<\Omega_1<n$.

There is only one below-resonant frequency of loads' getting stuck Ω_1 ($0<\Omega_1<1$), in this case, only at speeds, smaller than $\tilde{n}_2$ $(n < \tilde{n}_2)$, but at any parameters of the system.

It should be noted that the studied differential equations of motion of the vibratory machine have solutions, corresponding to the onset of auto-balancing. However, these solutions have not been studied.

Of all theoretically possible steady-state modes of motion of the vibratory machine, only stable steady-state motions will be implemented in practice. That is why in the future we plan to explore stability of the found dual-frequency modes of motion and to conduct computational experiments.

7. Conclusions

1. A single-mass vibratory machine with rectilinear translational motion of the platform and a vibration exciter in the form of a passive auto-balancer has the steady-state motion modes, equal to dual-frequency modes. At these motions, loads in the auto-balancer create constant imbalance, cannot catch up with the rotor and get stuck at a certain frequency. In this way, loads operate as the first vibration exciter, exciting vibrations at frequency of the loads getting stuck. The second vibration exciter is formed by unbalanced mass on the auto-balancer body. The mass rotates at rotor speed and excites more rapid vibrations with this frequency.

Despite a strong asymmetry of supports, the auto-balancer excites almost perfect dual-frequency vibrations. Deviations from the dual-frequency law are proportional to the ratio of loads' mass to the mass of the entire machine. That is why for real machines they do not exceed 2 %.

2. When the forces of external and internal resistance are small, when the mass of loads is much smaller than the

mass of the platform, etc., there are three characteristic rotor speeds. These speeds are larger than the resonance frequency of platform oscillations. In this case:

– at rotor speeds smaller than the first characteristic speed, there is only one frequency of the loads getting stuck, in this case, it is smaller than the resonance velocity of platform oscillations;

– at the above-resonance rotor speeds, located between the first and the second characteristic speeds, there are three frequencies of the loads getting stuck, among which only one is below-resonance;

– at the above-resonance rotor speeds, located between the second and the third characteristic speed, there are three frequencies of the loads getting stuck, in this case, they are all above-resonance;

– at the above-resonance rotor speeds, exceeding the third characteristic speed, there exists only one frequency of the loads getting stuck, in addition, it is above-resonance and close to the rotor speed.

Only at the rotor speeds smaller than the second characteristic speed there is always one, and only one, below-resonance frequency of the loads getting stuck.

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