1. Introduction

Separation systems carry out operations on collection, preparation and storage of oil. One of the main functions is the transport of well products under the action of reservoir pressure or at the expense of the energy of pumps to the point of oil preparation [1]. In these systems, there is a separation of gas from oil and supply of it to consumers, as well as a free water separation from well products (in the case of flooded oil).

The efficiency of the separation process to a large extent is determined by the methods and algorithms of the automatic control systems. The process of separation of oil proceeds under the influence of numerous obstacles and inherent in it complex internal communications [2]. Therefore, the actual task will be the development of effective systems of automatic control, which will be based on adequate mathematical models, which quantitatively and qualitatively will characterize the process of separation in general.

2. Literature review and problem statement

The production economy shows that oil separation processes should be as efficient as possible, since separated oil can contain partially captured water or gas and vice versa. This means that any gas-oil emulsion must be detached to the commercial state, the water content must not exceed 1% [1]. The reflection of the scientific problem of separation of oil can be found in the papers [2, 3]. However, the greatest problem of the current state of oil separation processes is the lack of integrated automated control systems, where the parameters of the flow of each component with the enabled function of flow management must be controlled. An integral part of such complexes are two-stage oil separation systems [4]. Typically, single-stage separation systems are used for oil separation at primary oil preparation plants. But the use of such equipment is fair and reasonable for oil with low water and gas content, that is, less than 1/3 of the total mixture. When the total content of water and gas is significantly higher than the oil content or equal, it is expedient to use two-stage separation systems, which will significantly increase the efficiency of the oil separation process [4, 5].

In the first stage of the creation of two-stage separation systems, the issue of mathematical modeling will be put forward [3, 10], which includes such important factors as productivity, gas factor, solubility factor, separation coefficient, geometric dimensions of the separator, etc. The study of oil separation by computer simulation was performed in the paper [6], but the disadvantage was the exclusion of water from the structure of the model, which significantly influences the key parameters: pressure, temperature, level, expense. The development of computer models of the separation process is necessary to provide engineers with valuable tools for obtaining more reliable qualitative and quantitative solutions for the further processing of oil and exploitation of oil fields. In the paper [7], a detailed description of the theory of motion of gas particles in a gravitational field has been made which significantly influences the performance of the separator, but this effect does not investigate the effect on multi-stage separation
systems. One of the first mathematical models that describe the process in terms of "input-output" and is suitable for the synthesis of automated control systems was the model suggested in the paper [8]. In the scientific paper [9], the mutual influence of the level of fluid and pressure in the separator was taken into account. But the disadvantage is that the last two papers are not sufficiently substantiated and are intended only for single-stage separation. Two-stage separation systems are considered to be the most effective, since the operation of the first stage significantly affects the efficiency of the second one. In the scientific paper [10], the work of the two-stage separation system is described in details, but according to the authors, it would be expedient to use a system with horizontally placed separators B-1 and B-2, as this can significantly affect the overall separation factor in subsequent calculations [11, 12]. Consequently, the issue of two-stage separation systems will remain inadequate. Therefore, based on the literature review, an imitation model of a two-stage separation system consisting of two horizontally placed separators and a vertical oil storage tank is suggested.

3. The aim and objectives of the study

The aim of the work is to develop a numerical simulation model of the two-stage oil separation model in the MatLab program, taking into account the features of the technological process for the synthesis of efficient systems for automatic control of the separation process and the further development of mathematical models in terms of "input-output". In other words, it is about determining the mutual influence between the technological parameters of each of the separators of the two-stage separation system.

To achieve this aim, it was necessary to fulfill the following objectives:
- the creation of a mathematical model of the material balance of a two-stage separation for the first B-1 and the second B-2 horizontally placed separators;
- the identification of parameters of the mathematical model of two-stage separation for further simulation in MatLab;
- the development of a simulation model of two-stage separation and examine it numerically in the MatLab program to change the key parameters of the level and pressure in each separator.

4. The object and methods of research of the separation unit

The object of research is the technological process of oil separation. The two-stage separation system (Fig. 1), which contains the first and second degree of separation [11], is taken into consideration.

Working pressure at the first stage of separation is created by reservoir pressure at the outlet of the well. Such pressures range from 4.1 to 8.3 MPa [11]. The technological scheme of the simulation model of the two-stage separation unit is shown in Fig. 1.

Production of wells of an oil and gas deposit at a pressure of $P_0=4.0$ MPa enters the horizontal separator B-1. Oil from the separator B-1 at a pressure of $P_1=1.6$ MPa enters the second degree of separation into a horizontal separator B-2 through the actuator (executive device) $ED_h$. Gas from the separator B-1 enters the compressor station "CCS" through the actuator $ED_g$ (Fig. 1).

![Fig. 1. Technological scheme of the simulation model of the two-stage separation unit](image)

Oil from the separator B-2 at a pressure of $P_2=0.6$ MPa enters the reservoirs of the GSR. Further oil through the pipeline is provided at the entrance to the oil pumping station and to the oil pipeline, where it is shipped to the tanker. Gas from B-2 at a pressure of $P_2=0.6$ MPa enters the "CCS" through the actuator $ED_g$. At the same time, gas from B-2 is supplied also for industrial and technological needs (ITN).

The main technical and technological parameters of the separation unit are given in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Device name</th>
<th>Parameter name</th>
<th>Unit of measurement</th>
<th>Parameters</th>
<th>Maximum permissible</th>
<th>Installed technologically</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Separator B-1</td>
<td>pressure temperature volume</td>
<td>MPa °C m$^3$</td>
<td>$P_1=1.6$ MPa $T_1=40^{+100}$ $V_1=100$</td>
<td>$P_1=1.6$ MPa $T_1=40^{+100}$ $V_1=100$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Separator B-2</td>
<td>pressure temperature volume</td>
<td>MPa °C m$^3$</td>
<td>$P_2=0.6$ MPa $T_2=40^{+100}$ $V_2=100$</td>
<td>$P_2=0.6$ MPa $T_2=40^{+100}$ $V_2=100$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>High-pressure gas line</td>
<td>diameter length pressure</td>
<td>mm m MPa</td>
<td>$D=89$ $L=1,700$ $P=8.0^{+15.0}$</td>
<td>$D=89$ $L=1,700$ $P=8.0^{+15.0}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gas line CCS</td>
<td>diameter length pressure</td>
<td>mm m MPa</td>
<td>$D=159$ $L=930$ $P=3.5^{+4.5}$</td>
<td>$D=159$ $L=930$ $P=3.5^{+4.5}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gas line ITN</td>
<td>diameter length pressure</td>
<td>mm m MPa</td>
<td>$D=219$ $L=5,740$ $P=0.52^{+0.57}$</td>
<td>$D=219$ $L=5,740$ $P=0.52^{+0.57}$</td>
<td></td>
</tr>
</tbody>
</table>

Some characteristics of oil and gas mixture together with oil and gas parameters after separation are shown in Table 2.
The necessary efficiency of the separation process is achieved by stabilizing such mode parameters as the level of fluid and pressure in the separator [12]. Therefore, material flows in a two-stage separation system are described using the equations of the material balance, respectively, for each separator individually.

### 5. Mathematical model of operation of a two-stage separation unit

The two-stage separation unit has two horizontal separators B-1 and B-2 (Fig. 1).

Since the first stage B-1 separator is horizontal and cylindrical, the degree of its filling will be given by the equations [14]:

\[
v_p(h) = \frac{1}{\pi} \left( \pi - \cos \left( \frac{h}{r_1} - 1 \right) + \left( \frac{h}{r_1} - 1 \right) \sqrt{1 - \left( \frac{h}{r_1} - 1 \right)^2} \right),
\]

(1)

\[
q_p(h) = \frac{2}{\pi r_1} \sqrt{1 - \left( \frac{h}{r_1} - 1 \right)^2},
\]

(2)

where \( h \) is the oil level in the separator B-1; \( r_1 \) is the radius of the separator B-1.

Consequently, the mathematical model for the first degree of separation will be as follows [14]:

\[
\frac{dP}{dt} = \frac{1}{1 - v_p(h)} \left( \frac{\alpha_s(U_1)}{\theta_s} \left( \frac{P^2 - P_1^2}{T_1} \right) \right)
\]

\[+ \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} \right),
\]

(3)

\[
\frac{dh}{dt} = \frac{1}{q_p(h) M_o} \times \left( \frac{\xi_f \xi_r \sqrt{\rho_f (P_2 - P_1)} - \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} \right).
\]

(4)

where \( v_p(h) \) and \( q_p(h) \) are calculated by formulas (1) and (2).

Other notations in equations (3) and (4) will be as follows: \( \xi \) stands for the gas separation factor; \( \theta = \frac{V_0}{2 R_0 T_1} \), where \( V_0 \) is the total volume of the separator B-1, \( Z \) is the gas compressibility factor, \( R_0 \) is the gas constant; \( T_1 \) stands for gas temperature in the separator B-1; \( P_1 \) is gas pressure in the separator B-1; \( M_0 = \rho_0 \), where \( \rho_0 \) is the density of oil; \( \alpha_s \) is the oil separation factor;

\[
\xi_f = \frac{1}{\sqrt{A_f}}, \quad A_f = \lambda l_f \frac{1}{2 F_f^2},
\]

where \( \lambda \) is the coefficient of friction resistance, \( D_f \) is the diameter of the inlet pipe, \( l_f \) is the total length of the section, which includes the equivalent lengths of the local resistances; \( F_f \) is the cross-sectional area of the pipeline; \( P_f, P_f \) are the density of oil and gas mixtures entering the separation system and the second stage of separation, \( \rho_0 \) stands for gas density; \( P_0 \) is the pressure at the inlet of the separator B-1; \( a_s(U_2) = K_{12} K_s(U_2) \xi_{sec.} \), where \( K_{12} \) is the shutter release factor, which is proportional to the command signal of the controller \( U_1 \), \( K_s(U_2) \) is the throughput of the ED_1 actuator (Fig. 1):

\[
\xi_{sec.} = \frac{\rho_0 T}{P_{co}},
\]

where \( P_{co}, T_{co} \) stand for gas density, temperature and pressure under normal conditions \( (T_{co} =273 \text{ K } \quad \text{and} \quad P_{co} = 0.1013 \text{ MPa}) \); \( P_{co} \) is pressure after shutting down of the regulating body (RB); \( \xi \) is the gas compressibility factor, which is calculated at temperature \( T \) and pressure

\[
\xi = \frac{P_f + P_{co} g h \cdot 10^6 - P_2}{2};
\]

\( a_s(U_2) = K_{12} K_s(U_2) \), where \( K_{12} \) is the proportionality factor between the transfer of the stem RB of the executive device ED_1 and the command signal \( U_2 \), \( K_s(U_2) \) is the throughput of the actuator ED_1.

The mathematical model of the material balance for the separator B-2 will be determined by the equations [14]:

\[
\frac{dP}{dt} = \frac{1}{1 - v_p(h)} \left( \frac{\alpha_s(U_2)}{\theta_s} \left( \frac{P^2 - P_1^2}{T_1} \right) \right)
\]

\[+ \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} \right),
\]

(5)

\[
\frac{dh}{dt} = \frac{1}{q_p(h) M_2} \left( \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} - \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} \right),
\]

(6)

\[
\frac{dh}{dt} = \frac{1}{v_p(h) M_2} \left( \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} - \alpha_s(U_2) \sqrt{\rho_f (P_1 + P_f g h \cdot 10^6 - P_2)} \right),
\]

(7)

\[
q_p(h) = \frac{2}{\pi r_1} \sqrt{1 - \left( \frac{h}{r_1} - 1 \right)^2},
\]

(8)

where \( h_1 \) is the oil level in the separator B-2; \( r_1 \) is the diameter of the separator B-2.
As the initial conditions for differential equations (3)–(6), we take the fixed values of the corresponding quantities – \( P_1(0) = P_{01}^{(0)} \), \( h(0) = h_{01}^{(0)} \), \( P_2(0) = P_{02}^{(0)} \) and \( h(0) = h_{02}^{(0)} \).

In equations (5) and (6), the following designations are taken: \( P_2 \) is gas pressure in the separator B-2; \( \rho_1 = 1 - \tau_{st1} \) \( G_1 \) stands for the gas separation factor, where \( \tau_{st1} \) is the efficiency indicator of the second stage of separation, \( G_1 \) is the gas factor of the second stage of separation; \( \theta_1 = -\frac{\nu_0}{V_{01}/R_{01}T_2} \) where \( V_{01} \) is the total volume of the separator B-2; \( T_2 \) is the gas temperature in the separator B-2; \( M_{01} = \rho_0 V_{01}/\rho \) is the mass of oil in the separator B-2 at its full filling; \( a_1(U_{02}) = K_{ac} K_{ar}(U_{02}) \) where \( K_{ac} \) is the proportionality factor between the transfer of the stem RB of the operating device \( ED_{B2} \) and the command signal \( U_{02} \). The transfer of the actuator \( ED_{B2} \) (Fig. 1): \( P_2 \) is an average pressure in reservoirs (Fig. 1); \( \theta_1 = -\frac{\nu_0}{V_{01}/R_{01}T_2} \) where \( T_2 \) is the gas temperature in the separator B-2; \( a_1(U_{02}) = K_{ac} K_{ar}(U_{02}) \) \( \xi_1 \), where \( K_{ac} \) is the gas transfer coefficient of the \( ED_{B2} \) actuator, which is proportional to the command signal of the controller \( U_{01}, K_{ac}(U_{01}) = \) the throughput of the \( ED_{B1} \) actuator (Fig. 1); \( P_3 \) is the gas pressure after the RB of the \( ED_{B2} \) actuator of the second stage of separation.

Thus, the mathematical model of a two-stage separation system, which has two cylindrical horizontal separators B-1 and B-2 in its composition, is given by systems of differential nonlinear equations (3)–(6). The peculiarity of the two-stage separation system is that the output (regulated) quantities \( P_3 \) and \( h \) of the first degree of separation, as well as the control action \( U_{02} \) act as a perturbation for the second degree of separation. Such interference with the separation stages worsens the efficiency of the separation unit and requires systemic solutions to eliminate or reduce the effect of the first stage on the efficiency of the second stage. Such solutions can be found by developing structural schemes that will make it possible to compensate or weaken the effect of the first degree of separation on the second stage.

In formulas (3)–(6), it is taken into account that the pressure is measured in MPa, and the hydrostatic pressure \( \rho gh \) in Pa. The linearization was carried out to solve the problem of mathematical models of the two-stage separation system [15].

6. Identification of the parameters of the mathematical model of the separation system

The initial data for calculating the parameters of the mathematical models of the separation system are given in Table 3.

The calculation of the compressibility factor \( z \) of natural gas is carried out according to the modified Benedict-Webb-Rabin equation [8]:

\[
z^3 - z^2 - az - b = 0,
\]

where

\[
a = \pi((0.1237/\tau) - (0.3468/\tau^2) - (0.1188/\tau^3));
\]
\[
b = \pi^2((0.0291/\tau^2) - (0.0273/\tau^3) + (0.0390/\tau^4));
\]
\[
\pi = (P + 1.33 	imes 10^{-4} P_0) / P_0 \) is the given pressure; \( \tau = (T_{st} + 273) / T_{kp} \) is the given temperature; \( P_{kp} \) is atmospheric pressure measured in mm Hg.; \( T_{kp} \) is temperature of natural gas measured in °C.

Table 3

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Designation</th>
<th>Value</th>
<th>Unit of measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure at the inlet of the separator B-1</td>
<td>( P_0 )</td>
<td>4.0</td>
<td>MPa</td>
</tr>
<tr>
<td>Gas pressure in the separator B-1</td>
<td>( P_1 )</td>
<td>1.6</td>
<td>MPa</td>
</tr>
<tr>
<td>Gas pressure in the separator B-2</td>
<td>( P_2 )</td>
<td>0.6</td>
<td>MPa</td>
</tr>
<tr>
<td>Oil density</td>
<td>( \rho_0 )</td>
<td>843</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Gas density at standard conditions</td>
<td>( \rho_{01} )</td>
<td>0.820</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Gas pressure in GSR tanks</td>
<td>( P_1 )</td>
<td>0.1</td>
<td>MPa</td>
</tr>
<tr>
<td>Gas pressure after ( ED_{B1} )</td>
<td>( P_{12} )</td>
<td>0.85</td>
<td>MPa</td>
</tr>
<tr>
<td>Gas pressure after ( ED_{B2} )</td>
<td>( P_{22} )</td>
<td>0.32</td>
<td>MPa</td>
</tr>
<tr>
<td>The volume of the separator B-1</td>
<td>( V_0 )</td>
<td>100</td>
<td>m³</td>
</tr>
<tr>
<td>The volume of the separator B-2</td>
<td>( V_{02} )</td>
<td>100</td>
<td>m³</td>
</tr>
<tr>
<td>Diameter of the separator B-1</td>
<td>( r_1 )</td>
<td>3.0</td>
<td>m</td>
</tr>
<tr>
<td>Diameter of the separator B-2</td>
<td>( r_{s1} )</td>
<td>3.0</td>
<td>m</td>
</tr>
<tr>
<td>Oil level in the separator B-1</td>
<td>( h )</td>
<td>1.51</td>
<td>m</td>
</tr>
<tr>
<td>Oil level in the separator B-2</td>
<td>( h_{s1} )</td>
<td>1.47</td>
<td>m</td>
</tr>
<tr>
<td>Diameter of the inlet pipeline</td>
<td>( D_1 )</td>
<td>0.219</td>
<td>m</td>
</tr>
<tr>
<td>The volume of oil supplied to the separation unit</td>
<td>( Q_1 )</td>
<td>3.71</td>
<td>m³/hour</td>
</tr>
<tr>
<td>Atmospheric pressure</td>
<td>( P_a )</td>
<td>762</td>
<td>mm Hg</td>
</tr>
<tr>
<td>Mass fraction of water</td>
<td>( w )</td>
<td>0.036</td>
<td>–</td>
</tr>
<tr>
<td>Productivity of the oil plant</td>
<td>( G_1 )</td>
<td>100</td>
<td>tons/day</td>
</tr>
<tr>
<td>Productivity of the gas plant</td>
<td>( Q_{gg} )</td>
<td>10²</td>
<td>nm³/day</td>
</tr>
</tbody>
</table>

The relative gas density by air is calculated by the formula [8] \( \rho_{rel} = \rho_{a1}/1.205 \), where \( \rho_{a1} \) is the density of natural gas under standard conditions.

The real root of the Benedict-Webb-Rabin equation determines the value of the gas compressibility factor.

The calculation of the parameters of the mathematical model of the separation system, given by the equations (3)–(6), was carried out according to a program written in the algorithmic language of the MatLab environment. The results of the calculations are summarized in Table 4.

The productivity of the separation unit for oil is \( G_{ol} = 100 \) tons/day, for gas \( Q_{gg} = 10^{15} \) nm³/day (Table 3), which we shall express in units of mass per unit of time according to the following formula

\[G_{gg} = \frac{Q_{gg}}{\rho_{gg} \cdot \frac{c}{k}},\]

where \( k \) is the conversion factor, which makes it possible to express \( G_{gg} \) in kg/sec \((k=86.400)\).

Then we will determine the mass fraction of gas in the oil and gas mixture as follows

\[x = \frac{G_{gg}}{G_{gg} \cdot k_1 + G_{gg}},\]

where \( k_1 = 10^{-5}/k \).
We will determine the density of the oil and gas mixture \( \rho_0 \) as far as we know the proportion of gas in the oil and gas mixture \( x \). The results of the calculations by the formula (10) are recorded in Table 4.

Since gas is supplied to the first stage of separation in the amount of \( G \) and \( G=\frac{G_0}{k_1}+G_0 \), then by comparing with the formula (10), we arrive at the conclusion that \( G_0=x \).

The efficiency of the first stage of gas separation \( \eta_{f1}^{(i)} \) can be calculated using the following formula

\[
\eta_{f1}^{(i)} = Q_f Q_w (1-w) (G_0 - \alpha_1 P_1),
\]

where \( \alpha_1 \) is the solubility factor of natural gas in oil.

Formula (11) can be represented as follows:

\[
Q_f \rho_s = Q_f \rho_g (\rho_g / \rho_f) (1-w) (G_0 - \alpha_1 P_1),
\]

where \( \rho_f \) is the density of the gas for the conditions of separation of the first degree.

Since \( G_0=\frac{G_0}{k_1}+G_0 \) and \( G_0=\frac{G_0}{k_1} P_0 P_1 \), taking into account that the equation (12) will take the form:

\[
G_0 = G_f (1-w) \left( \frac{G_0}{\rho_g} \right) \left( \frac{G_0}{\rho_f} \right) (\alpha_1 P_1).
\]

The density of the gas for the conditions of separation in B-1 is calculated by a formula that is similar to the formula:

\[
P_c \left( \frac{P_c}{T_c} \right) \gamma_{cs},
\]

where \( \gamma_{cs} = \left( \frac{P_c}{T_c} \right)^{1/2} \).

The gas compressibility factor \( z \) is calculated from the Benedict-Webb-Rabin equation at a pressure \( P_1 \) and temperature \( T_1 \) while \( T_1 = T_0 \). The solubility factor of natural gas \( \alpha_1 \) was determined from the graphical dependence, which is given in [2] (Table 4).

The productivity of the B-2 separator for gas \( G_{f2} \) will be determined as the difference between the overall performance of the separation unit and the performance of the separator B-1

\[
G_{f2} = G_f - G_{f1}.
\]

The calculated values \( G_{f1} \) and \( G_{f2} \) are recorded in Table 4. Table 4 contains the data on the performance (volumetric) of \( Q_{f1} \) and \( Q_{f2} \) of B-1 and B-2 separators, which are brought to normal conditions under the following formulas

\[
Q_{f,i} = k(G_{f,i}/\rho_0), \quad i=1,2,
\]

where \( Q_{f,i} \) has a dimension of \( \text{nm}^3/\text{day} \).

Recalculation of gas density \( \rho_0 \) to normal conditions (\( P_c = 0.1013 \text{ MPa}, T_c = 273 \text{ K} \)) was carried out according to the following formula:

\[
\rho_0 = \rho_c \left( \frac{P_c}{T_c} \right) \gamma_{cs}.
\]

Taking into account that the established mode of the separation system is considered, we find that:

\[
\gamma_{cs} = \frac{G_f}{\sqrt[1/2]{\rho_f (R_f - R)}}.
\]

Oil and gas mixture is supplied to the second stage of separation, which we calculate as the difference between the mass amount of oil and gas mixture \( G_f \), supplied to the separation system and the mass amount of oil in the gas mixture \( G_{f1} \):

\[
G_f \rho_s = G_f \rho_g (\rho_g / \rho_f) (1-w) (G_0 - \alpha_1 P_1).
\]
separation system and the amount of gas \( G_{g1} \) that has been isolated at the first stage of separation,

\[
G_{g1} = G_f - G_{g1}.
\]  

(19)

Then we determine the performance of the separator B-2 for oil by the formula, which is similar to the formula (19):

\[
G_{g2} = G_{f1} - G_{g2}.
\]

(20)

The mass fraction of gas in the oil and gas mixture entering the second stage of separation will be as follows:

\[
x_1 = \frac{G_{g2}}{G_{f1}}.
\]

(21)

Since the established mode of the separation system is considered, we obtain the following system of algebraic equations from the equations (3)–(6), which describe the dynamics of the separation system:

\[
\alpha_1 \left( U_1 \right) \frac{1}{\theta_f} \sqrt{\frac{P^2_f - P^2_{g1}}{Z_{\alpha}}} + + \alpha_2 \left( U_1 \right) \frac{P_f}{M_0} \rho_f \left( P_f + \rho_f gh \cdot 10^{-6} - P_f \right) = \frac{\rho_{g1}}{\rho_f} \sqrt{\frac{P^2_f - P^2_{g1}}{Z_{\alpha}}} - \alpha_1 \left( U_1 \right) \frac{1}{\theta_f} \sqrt{\frac{P^2_f - P^2_{g1}}{Z_{\alpha}}} - - \alpha_2 \left( U_1 \right) \frac{P_f}{M_0} \rho_f \left( P_f + \rho_f gh \cdot 10^{-6} - P_f \right) = 0,
\]

(22)

\[
\alpha_3 \left( U_1 \right) \frac{\rho_{g1}}{\rho_f} \sqrt{\frac{P^2_f - P^2_{g1}}{Z_{\alpha}}} - - \alpha_3 \left( U_1 \right) \frac{P_f}{M_0} \rho_f \left( P_f + \rho_f gh \cdot 10^{-6} - P_f \right) = 0.
\]

(23)

The system of equations (22)–(25) is linear according to the unknown values \( \alpha_2 \left( U_1 \right), \alpha_3 \left( U_1 \right), \) and \( \alpha_4 \left( U_1 \right) \).

We introduce the following designations:

\[
\begin{align*}
\alpha_1 & = \frac{1}{\theta_f} \sqrt{\frac{\left( P^2_f - P^2_{g1} \right)}{Z_{\alpha}}} , \\
\alpha_2 & = \frac{P_f}{M_0} \rho_f \left( P_f + \rho_f gh - P_f \right) , \\
\alpha_3 & = \rho_{g1} \left( P_f + \rho_f gh - P_f \right) , \\
\alpha_4 & = 0, \quad \alpha_{41} = 0, \\
\alpha_{42} & = 0, \quad \alpha_{43} = 0, \quad \alpha_{44} = 0.
\end{align*}
\]

(24)

The system of equations (22)–(25) was solved by the Gauss's reverse method with the choice of the maximal element [13].

When recording equations (22)–(25), it was taken into account that the pressures \( P_1, P_2, P_3, P_{g1} \), and \( P_{g3} \) are measured in MPa, and the hydrostatic pressure \( P = gh \) is measured in Pa.

When calculating the values of \( \theta_1 \) and \( \theta_3 \) (Table 4), we assume that the temperatures in the separators B-1 and B-2 are the same. The gas constant \( R_g \) for natural gas, which is included in the expressions \( \theta_1 \) and \( \theta_3 \) (Table 4), is calculated by the following formula [8]:

\[
R_g = (288.15/\Lambda) \cdot 10^{-6},
\]

(27)

where \( R_g \) is measured in MJ/kg·K.

7. Results of studies of a two-stage separation unit

The mathematical model (3)–(6) of the separation system will be solved with the following assumptions:

- pressures \( P_0 \), \( P_{g0} \), and \( P_{g3} \) are accepted as stable;
- the temperature of the gas stream entering the separation system and the temperature of the products in the separators B-1 and B-2 are the same and unchanged in time for the period of the transition time (\( T_0 - T_2 = T_1 \));
- we neglect the influence of temperature on the density of oil;
- the gas compressibility factor is computed at pressure.
Industry control systems

\[ \bar{p}_d = \frac{p_1 + p_2}{2} \]

and temperature \( T_1 \):

- gas in the separator B-2 is considered ideal;
- we consider mass particles of gas \( x \) and \( x_1 \) in the separators B-1 and B-2 unchanged.

The input values that cause the change in the output values \( P_1 \) and \( P_3 \) and levels \( h_1 \) and \( h_2 \) in the separators B-1 and B-2 will be considered as changes in the values of \( a_x \) (\( U_1 \)), \( a_o \) (\( U_2 \)), \( a_s1 \) (\( U_3 \)) and \( a_s2 \) (\( U_4 \)).

We use the Runge-Kutta method [13] to solve the system of differential equations that describe the dynamics of the separation unit, which in the vector form implements the following iterative procedure:

\[
\bar{p}(\bar{y}) = \bar{f}(\bar{y}),
\]

\[
\bar{y}_{k+1} = \bar{y}_k + \frac{h}{2} \bar{f}(\bar{y}_k + \frac{h}{2} \bar{f}(\bar{y}_k)),
\]

\[
\bar{y}_{k+1} = \bar{y}_k + h \bar{f}(\bar{y}_k),
\]

where

\[
\bar{y} = \begin{bmatrix} P \cr h \cr p \cr t \end{bmatrix}
\]

is the vector of output quantities;

\[
\bar{f}(\bar{y}) = \begin{bmatrix} \frac{1}{1 - \nu \chi}(\xi_1 \sqrt{\rho_1 (p_1 - p_2)} - \frac{\chi}{2} T_1) - \alpha_o (U_1) \left( \frac{P_1^2 - P_2^2}{\theta_2} \right) - \frac{P_1 - \alpha_s(U_1) \sqrt{\rho_1 (p_1 + p_2 gh - 10^a - p_2)}}{\theta_2} - \frac{1}{q_p(h) M_1} \left( \alpha_o(U_1) \left( \sqrt{\rho_1 (p_1 + p_2 gh - 10^a - p_2)} \right) \right) \end{bmatrix}
\]

is a vector-function, the components of which are right parts of the system of differential equations (3)–(6).

The step of discreteness \( h_t \) is calculated by the formula \( h_t = (t_f - t_0)/n \), where \( t_0, t_f \) is the start and end time; \( n \) is the number of iterations in the computational process.

The program for solving the system of differential equations (3)–(6) is written in the algorithmic language of the MatLab package.

As an example of solving the system of differential equations (3)–(6), consider the change in the output values \( P_1 \), \( h \) and \( h_1 \) in time, depending on the change in the input value \( a_o(U_2) \), which is defined as follows:

\[
a_o(U_2) = a_o(U_2^{(0)}) + \Delta a_o(U_2). \tag{29}
\]

The increment of the value \( \Delta a_o(U_2) \) will be calculated as

\[
\Delta a_o(U_2) = \alpha_o(U_2^{(0)}) - \alpha_o(U_2^{(0)}). \tag{30}
\]

Taking into account the value \( \Delta a_o(U_2) \), the formula (30) will take the form of:

\[
a_o(U_2) = a_o(U_2^{(0)}) (1 + \chi). \tag{30}
\]

The values of \( a_x(U_1), a_o(U_2), a_s1(U_3) \) and \( a_s2(U_4) \), in essence, are the throughput characteristics of the regulatory bodies of the corresponding actuators (Fig. 1).

Table 5 establishes the correspondence between the parameters of the mathematical model (3)–(6) of the separation system and machine variables. In the same table, the numerical values of the values included in the model (3)–(6) are given.

In the process of solving the mathematical model (3)–(6), the following values of variables selected by the user were selected:

- the number of iterations \( n = 500 \);
- the duration of the transition process \( t_r = 800 \) sec;
- the change value of \( a_o(U_2^{(0)}) = \chi = 1.0 \).

The result of a numerical solution of a mathematical model is shown in Fig. 2–4.

Fig. 2. The change of pressure \( P_1(t) \) in the separator B-1 as a function of time \( t \)

Fig. 3. The change of the fluid level \( h(t) \) in the separator B-1
Correspondence between the parameters of the mathematical model and the machine variables

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Designation</th>
<th>Machine variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The degree of filling of the separator B-1</td>
<td>( v_{o}(h) )</td>
<td>( \text{Vp} )</td>
<td>0.5042</td>
</tr>
<tr>
<td>The degree of filling of the separator B-2</td>
<td>( v_{p1}(h_{1}) )</td>
<td>( \text{Vp1} )</td>
<td>0.4823</td>
</tr>
<tr>
<td>Density of oil and gas mixture</td>
<td>( \rho_{f} )</td>
<td>( \text{ro} )</td>
<td>70.47</td>
</tr>
<tr>
<td>–</td>
<td>( \theta_{1} )</td>
<td>( \text{teta} )</td>
<td>830.2487</td>
</tr>
<tr>
<td>The intensity of change in the degree of filling of the separator B-1</td>
<td>( q_{1}(h) )</td>
<td>( \text{Q} )</td>
<td>0.4243</td>
</tr>
<tr>
<td>The intensity of change in the degree of filling of the separator B-2</td>
<td>( q_{1}(h_{1}) )</td>
<td>( \text{q1} )</td>
<td>0.4243</td>
</tr>
<tr>
<td>Gas solubility factor</td>
<td>( \alpha_{1} )</td>
<td>alpha1</td>
<td>0.44</td>
</tr>
<tr>
<td>Productivity of the separator B-1 by gas (mass)</td>
<td>( G_{g1} )</td>
<td>( \text{Gg1} )</td>
<td>0.6453</td>
</tr>
<tr>
<td>Productivity of the separator B-2 by gas (mass)</td>
<td>( G_{g2} )</td>
<td>( \text{Gg2} )</td>
<td>0.3037</td>
</tr>
<tr>
<td>–</td>
<td>( a_{s}(U_{1}) )</td>
<td>alphag, ( U_{1} )</td>
<td>0.4761</td>
</tr>
<tr>
<td>Gas separation factor</td>
<td>( \varepsilon_{d} )</td>
<td>eps.g</td>
<td>0.3064</td>
</tr>
<tr>
<td>Productivity of the separator B-1 by liquid</td>
<td>( G_{l1} )</td>
<td>( \text{Gl1} )</td>
<td>1.4611</td>
</tr>
<tr>
<td>Separation factor of the separator B-1</td>
<td>( \varepsilon_{f1} )</td>
<td>eps_f1</td>
<td>0.6936</td>
</tr>
<tr>
<td>Productivity of the separator B-2 by oil</td>
<td>( G_{o2} )</td>
<td>( \text{Go2} )</td>
<td>1.1574</td>
</tr>
<tr>
<td>Density of liquid in the separator B-1</td>
<td>( \rho_{f1} )</td>
<td>( \text{ro} )</td>
<td>60.30</td>
</tr>
<tr>
<td>Mass of liquid at ( h=D ), where ( D ) is the diameter of the separator B-1</td>
<td>( M_{0} )</td>
<td>( \text{Mo} )</td>
<td>6.0301·10⁷</td>
</tr>
<tr>
<td>–</td>
<td>( a_{s}(U_{1}) )</td>
<td>alphaN, ( U_{2} )</td>
<td>0.1881</td>
</tr>
<tr>
<td>–</td>
<td>( \varepsilon_{d1} )</td>
<td>eps_g1</td>
<td>0.2079</td>
</tr>
<tr>
<td>Gas separation factor (second separation segment)</td>
<td>( \theta_{1} )</td>
<td>( \text{teta} )</td>
<td>829.05</td>
</tr>
<tr>
<td>Mass of liquid at ( h=H_{1} ), where ( D_{1} ) is the diameter of the separator B-2</td>
<td>( M_{01} )</td>
<td>( \text{Mo1} )</td>
<td>84.300</td>
</tr>
<tr>
<td>–</td>
<td>( a_{s}(U_{1}) )</td>
<td>alphaN1, ( U_{2} )</td>
<td>0.0703</td>
</tr>
<tr>
<td>–</td>
<td>( a_{s}(U_{1}) )</td>
<td>alphag1, ( U_{1} )</td>
<td>10.1294</td>
</tr>
<tr>
<td>Oil density</td>
<td>( \rho_{o} )</td>
<td>( \text{ro} )</td>
<td>843</td>
</tr>
</tbody>
</table>

The above graphs (Fig. 2–5) for the steady-state operation of the two-stage separation system are the result of a program that is written in the Matlab algorithmic language.

8. Discussion of the results of the study of the simulation model of the process of two-stage oil separation

The mathematical model of the process of two-stage separation is obtained, which, in contrast to the known models [10, 13], takes into account the interaction of the first and second stages of separation. The simulation model of the two-stage separation system is investigated by a numerical method in the MatLab’s mathematical laboratory to change the key parameters of the level and pressure in each separator. The results obtained in the form of graphs (Fig. 2–5) describe the dynamics of transients for each separator individually, which collectively reflects the work of the system in general.

Due to the combination of two horizontally placed B-1 and B-2 separators and an oil storage tank, functional dependencies were obtained that allow establishing the interconnection of the technological parameters, in other words, the change in the value of the parameters at the second stage of separation will depend on the first one and vice versa. The advantage of such use of technological equipment is the possibility of improving the one-stage technological process of oil separation, in particular, increasing the overall oil separation factor. The study was conducted with the following assumptions:
- pressures \( P_{0}, P_{g2} \) and \( P_{g3} \) are accepted as stable;
- the temperature of the gas stream entering the separation system and the temperature of the products in the...
separators B-1 and B-2 are the same and unchanged in time for the period of the transition time \((T_0 = T_2 = T_1)\):
- the gas compressibility factor is computed at pressure and temperature \(T_1\);
- we considered gas in the separator B-2 ideal;
- we considered mass particles of gas \(x_1\) and \(x_2\) in the separators B-1 and B-2 unchanged. The disadvantage is the neglect of the influence of temperature on the density of oil.

This simulation model is investigated by a numerical method in MatLab and can be used to synthesize effective control systems for the process of two-stage separation and to create mathematical models in terms of “input-output”. Difficulties in further research may be low informing about the processes associated with oil production and not taking into account new perturbations that can influence the system in general.

**9. Conclusions**

1. A mathematical model of the material balance of the two-stage separation for the first B-1 and the second B-2 horizontally placed separators is created, which takes into account the interaction and mutual influence of the first and second stages of separation. Automated control systems that operate on the principle of negative feedback were used to stabilize the main technological parameters of oil separation. Since the deviations of the regulated quantities from their prescribed values are small, this allowed linearizing the nonlinear model of the two-stage process of oil separation.

2. The parameters of the mathematical system of two-stage separation for simulation modeling are identified. All values of physical quantities are obtained for the steady mode of operation of the two-stage separation system. During the identification of the parameters, it was assumed that the pressures \(P_1, P_2, P_3, P_4, P_5\) and \(P_6\) are measured in MPa, and the hydrostatic pressure \(P_{gh}\) is measured in Pa, and it is assumed that the temperatures in the separators B-1 and B-2 are the same when calculating the values of \(\theta_1\) and \(\theta_2\).

3. As a result of the research of the newly created simulation model in the MatLab program, it was determined that the increase in pressure \(P_1\) is the increase in the fluid level \(h\) in the separator B-1 from 1.51 m to 1.6319 m. It was also established that an increase in the hydraulic resistance due to the closure of the regulatory body of the executive mechanism of the ED, which is mounted on the initial line of the separator B-1, causes an increase in pressure \(P_2\) from 1.6 MPa to 3.9970 MPa. On the basis of changes of the technological parameters \(P_1\) and \(h\) in the separator B-2, it is obvious that the liquid level \(h_2\) is from 1.47 m to 1.4448 m due to the decrease in the flow of liquid from the separator B-1. Such a decrease in the liquid level in the separator B-2 entails a reduction in the pressure \(P_2\) from 0.6 MPa to 0.3210 MPa. This means that the received scientific result in the form of functional dependencies allows us to establish the relationship of technological parameters, in other words, the change of the value of the parameters at the second stage of separation will depend on the first one and vice versa. Thus, the applied aspect of using the obtained scientific result is the possibility of improving the typical technological process of oil separation.

**References**