1. Introduction

The European integration of Ukraine, covering all aspects of functioning of the state, cannot but consider, first of all, priority vectors of development, including strategic industries. Concerning Ukraine, the focus should be mainly on metallurgy. Obviously, in the conditions of depreciation and obsolescence of assets occurring today in the metallurgical industry, enterprises have to find reserves that enable them to reduce their production costs. This in itself is a complex organizational and technical problem. However, in this aspect, it should be also taken into account that the solution to the above situation is impossible without systemic state support. Such support should be provided not only on a technical level, but also within the appropriate legal support for introducing products to European markets. Thus, the attitude of the state to producers in the conditions of reforms has to be changed.

It is necessary to note another problem in the metallurgical industry, which is not associated with the cost of production, but directly related to the effectiveness of European integration in the context of taking serious positions in the market. It is about the possibility of entering the European markets, viewed in terms of not only political, economic or legal aspects, but, first of all, the quality of products. The latter is regulated by the relevant standards, the implementation of which in the realities of Ukrainian metallurgical production has to imply the possibility of using internal reserves of enterprises.
Control processes

2. Literature review and problem statement

“Ukrainian metallurgy is 200 thousand jobs directly in the sphere and 500 thousand in related fields. It is 25 % of Ukrainian exports. And when Ukrainian metallurgy is unstable, production decreases due to a lack of raw materials, this immediately affects almost each Ukrainian – due to drop in the hryvnia exchange rate” [1]. Therefore, the Law No. 6382 of 14.04.2017 adopted by the Verkhovna Rada provides for the extension of the increased export duty on ferrous scrap in the amount of 30 euro/ton for another 2 years. Thus, the adopted Law of Ukraine No. 1435-VIII “On Amendments to Some Laws of Ukraine Concerning Reduction of the Deficit of Ferrous Scrap in the Domestic Market” of 12.07.2016 has allowed stabilizing Ukrainian metallurgy, saving 25 thousand jobs and attracting 1.5 billion UAH of taxes to the state budget through high-value-added production [1]. However, it should be noted that introduction of duties on Ukrainian metal products by the European Union can lead to a loss of up to half a billion euro a year, job cuts and reduction of assignments to the budget. This opinion is shared by the Federation of Metallurgists of Ukraine [2]. This, in turn, may lead to the fact that Ukrainian metal products will become uncompetitive in the EU markets. These forecasts prioritize the problems associated with finding ways to reduce resource and energy consumption primarily for those technological processes of metal production, which are the most costly. These are basic metallurgical processes – melting, implemented in electric arc and induction industrial furnaces. The first of them allow high-quality processing of different sorts of charge of any dimensions, but do not provide minimum burn-off losses of materials. The second ones provide the possibility to minimize resource losses and are more cost-efficient and environmentally friendly. Therefore, it is expedient to recognize the research aimed at finding ways to optimize melting processes in induction furnaces in order to obtain high-quality products with minimum production costs as more promising. Speaking about the induction melting process control in the context of obtaining high-quality alloys, according to the authors of [3], it is necessary to choose a technological parameter that “link” the electric power and technological components of melting. Such factor, in the opinion of the authors, should be stirring of the melt in a crucible of the induction furnace, caused by electrodynamic forces. This significantly affects electric, power and thermal parameters of the unit, especially in the case of crucible holding furnaces [4]. The authors of this paper note that when creating a parameter control system of induction furnaces, identification of the furnace or holding furnace is necessary to obtain their refined model, without taking a number of serious assumptions. The latter include the need to develop and study power controllers, power and balancing factors for induction crucible furnaces [5]. However, the above works placed no emphasis on issues of mathematical modeling, and conclusions on the results presented are more qualitative in nature.

A number of authors [6] consider the stage of identification of electric furnaces – power-engineering systems – as the problem of obtaining mathematical models, constructed according to the principles of using a number of partial models and revealing the key interrelations between them. This is an alternative approach to constructing a general model that integrates interrelations, processes, characteristics and parameters of energy-saving and managerial nature. However, modeling issues are considered for electric arc melting and the results given in it are not applicable directly to induction melting.

Some aspects concerning control of redox processes in induction melting are discussed in [7, 8]. In particular, they revealed the patterns of carbon change at high temperatures, typical for melting of ferrous alloys in induction furnaces, as basic processes subject to control by melting technologies. The values of the pre-exponential factor and activation energy of the heterogeneous reaction of carbon oxidation in a carbon dioxide medium calculated in them can be used in a number of mathematical models in problems of melting control.

In [9, 10], the problems of melting control when the chemical composition is chosen as an input variable, alloy properties – as an output variable, and the final state and speed – as control quality criteria are considered. These approaches are good in case of a multi-alternative description of the final state and if there is a need to take into account a compromise between the resulting alloy quality and line speed. However, the results do not contain data on testing of induction furnaces, in particular when controlling duplex processes of induction melting.

Obviously, the information on the physicochemical processes occurring in induction melting poorly coordinate in the described sources with significant factors – melt temperature and control factors, for example, the input power. In this regard, the research on the formalization of the induction melting process presentation and its subsequent use to search for the optimum process control is of interest.

3. The aim and objectives of the study

The aim of the paper is to select and justify the effectiveness function of quality control of metal products on the example of a controlled induction duplex melting process. This will allow the system approach to forming a set of measures aimed at strengthening the position of Ukrainian metal products in the EU market.

To achieve this aim, it is necessary to accomplish the following objectives:
– to formalize the principle of operation of a control object in industrial conditions;
– to develop a mathematical model of the melting process in induction furnaces with the assumptions allowing to simplify the process description without affecting the model effectiveness;
– to develop a mathematical model of the heat treatment process in induction holding furnaces with the assumptions allowing to simplify the process description without affecting the model effectiveness;
– to identify the factors of organizational and legal nature, accounting of which is mandatory for developing the quality control effectiveness function.

4. Development of a mathematical model of the induction duplex process as a control object

4.1. Formalization of the operation principle of the control object in industrial conditions

A standard technological process of induction melting is carried out at the melting department of the foundry and
is an element of the overall production chain of shaped castings. Fig. 1 shows the flow chart of operation of a standard department, equipped with induction crucible furnaces for melting and induction holding furnace for heat treatment.

The algorithm of the induction melting process is shown in Fig. 2.

Melting process control in IST is carried out in the manual or semi-automatic mode in the control cabinet and lies in selecting the transformer power and introducing the necessary number of corrective additives depending on the actual state of the melting process. The melting process state is estimated by the results of temperature measurements and chemical analysis of the alloy. Sampling for determining the chemical composition is carried out during the melting process in accordance with the melting instruction manual, approved by the enterprise. Process control is considered effective if the value of the content of each chemical element falls within the specified range of specifications. Mathematical models describing the chemical composition dynamics are absent, control is situational, the choice of the control effect is based on the actual values of the content of chemical elements obtained from the rapid diagnostics laboratory. The time drift between the sampling (time point $τ$) and obtaining the results (time point $τ+1$) is not taken into account. Thus, the decision on control is taken at the time point $τ+1$, based on the data obtained. Such a process control option in actual industrial conditions cannot be considered effective. The reason is, first of all, the lack of an adequate mathematical model of the process. Therefore, one of the main problems to be solved and preceding the search for optimum process control is constructing a mathematical model describing the time variation of the chemical composition in specific conditions. This model allows predicting the state of the system and calculating the time points to perform control functions. The latter should be such as to transfer the system from the actual initial state estimated by the set

$$\{x_0(0) = x_{\tau+1}(a)\}$$

to the specified state determined by the required chemical composition of the alloy for minimum time. Here, $x_i$ is the content of the $i$-th chemical element in the alloy, $e_{\tau}(a)$ is the time point to deliver the melt to the pouring area in a conveyor.

When selecting the control, it is also necessary to consider the time drift noted above. Thus, it is expedient to take the kinetic equations describing the time variation of the content of each element in the melt depending on temperature as a mathematical model. This model and the results of the analysis at the time point ($τ+1$) received from the rapid diagnostics laboratory, valid for the time point $τ$ allow determining the actual initial state of the system

$$\{x_0(0) = x_{\tau+1+1}(a)\}$$

which can be used to synthesize the optimum control.

In this case, two control problems should be distinguished:
- control at the melting stage;
- control at the heat treatment stage.

The first problem, in turn, is divided into two sub-problems. One of them is to find the control that transfers the charge from the solid state to the molten state for the shortest time, or from the liquid-solid state to the liquid state. In other words, it is about the maximum melting speed in IST. The second one is to find the control that transfers the melt from the actual state (determined by temperature and chemical composition) to the specified state, regulated by standard documentation for the shortest time. This requirement is important since it allows minimizing the costs associated with heat treatment of the melt.

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**Fig. 1.** General process flow chart of the melting department of the foundry with induction furnaces: 1 – induction furnaces (IST), 2 – induction channel holding furnace (IchT). The arrows show the direction of the melt.
Thus, the problem of control of the process under study at the heat treatment stage is the problem of speed and can be formulated as follows: to determine the optimum control $u^{\text{opt}}$ that transfers the process from the initial state $x_i(t(0)) = x_i(0)$ to the final state, located on the surface $f^{\text{min}}(x_i; x_k) = 0$ for minimum time. This surface is an analytical description linking the chemical composition of the alloy, satisfying a set of given values of output variables $y_i$, and can be determined on the basis of mathematical models “composition – property”. In this statement, the initial state of the heat treatment process control is formed by the chemical composition of the alloy obtained immediately after melting of the charge, or after holding it in the induction furnace.

The mathematical description of the process is the equation of the rates of the chemical reactions occurring in the melt of a given mass at a given temperature. If the results of process monitoring revealed that under the existing melting conditions for obtaining the desired alloy grades, the content of some elements remains practically unchanged, their number in the melt during holding can be considered a constant. In this case, the calculation of the charge needs to be carried out so that after melting the content of these elements is within the specified tolerance. This simplification allows reducing the number of factors varying in the process and the dimensionality of the response surface describing the final state of the process.

4.2. Development of the mathematical model of the melting process in IST induction furnaces

The mathematical model of the induction melting process should describe the state of the system in time in the chosen control. In this case, such a model should be constructed for two stages of melting (Fig. 2): actually melting (melting of the solid charge) and holding the melt at a given temperature (or heating from the liquid state during the “hot heel” melting). Construction of models for both stages implies the possibility of controlling the melting parameters – state variables. The key parameters determining both the melting process and its safety are the melt temperature in the crucible, lining temperature, coolant temperature, state of the induction furnace lining and the inductor insulation, as well as the values of electric signals in the form of current and voltage on the inductor. Fig. 3 presents the schematic diagram of the control object at the first stage of melting.

At the first stage of melting, the mathematical model should include:
- components of the charge (amount, fraction) as the vector of input variables $V$;
- vector with components – the amount of liquid melt and the chemical composition as the vector of output variables $Y$;
- system temperature as the state variables $z_i(t)$;
- input power as the control $u$.

In such a context, the equation of state is an ordinary differential equation describing the variation of system temperatures: melt temperature, lining temperature, cooling...
water temperature. However, considering that lining and cooling water temperature control loops are different, only melt temperature control is to be considered. Temperature control is determined by the initial furnace charging. If, with a constant voltage at the furnace terminals, the input power is approximately constant, the energy supplied to the charge causes either melting of solid pieces of the charge or heating of the “heel”. Therefore, feeding of new portions of the charge is equivalent to a sudden (abrupt) temperature drop. If the size of the charge pieces is small, we can assume that it melts almost instantly. This means that the solid charge melts in a negligible time period and the temperature that is lower by $\Delta T$ than before feeding is instantly set.

In case of the “hot heel” melting, the solid charge absorbs the heat of the melt and the melting process is much faster and more energy-efficient. Therefore, it is advisable to consider both options.

Decrease in the melt temperature due to the addition of the mass $g_c$ can be designated as $\Delta T$, assuming that the charge is small-sized and melts in a negligible time period. If $T > T_m$, then

$$g_c (c_s + T) = G_c \Delta T,$$

(1)

If $T = T_m$, then

$$g_c (c_s + T) = G_c (c_s \Delta T + c_s).$$

(2)

Here, $G_c$ is the furnace capacity at the time point $t$, $g_c$ is the charge portion fed at the time point $t$, $T$ is the temperature of the bath (melt), $T_m$ is the melting point, $c_s$ is the specific melting heat, and $c$ is the specific heat.

Then

- with $T > T_m$

$$\Delta T = \frac{g_c (c_s + T)}{G_c},$$

(3)

- with $T = T_m$

$$\Delta T = \frac{g_c (c_s + T)}{G_c} \left( \frac{c_s + c}{c} \right).$$

(4)

Given the fact that the melting rate is assumed to be high, since, as noted above, it is assumed that melting occurs in a short period of time, it can be considered that the rate is proportional to the power supplied to the furnace. Therefore, the corresponding equation for the melting rate is presented in the form

$$\dot{G} = k_1 N,$$

(5)

where $k_1$ is the coefficient of proportionality, depending on the power of the unit and reactive power compensation value.

The equation (5) can be transformed as follows:

$$\frac{dG}{dt} = \frac{dG}{dT} \frac{dT}{dt} = k_1 N.$$}

(6)

Expressing the melting rate from the equation (6) as a function of power (control $u$), we obtain the equation of state

$$T = k_2 N.$$}

(7)

Here

$$k_2 = \left( \frac{dT}{dN} \right)^{-1}$$

is the reciprocal of the melting rate.

The description of the form (7) does not contradict the physical principles of the melting process, since the dependence of the temperature growth rate on the input power is obvious. At the same time, the proportionality adopted as a simplifying assumption is proved by the assumption of high melting rate.

In terms of ensuring that the chemical composition falls within the specified range, melting process control should involve holding until the melt is delivered to the ladle and transported to the induction holding furnace. This is the second stage of melting in the induction crucible furnace. In this case, the schematic diagram of the control object may take the form shown in Fig. 4.

A distinctive feature of this scheme compared to the previous one (Fig. 3) is the description of the vector of input variables, the vector of output variables and state variables. These differences are as follows.

The vector of input variables $Y$ consists of the components of the chemical composition of cast iron and its temperature after complete melting of the charge in the induction furnace. The vector of output variables $X$ consists of the components of the chemical composition of cast iron and its temperature when the melt is delivered from the induction furnace to the ladle to be transported further to the induction holding furnace. The state variables $z(t)$ are the values of the chemical composition of the melt in the control process during the melt holding in the induction crucible furnace.
4. 3. Development of the mathematical model of the heat treatment process in the IChF induction holding furnace

For constructing a mathematical model of the process of heat treatment in the induction holding furnace and subsequent synthesis of the optimum control of the corresponding stage of induction melting, it is necessary to consider the goals of the functioning of the melting facility. On the one hand, the induction holding furnace is designed to hold the melt of a predetermined amount that should fully meet the need for the melt in the foundry conveyor. That is, the facility is the accumulator of liquid alloy and the drop of the melt level in it when a certain portion is delivered to the conveyor requires the level to be restored by filling with the melt delivered from the induction furnace. On the other hand, the task of the induction holding furnace is to maintain the temperature regime of heat treatment at a predetermined level for the specified time. The latter is determined, first, by the degree of completeness of the physicochemical processes occurring in the melt of the actual volume at a given temperature, and, second, by the need to deliver a certain amount to the conveyor if needed.

Both these functional purposes of the induction holding furnace require two mathematical descriptions, suitable for two different control problems, respectively. The first assumes the need for optimum control of the melt level, and the second – the maximum speed of bringing the melt to a specified quality. In the latter case, it is assumed that as the melt is delivered to the conveyor, its quality has to correspond to the preset one. The diagram of the control object for the first problem is shown in Fig. 5.

![Diagram of the control object for solving the problem of optimum control of the melt level](image)

Fig. 5. Diagram of the control object for solving the problem of optimum control of the melt level: \( h_0 \) – the initial melt level, \( \Delta h \) – the height difference of the melt surface in the furnace, \( Q_1 \) – the melt flow rate in the ladle (or induction furnaces) corresponding to the compensated need for the melt of the induction holding furnace, \( Q_2 \) – the melt flow rate when delivered from the induction holding furnace to the foundry conveyor (to the pouring area), \( F \) – the internal cross-sectional area of the holding furnace, \( d \) – the holding furnace diameter, \( M \) – the controller actuator – the drive for switching the transformer voltage stages

The main process parameter in this consideration of the induction holding furnace is the melt level. The principle of operation of the melt level control system can be described as follows. An increase in the flow rate of the melt delivered to the conveyor, \( Q_2 \), causes a sharp drop in the melt level in the induction holding furnace from the initial height \( h_0 \) by the value \( h \). Such a change is perceived by the system as a single abrupt input action and the control signal actuates the controller switching the transformer voltage stage of the IST induction furnace. The transformer power, which is the control action, should be optimum. It is understood as the possibility of melting for the minimum period of time and holding of the melt of a given quality and amount. The latter should compensate for the amount of the melt delivered from the induction holding furnace to the foundry conveyor by an amount that compensates for the melt \( Q_2 \), i.e., there is an increase in the flow rate \( Q_1 \). If we simply assume that the internal surface of the furnace has a cylindrical form, the mathematical model describing the process of the melt level variation can be represented as follows:

\[
F \Delta h = Q_1 - Q_2. \tag{8}
\]

If the actuator is an electric motor and the rotational speed of the motor shaft is proportional to the value and sign of the applied voltage, the analytical dependence of the melt flow rate variation on the voltage applied to the motor has the form

\[
Q = kU, \tag{9}
\]

where \( k \) is the integrated coefficient taking into account the influence of the characteristics of a specific electric drive.

It is expedient to perform the replacement of variables:

\[
x = \frac{Q}{Q_1}, \quad \Delta \Delta Q = Q_2 - \Delta Q_1 \quad \text{variation of the flow rate of the melt delivered in the "induction furnace – ladle – induction holding furnace" chain;}
\]

\[
x = F \Delta h \quad \text{variation of the melt volume in the induction holding furnace with increasing flow rate} \quad Q_2; \]

\[
q = \Delta Q_2 - \Delta Q_1 \quad \text{variation of the melt when delivered from the induction holding furnace to the foundry conveyor;}
\]

\[
u = kU \quad \text{control proportional to the voltage applied to the motor.}
\]

Then we can write down a system of differential equations (SDE), describing the melt flow rate variation in the induction holding furnace:

\[
x_1 = u, x_2 = x_1 - q. \tag{10}
\]

The SDE (10) can be taken as the mathematical model for the synthesis of the optimum control of the melt level in the induction holding furnace. As shown in [11], the structure of the mathematical model describing the bath temperature control can be similar. The solution of SDE (10) has the form of (11), and the law of optimum control is described by the form of (12):

\[
x_1 = \frac{1}{2\delta_0} \left( x_1 - q \right)^2 \text{sgn}(q - x_1), \tag{11}
\]

\[
u_{sy} = \delta_0 \text{sgn} \left[ \frac{1}{2\delta_0} \left( x_1 - q \right)^2 \text{sgn}(q - x_1) - x_2 \right], \tag{12}
\]

where \( \text{sgn}(q - x_1) \) and \( \text{sgn} \) are the sign functions.

This means that the conclusion can be drawn on the possibility of using the unified mathematical description for technologically different, but identical in terms of control,
“standard” melting process parameters at the stage of heat treatment.

5. Integrated approach to the development of the control effectiveness function taking into account the factors of organizational and legal nature

The above approach to the formalization of the description of the metallurgical process is based only on the technological component. Its ultimate goal is to obtain products that meet European standards. For example, for gray cast iron, the production technology of which in metallurgical duplex processes is described above, this can be DIN 1691 (Germany), BS 1452 (UK). However, it is one of the criteria. In order to evaluate Ukrainian metal products in terms of the potential possibility of introducing them to European markets, it is necessary to consider other criteria, including economic and legal ones. These criteria can be reduced to some control effectiveness function that has a meaning of the risk function used in the statistical game theory. Such a function may have, for example, the following form:

\[ \rho = \rho(\kappa_t, \kappa_e, \kappa_l) \]  

(13)

where \( \kappa_t \) is the coefficient of technological effectiveness, \( \kappa_e \) is the coefficient of economic capacity, \( \kappa_l \) is the coefficient of legal capacity.

The concept of economic capacity has a meaning of potential losses associated with the introduction of metal products with the quality provided by the process control possibilities adopted at the enterprise to the market. This means that a probabilistic assessment of the enterprise capacity is carried out in terms of ensuring the quality required by European standards.

The concept of legal capacity has a meaning of the possibilities of legal regulation as a mechanism of promotion of Ukrainian metal products to the European market. And here problems arise. As an example, the materials of [12] devoted to the study of legal aspects of ferrous scrap export can be cited. In particular, the following is noted. “In April 2015, quotas on the export of ferrous scrap from Ukraine – 1.25 million tons were established. However, except the approval of the volume of scrap exports from Ukraine in 2015, no significant progress has occurred. The existing “foundation” for regulating this issue includes certain provisions of the Law of Ukraine “On scrap metal” (Law 619-XIV) and some bylaws. Thus, Article 9 of the Law 619-XIV provides that scrap metal export contracts are subject to registration in accordance with the procedure established by the Cabinet of Ministers. The procedure of registration of foreign economic contracts (agreements) for foreign economic operations with scrap metal has been approved by the Decision of the Cabinet of Ministers of Ukraine in 2002 and was last changed in 2013. Paragraph 3 of the Procedure establishes that the Ministry of Economic Development makes decisions regarding the registration or refusal of registration of contracts within the scrap export quotas for the corresponding year. In addition, Paragraph 5 of the Procedure prohibits disclosure of information contained in the documents submitted for registration of foreign economic contracts. Officials, guided, among other things, by this paragraph, refuse to disclose any information, which makes the procedure closed to the public.

Scrap export quotas for the corresponding year as a non-tariff regulation measure are established in order to prevent a deficit in the domestic market. Ukrainian metalurgists fully support such a system of scrap metal export limitation, through which scrap suppliers are forced to work in the domestic market at prices far lower below the world prices”. According to the author of [12], all this gives grounds to believe that “under the pretext of “protection” of the domestic market, a closed corruption procedure has been created in due time”. It can be assumed that similar conclusions are appropriate concerning other metal products. In terms of the control selection, it is important here that in this case there is the influence of the uncertainty factor introduced by legal aspects in the development of the control effectiveness function (13).

Thus, the coefficients in the description (13) contain uncertainties and it can be assumed that their values should be determined with the help of expert estimates. This circumstance also introduces an effect of uncertainty, so appropriate membership functions can be used for evaluation. With this approach, the risk function is replaced with the control effectiveness function (13), and classical solutions of the statistical game theory are used to select an optimum control strategy.

6. Discussion of the results of the development of the control effectiveness function

The coefficient of technological effectiveness included in the description of the proposed control effectiveness function (13) depends on the chosen control of duplex induction melting processes. The mathematical description (7) allows selecting the optimum process control at the melting stage. In this case, bath temperature variation due to feeding new portions of the charge affects only the initial state in the mathematical description of the control object. Concerning the use of the mathematical description for the heat treatment stage, the following circumstance should be noted. Temperature variation entails the variation of the rate constants of chemical processes – redox reactions involving chemical elements. In this case, various options of kinetic equations are possible, depending on the order of reactions. As a rule, temperature increase in such a system increases the process rate. However, one should consider the presence of the exothermic effect of some reactions, hindering the process rate with increasing system temperature. This effect is a manifestation of feedback in the control of physiochemical processes. Temperature increase due to increase in the power supplied to the furnace is an effective way to transfer the system to the desired state. The latter is regulated by the specified tolerance for the chemical composition. The equation (9) is simplified, but the possibility of its application can be explained by the same reasons that have been described with reference to the construction of the mathematical model of the melting process in the induction furnace. Thus, an increase in voltage at the furnace terminals (input power) leads to a proportional increase in the amount of molten metal. This, in turn, leads to an increase in the flow rate of the melt delivered from the induction furnace to the ladle, and from the ladle to the induction holding furnace.
Such an approach to the mathematical description of the induction melting process allows using the same mathematical model, but with different variables when searching for optimum control. If in the first case it is the melt level, in the second case it is temperature. At the same time, it should be noted that in the first case, the quality control criterion is the possibility to meet the performance requirements of the foundry conveyor, and in the second – the quality requirements of the alloy. These are obvious advantages of the proposed approach.

The disadvantage of the proposed approach is a considerable simplification with respect to the parameters included in the mathematical models. The simplifications do not allow making serious theoretical conclusions about the predicted behavior of the control object, but they are justified in terms of practical implementation of the chosen control. Therefore, it is of theoretical interest to refine the parameters included in the mathematical description of the induction melting control process. This requires additional research and is also associated with probable complication of the structure of the mathematical description of the control object. However, such a description should be considered as part of effective quality control, in particular through the formation of the process effectiveness coefficient.

The proposed control effectiveness function is essential, since it contains several criteria that are different in nature, but important in fact. The mathematical descriptions (7) and (9) allow finding the optimum control that maximizes the required quality parameter. This also leads to the maximization of the process effectiveness coefficient in the control effectiveness function used to select an optimum strategy based on the statistical game theory. Herewith, the control effectiveness function is used instead of the risk function.

The coefficients of economic and legal capacity, which can be determined on the basis of expert estimates, allow considering the uncertainty factors associated with risks, including legal regulation. One of the examples confirming the complexity of estimating these values can be the results of the analysis given in [12] concerning the effectiveness evaluation of legal regulation of the export of metal products: "... current resolutions of the Cabinet of Ministers, the orders of the Ministry of Economic Development, establishing the order of negotiation of export contracts, are questionable with the legal point of view. The Law No. 619-XIV stipulates that scrap metal export contracts are subject to registration. Registration, in essence, is a "technical" function, unlike "negotiation" or "approval", which create a lot of opportunities for the in essence, is a "technical" function, unlike "negotiation" or "approval", which create a lot of opportunities for the in essence, is a "technical" function, unlike "negotiation" or "approval", which create a lot of opportunities for the in essence, is a "technical" function, unlike "negotiation" or "approval", which create a lot of opportunities for

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This complexity of the quantitative evaluation of coefficients in the description (13) is a weak point of the approach. Nevertheless, at the conceptual level, it can be considered reasonable, and additional research on the structure selection and quantitative evaluation of coefficients seems promising. Such additional research can become a theoretical basis for introducing practical methods for implementing the principle of liberalization, approved in the basic agreement of the EU, in terms of strengthening the position of Ukrainian metal products in the EU market.

### 7. Conclusions

1. It is shown that the construction of the mathematical description of the induction duplex melting process as a control object must be preceded by a distinction between two control problems: control at the melting stage and control at the heat treatment stage.

2. The mathematical model, simply describing the control object at the induction melting stage, is an ordinary differential equation. Its feature is that the heating rate is proportional to the input power, and the proportionality factors depend on the furnace power and the reciprocal of the melting rate. Such a description does not contradict the physical principles of the melting process, since the fact of dependence of the temperature growth rate on the input power is known, and the proportionality, accepted as a simplifying assumption, is justified by the assumption of fast melting rate.

3. The system of ordinary differential equations in a universal form can be used as a mathematical model for the synthesis of the optimum control of the melt level in the induction holding furnace and bath temperature at the heat treatment stage. This allows using the unified mathematical description for technologically different, but identical in terms of control, “standard” melting process parameters at the stage of heat treatment.

4. Factors of organizational and legal nature, accounting of which is mandatory for constructing the quality control effectiveness function, can be represented by the coefficients of economic (κ₁) and legal (κ₂) capacity. The greatest uncertainty is present in the assessment of the legal capacity associated with the possibilities of legal regulation as a mechanism of promotion of Ukrainian metal products to the European market. Thus, the integrated effectiveness function of quality control of metal products can be expressed through the coefficient of technological effectiveness, the maximum of which depends on the chosen control, and the coefficients of economic and legal capacity. Such quality control effectiveness function has a meaning of risk function, so it can be used to select a control strategy based on the statistical game theory.

### References


