1. Introduction

Investing in innovative projects, for example, in the field of information technologies and cybersecurity is in many cases characterized by a high degree of uncertainty and risk. To improve efficiency and optimize procedures of project assessment and decision making, associated with investing, "intelligent" data analysis systems are now involved increasingly [1]. In addition, various decision support systems (DSS) have gained good reputation for solving similar problems [2]. Filling of DSS and their separate modules, directly responsible for analysis and solution of problems, is performed by introduction of the units that contain programmed algorithms into economic-mathematical models. However, few DSS enable optimization of procedures, associated with finding multi-variant strategies in mutual financial investment of projects [3, 4]. In this context, it is a relevant task to develop new economic-mathematical models for DSS, which allow adequate description of actual economic processes, caused by increasing competitiveness of various companies, corporations and states.

2. Literature review and problem statement

The problem of effective financial investment and control over this process is one of the most important in the financial sector [1]. However, studies very often are of purely economic nature [2] and do not take into account trends, associated with implementation of information technologies in the processes of control and decision-making [3]. Economic aspects of the processes of mutual investment are described in papers [4–6]. A shortcoming of these studies is the lack of relevant recommendations on formulation of strategies for mutual financial investment.

A separate direction of research in this area was represented by papers, dedicated to application of various expert systems [7–9] and decision support systems (DSS) [10, 11] for selection of rational investment strategies [12]. The approaches, developed by the authors [8, 9, 11], do not make it possible to find effective recommendations and investment management strategy while solving this problem [12, 13]. In articles [14, 15], the models of assessment of mutual investment processes were proposed. The authors do not take into
account the effect of changes of some factors influencing effectiveness of investment projects, in particular, the issue of strengthening or weakening of the investor's currency was not considered. Papers [16, 17] proposed the models of selection of strategies of investment processes, which are based on the theory of differential games. However, the authors did not develop the tools for solution of differential games for mutual investment problems. Article [18] considered the possibility of using the game theory in DSS for estimation of attractiveness of investment projects. Software implementation was not described by the authors, no specific recommendations for formulation of strategies for mutual financial investment were made either.

This fact necessitates development of new DSS models, which would give the opportunity to find optimal strategies of mutual financial investment. This may be achieved, in particular, through application of the methods of the theory of differential quality games with some terminal surfaces.

Thus, as analysis of research in this area showed, the problem of further development of models and relevant algorithms for decision support systems in problems of continuous mutual investment remains relevant. The latter is especially important for cases where it is necessary to take into account many parameters that describe the process of mutual investment.

3. The aim and objectives of the study

The aim of present research is to develop a model for DSS of continuous mutual investment, taking into account many parameters, which adequately describes the investment process. The special feature of the model is that such an investment process is described by bilinear differential equations with arbitrary coefficients with bilinear functions.

To accomplish the set goal, the following tasks had to be solved:

– to develop a model for finding investment control strategies for different ratios of parameters of the investment process;

– to implement the model in a high-level programming language in DSS module and conduct computational experiment.

4. The model of mutual financial investment for a decision support system

An investor from the country, where the United States dollar is the currency, having free capital, is trying to select the most preferable options for its investment. To do this, he chooses the counterparty, i.e., an object for investing his funds in a country, where the currency is euro. This object can be, for example, the economy of another country, or a corporation or, for example, information and communication systems, and so on. There is interaction of the investor and its counterparty. Entering this interaction, the investor and the counterparty seek to achieve their goals. The investor seeks to increase his capital, while the counterparty aims to improve his financial and economic indicators, one of which can be his capital. Subsequently, without decreasing generality, we will assume that the counterparty also seeks to increase its capital. However, the interests that do not coincide, non-optimal control and existence of uncertainty do not always make it possible to achieve interaction of both sides at the same time. If the investor faces such problem regularly, it is advisable to use DSS.

We will formalize the investment process in the assumption that the investor is the investment company, the counterparty is the financial corporation in another country.

We will give description of the “basic” process – the process of interaction between the investment company of one country and the corporation of another country. The investment company, having some free resources (its investment capital) increases them by \( n \) times (where \( n \) is rate of increase of the company’s resources) and then decides which of these resources will be invested in active operations. These operations include allocation of resources for investment projects of the corporation and payment of the debt the company has by that point of time. We will assume that the same things are done by the corporation with respect to this investment company. We will note that if the corporation does not invest its resources in the investment company, then, as it will follow from the stated below, that will be a particular case of the variant with investment of the corporation's capital in the investment company. Described above interactions between the company and the corporation will act under the following assumptions:

a) the investment company controls financial resources \( x \), estimated in dollars (USD);

b) the corporation controls financial resources \( y \), estimated in euros (EUR);

c) during interaction, the ratio of the dollar to the euro (dollar’s exchange rate) \( k_d \) remains constant.

If these assumptions are met, the interaction happens as follows.

First, the investor determines the share of resources, allocated for mutual active transactions with the counterparty. After the investment company and the counterparty have located for mutual active transactions with the counterparty, the investor is ready to implement the model in a high-level programming language in DSS module and conduct computational experiment.

The following system of differential equations is solved:

\[
\frac{dx(t)}{dt} = -x(t) + a_1(t)x(t) + \left[1-\beta_1(t)\right]u(t)u_1(t)x(t) + \left[1-\left((a_2(t)+r_2(t))\left(1-\beta_2(t)\right)\right)\right]v(t)u_2(t)y(t)/k_d; \tag{1}
\]

\[
\frac{dy(t)}{dt} = -y(t) + a_2(t)y(t) + \left[1-\beta_2(t)\right]u(t)u_2(t)y(t) + \left[1-\left((a_1(t)+r_1(t))\left(1-\beta_1(t)\right)\right)\right]u(t)u_1(t)x(t)/k_d. \tag{2}
\]

Thus, at the point of time \( t \) magnitude \( dx(t)/dt \) of the company (in dollars) will be equal to the sum of the following summands:

– magnitude \( a_1(t)x(t) \), magnitude of per cent \( a_1(t) \)\( x(t) \) for invested financial resources \( 1-\beta_1(t) \)\( u(t)u_1(t)x(t) \) of the company;

– magnitude \( r_1(t)\left(1-\beta_1(t)\right)u(t)u_1(t)x(t) \), characterizing the share of “returned” investment resource \( 1-\beta_1(t)\)\( x(t) \) of the company;

– magnitudes of “non-returned” assets (investment) \( \left[1-\beta_2(t)\right]u(t)u_2(t)y(t) \) of the corporation in dollars;

– magnitude \( \beta_2(t)k_d v(t)u(t)x(t) \), characterizing payment of the corporation’s debt to the company.

From this sum we deduce:

– magnitude of financial resource \( x(t) \);

– magnitude \( u(t)\beta_2(t)u_2(t)x(t) \), allocated to pay the debt of the company to the corporation by moment of time \( t \);
Control processes

- magnitude \( u(t) (1-\beta_1(t)) a_1(t)x(t) \), allocated for transactions of the company (investment) at the moment of time \( t \);
- magnitude \( a_2(t) (1-\beta_2(t)) k_d t v(t) a_2(t) y(t) \) is the amount of per cent for investment resources \((1-\beta_2(t)) k_d x(t) v(t) a_2(t) y(t) \) of the corporation.

Magnitude \( d y(t)/d t \) (in Euro) at the moment of time \( t \) will be equal to the sum of all the following summands:

- magnitude \( a_3(t) (1-\beta_3(t)) x(t) a_3(t) y(t) \), amount of per cent \( a_3(t) (1-\beta_3(t)) x(t) a_3(t) y(t) \) for invested financial resources \((1-\beta_3(t)) x(t) a_3(t) y(t) \) of the corporation;
- magnitude \( r_3(t) (1-\beta_3(t)) v(t) a_3(t) y(t) \), characterizes the share of "returned" investment resources \((1-\beta_3(t)) v(t) a_3(t) y(t) \) by the company to the corporation;
- magnitude of "non-returned" assets (investment) \((1-\beta_3(t)) x(t) a_3(t) y(t) \) to the company by the corporation.

From this sum we deduce:

- magnitude of financial resource \( y(t) \), magnitude \( r(t) (1-\beta_2(t)) a_2(t) y(t) \), allocated by the company to pay debts it has to the company by the moment of time \( t \);
- magnitude \((1-\beta_3(t)) k_d t v(t) a_3(t) y(t) \), allocated by the corporation for conducting transactions (investment) of the corporation at the moment of time \( t \);
- magnitude \( r_4(t) (1-\beta_3(t)) k_d t v(t) a_3(t) y(t) \) is the per cent payment for investment resources \((1-\beta_3(t)) k_d t v(t) a_3(t) y(t) \) of the company.

Interaction finishes when the conditions are met:

\[
(x(t), y(t)) \in M_0, \tag{3}
\]

\[
(x(t), y(t)) \in N_0, \tag{4}
\]

where \( M_0 = \{(x, y) \in \mathbb{R}_+^2, x > 0, y > 0 \} \), \( N_0 = \{(x, y) : (x, y) \in \mathbb{R}_+^2, x > 0, y > 0 \} \).

From an economic perspective, these terms are interpreted as follows. Expression (3) corresponds to the state of loss of investment resources (capital) of the corporation, and the investment company increased its capital by the magnitude of the capital of the corporation. Expression (4) corresponds to the loss of the capital of the investment company and the corporation increased its capital by the magnitude of the capital of the investment company. If neither the first, nor the second condition is met, interaction of the investment companies and the corporations continues.

Here arises the question about determining the time of probabilities of loss of capital (investment resources) based on information about primary resources (capital), currency exchange rate, rate of growth of resources of the investment company and the corporation, interest rates on allocated capital, levels of payable and receivable accounts. The answer lies in the area of the theory of differential quality games [4–7] for interacting objects. To find the preference regions, the differential quality game with two terminal surfaces is solved. The solution lies in determining of sets of preferences of the parties, as well as strategies (control influences) of the parties, by applying which it is possible to receive the outcomes, preferred by each side.

Under this approach, a set of preferences of one party is in its essence a set of loss of capital for the other party. Indeed, preservation of capital is the preferred outcome and its loss is an undesirable outcome for either party of such interaction. However, there may be cases where one of the interaction parties could act in the worst way in relation to the other party, which ultimately brought another party to a loss of capital. In this case, a set of original states of resources of the interaction parties with the property that there are strategies (control influences) of one party, leading the other party to the state of the loss of capital, can be called a set of loss of capital for the other side.

We will note that the initial interaction is not limited to model of the differential game. Similarly, it can be possible to simulate interaction, reflecting the operation of several investment companies, corporations; it is possible to take into account incompleteness of information that companies, corporations, etc. have. That is, it is possible to use the apparatus of differential games for group interaction and for interaction with incomplete information.

In the research we will limit ourselves to consideration of a simple variant of interaction, which allows making qualitative conclusions about the financial state of entities. In the decision the ratio of interaction parameters is taken into account, and possible moment of capital loss by entities of interaction is modeled.

Solution to the problem

For convenience of presentation, subsequently we will “identify” the investment company with player (I), and the corporation – with player (II). Interaction will be considered within the scheme of a positional differential game with complete information [4–7]. Within this scheme, interaction gives rise to two problems – from the point of view of player-ally 1 and from the point of view of player-ally 2. Because of symmetry, it is enough to consider only one of them, for example, from the point of view of player-ally 1. To do this, we will determine pure strategies of player-ally 1. Let \( T = [0, L] \) designate temporary segment, i.e. the set, characterizing the region of changing of temporary parameter; \( L \) will designate a positive real number.

Definition

A pure strategy of player-ally 1 is function \( u : T [0,1] \times [0,1] \rightarrow [0,1] \), putting to the state of information (position) \( t, (x(0), y(0)) \) value \( u(t, (x(0), y(0))) \), that satisfies \( 0 \leq u(t, (x(0), y(0))) \leq 1 \). The pure strategy of player-ally 1 is a function (rule), putting to the state of information at moment \( t \) value \( u(t, (x(0), y(0))) \) which determines the magnitude of resource (capital) of player 1, which he allocated for “investment” of player 2. In relation to awareness of the player-opponent (in the context of the positional play), no assumptions are made, which is equivalent to the fact that the player-opponent chooses his control influence \( u(t) \) based on any information. After determining of strategies in problem 1, it is necessary to determine a set of preference for player 1. Bearing in mind that for presentation of the proposed approach it is sufficient to give only a high-quality description, set of preference \( W_1 \) of player 1 will be presented as follows.

\( W_1 \) is the set of such initial resources \( (x(0), y(0)) \) of the players, which have the property: for these initial states, there exists the strategy of player 1, which for any implementations of strategies of player 2 at one point of time \( t \) “brings” the state of system \((x(t), y(t))\) to the state, in which condition (1) will be satisfied. In this case, player 2 does not have any strategy that can “lead” to the implementation of condition (2) at one of the previous moments of time. The strategy for player 1 that has the specified property is called optimal. Solution of problem 1 lies in finding a set of preference of player 1 and its optimal strategies. Similarly, the problem is stated from the point of view of player-ally 2.
to the symmetry of problem statement, it is sufficient to solve only problem 1, because problem 2 is solved similarly.

Let us assume that for any moment of time $t$ the following conditions are satisfied:

$$a_1(t)-a_2; a_2(t)-a_2; b_1(t)-b_1; b_2(t)-b_2; r_1(t)-r_1; r_2(t)-r_2.$$ 

Let us designate through $q_1$ and $q_2$ the following magnitudes:

$$q_1=1-\alpha_1-\alpha_2-\alpha_1-\alpha_2-\alpha_1-\alpha_2,$$

Four cases are possible:

a) $q_1 \geq 0$; $q_2 \leq 0$;

b) $q_1 < 0$; $q_2 > 0$;

c) $q_1 > 0$; $q_2 \leq 0$;

d) $q_1 \leq 0$; $q_2 > 0$.

In addition, there are various ratios of other interaction parameters, for instance, growth rates $a_1$, $a_2$ and other parameters.

Solution of problem 1 is found with the help of the tools of the theory of differential games with complete information [4–7], which allows finding the solution of the game at various ratios of game parameters. Let us present the solution of the game, i. e. set of preference $W_1$ and optimal strategies of player 1.

Let us consider case a). In this case, we will obtain:

$$W_1=\{(x(0), y(0)):\ (x,y)\in\text{int} \ R^2, y(0) < g^*(x(0)), (5)$$

where

$$g^* = \left[ q_1 \alpha_1 + \alpha_2 - q_1 \alpha_1 - \alpha_2 \right]/\left[2q_1 \alpha_2\right] + \sqrt{\left[\left(2q_1 \alpha_2\right)^2 + \left(q_1 \alpha_1\right)\right]/\left(q_2 \alpha_2\right)}.$$ 

where $u(x, y) = 1 - g^* \cdot x$, $x < f \cdot x(0)$, and is not determined, otherwise.

In cases b) and c) set $W_1$ is empty.

In case d) and $a_2 > a_1 + q_1$, we obtain

$$W_1=\{(x(0), y(0)): (x(0), y(0))\in\text{int} \ R^2, y(0) < f \cdot x(0)\},$$

and is not determined, otherwise.

As it was already noted, the problem from the position player-ally 2 is solved similarly. And regions of preference from the position player 2 are “adjacent” to the preference regions of player 1. These regions are divided by balance beams, which have the property that if couple of states $(x(0), y(0))$ belongs to the balance beam, the players have strategies that allow them to stay on the balance beam for all subsequent points of time. While solving the problems using the proposed game methods, there are balance beams in the space of variables $(x, y)$.

That is, if interaction starts from these states, the players have strategies that allow them to remain on the balance beam. This means that at assigned $(x(0), y(0))$, it is possible to find the ratio on the parameters of interaction. In this case, couple $(x(t), y(t))$ will be on the balance beam.

If initial states (resources) are not located on the balanced interaction beam, it is possible to try to modify communication parameters for initial resources to be on the balance beam. This will enable the parties to continue their interaction as long as you want.

It should be noted that there may be situations when interaction parameters changed. Then it is possible to carry out the stated procedure at new parameters and find the optimum strategies for interaction between the parties, i. e. the proposed scheme of control of interaction of the vertically integrated company and its counterparty is adaptive.

Comment 1. The “stronger” currency affects “an increase” in preference zones (comparison by inclusion of sets) and “a decrease” of investor’s risk zones from the economy with a “stronger” currency and vice versa. This means that an investor with a “weaker” currency should leave the areas of financial resources, which are subject to risk of losing capital due to “weakening” of currency of the investor-country.

Comment 2. Considered example of the simplest interaction allows making the following conclusion about the fact that in the space of initial resources, there are preference areas for players. Therefore, if resources are in the preference area of any player, it is disadvantageous for this player to avoid interaction with another player. Player 2 is supposed to be able to change the ratio of resources in the absence of interaction as a result of autonomous operation. For example, this is possible to implement by using the advantage of technology. And then, after starting interaction, one may gain the advantage in this interaction and “bring” the other player to loss of capital.

5. Computational experiment on the choice of strategy for mutual financing

Based on the proposed model of continuous mutual investment, DSS software module “SSDMI” was implemented (Fig. 1).

Module “SSDMI” can be used both as an independent software product and as an auxiliary unit of decision support system “DMSSCIS”, which, in particular, allows assessment of investment risks in information security systems of large enterprises [19, 20].

Computational experiments were conducted (Fig. 2, 3). Data from the State Statistics Committee of Ukraine over the period from 2011 to 2016 were taken as source data. During analysis of operation of “SSDMI”, correctness of the algorithm implementation was monitored.
Fig. 1. General outlay of the software module “SSDMI”

Fig. 2 shows the form for interpretation of the results of decision making support, provided by DSS “SSDMI”. X axis in the graph means “USD, million”. Y axis means “UAH, million”. Tangent of the slope angle is equal to “3.5”. That is, the balance beam is determined by ratio value \( y = g^* x \), \( g^* = 3.5 \). The area, highlighted in blue color, corresponds to \( W_1 \). The area, highlighted in light yellow, corresponds to \( W_2 \).

Fig. 2. General view of tab of graphic interpretation of results of DSS operation on mutual investment

Fig. 3 shows results of DSS operation for modeling investments at a particular enterprise of the IT sector. The trajectory of the investors’ motion is shown in a red line with blue markers in \( W_1 \) area. In the graph, it is accepted: \( x \) – financial resource of investor 1; \( y \) – financial resource of investor 2. The graph is plotted in the course of computational experiment for the following set of states: \( (x(0), y(0)) = (1.5, 4.77) \); \( (x(1), y(1)) = (2.0, 4.583) \); \( (x(2), y(2)) = (2.5, 4.33) \); \( (x(3), y(3)) = (3.0, 4.0) \); \( (x(4), y(4)) = (3.5, 3.57) \); \( (x(5), y(5)) = (4.0, 3.0) \).

In Fig. 3, the balance beam (shown in blue with red handles) is the same as in the first graph, i.e. \( y = g^* x \), \( g^* = 3.5 \).

Fig. 3. Results of DSS operation for selection of strategies for mutual investment at an enterprise in IT sector

The obtained results demonstrate effectiveness of the proposed approach. During testing of program “SSDMI”, correctness of the obtained results was established. Approximation of DSS “SSDMI” was also executed for actual investment projects in IT sector in the city of Kyiv.

6. Discussion of results of computational experiment

The considered procedure of interaction is the process of prediction of investment outcomes. It is natural, that in this case, the prediction data, obtained with the help of DSS did not always coincide with the actual data. We will note that this is an objective reality and it is impossible to get rid of it. This is a definite drawback of the approach, outlined in this article. We can only strive to reduce discrepancies, in particular to improve the tools of prediction of investment processes due to information technologies (intellectual data analysis, DSS, ES, etc.). If these tools are reasonably supplemented by the IT, mentioned above, it will be regarded as an attempt to make the investment process more efficient. The considered approach allows us to do this because it gives the possibility to select variable parameters of the investment process so that it could become balanced. Results of the computational experiment and practical data of evaluation of investment projects were compared to the results, described in the papers [1, 2, 12, 19, 21]. In the course of comparative analysis, it was determined that the proposed tools allow participants of the investment process to substantially improve indicators of performance and predictability of their activities.

Further prospects of developments of this study include transfer of accumulated experience to Android platform when designing DSS. This will increase mobility in making decision on mutual investment.

7. Conclusions

1. The procedure of interaction of investors on the micro- and macro-level, which allows predicting investment
outcomes, was considered. For this purpose, the model, within which there are investment management strategies at different ratios of parameters of the investment process, was proposed. Application of the model in decision support systems, in contrast to the existing models, gives specific recommendations in the process of making management decisions in the investment process. The essence of the model lies in the fact that it makes it possible to determine the optimal investment strategy and to predict the outcome of the investment process at any ratios of parameters of the investment process. For the case when a prediction is not beneficial for investors, we propose using the algorithm of adjustment of the investment process parameters so that the parties could reach a result acceptable for them. From the mathematical point of view, the model is based on solution of a bilinear differential quality game with several terminal surfaces. A special feature of this differential game is that the right part of the system of differential equations is bilinear functions with arbitrary coefficients.

2. Software module of DSS “SSDMI”, which implements the proposed model based on application of methods of the theory of differential games, was described. A new class of bilinear differential games, which allowed us to describe adequately the investment process, was considered. DSS enables us to reduce divergence of prediction data from the actual return on investment. It is shown that if the proposed DSS tools are reasonably supplemented, this will be an attempt to make the investment process more efficient. The computational experiment was conducted, which proved workability of the model and software program “SSDMI”, specifically for investment projects in information technologies, information and communication and cyber security systems.

References

1. Introduction

Targeted improvement of operational efficiency of an industrial enterprise is possible only in the case when all its resource-intensive technological processes are optimized. In this case, optimization criteria of the managed systems must be systematically substantiated, inter-coordinated, and have to ensure maximization of financial capacity for the owner of results of operational processes (super-system).

Despite the fact that such a statement of the optimization task seems obvious, at present, managed systems integrate as optimization criteria a variety of indicators that are subjectively defined as the criteria of optimization [1]. Such indicators turn managed systems into extreme systems rather than making them optimal [2].

Among the many classes of managed systems, a special place is taken by displacement systems. The systems of this class are extremely diverse and comprise hoisting-and-transporting mechanisms [3], conveyor mechanisms for continuous transportation [4], motor transport systems [5]. They also include systems for transporting liquid and gaseous products. All these objects perform a function of the connecting link between the systems of a transformative class and the buffering systems.

A special feature of such objects is that the choice of the best parameters of a technological process is affected by...