1. Introduction

The task to improve operation efficiency of traction rolling stock has remained relevant over the entire period of existence of railways. Scientific research and new technical solutions have led to significant progress in this direction in recent years. Given the advances in the fields of chemistry, materials science, industrial electronics, etc., the latest element base and materials became available for rolling stock and significantly decreased the costs of transportation, increased speed and reliability of rolling stock. It is worth noting that an up-to-date locomotive is a complex, multi-purpose developed system. This predetermines the relevance of research into improvement of rolling stock via determining and formal description of internal relations between elements of a given system.

2. Literature review and problem statement

At present, most research aimed at enhancing effectiveness of rolling stock operation are conducted in the following main areas: improvement of efficiency of energy conversion and power transmission [1], improvement of onboard control systems, enhancement of conditions for implementation of traction and braking efforts of rolling stock [2]. One of the main factors for increasing the quality of traction of wheels and rails is the development of more advanced structures of crew pieces of rolling stock [3–5]. Today, introduction of new locomotive control systems as the factor of energy efficiency is directly related to intelligent technologies [6, 7]. However, despite a wide range of issues, addressed in these and many other scientific papers, there are still reserves for an increase in efficiency of rolling stock. One of these reserves is the improvement of interaction between traction elements of the power circuit. The grounds for such problem statement include modern technological requirements for traction electric motors, wheelsets, traction reducers and other sections of power transmission [8].
from wheelsets on rails [9], alignment of currents by parallel branches of the power circuit [10], were developed and implemented, the design of bearing elements of rolling stock [11] was improved, interaction of the wheel rolling surface with rails and brake pads [12] was improved and so on. However, these developments have not enabled the full use of the traction mass of locomotives and maximal efficiency of their operation under all modes of operation [13, 14]. A possible cause of this situation is the lack of a comprehensive approach to increasing the coefficient of traction mass in the existing studies. In other words, high quality methods for elimination of particular harmful influences on implementation of traction effort were developed. But their relationship with the design features of certain locomotives, which can both increase and decrease effectiveness of each method, has not been determined yet. Thus, the problem of a comprehensive consideration of particular transmission parameters and their overall impact on the traction effort of direct current locomotives has not been sufficiently solved so far.

3. The aim and objectives of the study

The aim of present research is to develop a comprehensive mathematical model of current distribution in the power circuit of a direct current locomotive for substantiation of improvement of traction conditions and enhancing efficiency of locomotive operation.

To accomplish the set goal, it is necessary to solve the following problem:

– to explore the influence of non-uniformity of current distribution in a power circuit on power efficiency of operation of locomotives;

– to define parameters of wheel-motor units that affect non-uniformity of current distribution;

– to develop the method for analytical calculation of the total impact of design, technological and operational factors on current distribution in a power circuit;

– to determine a condition for elimination of non-uniformity of current distribution.

4. Materials and methods of research

4.1. Study of influence of the non-uniformity of current distribution in a power circuit on energy efficiency of locomotive operation

The study of current distribution was performed at the locomotive DE1 using the on-board diagnostics systems “Magistral-DE1”, which is installed in each section. It was determined that the non-uniformity of current distribution occurs in parallel (P) and series-parallel (SP) connections of a power circuit. Distribution of currents with variable speed is represented in Fig. 1, 2. Here, I1, I2, I3, I4 are the currents in their respective branches.

According to research, at an increase in motion speed, the difference in powers of parallel branches decreases (Fig. 3).

Deviation of currents influences the locomotive’s traction power, in particular contributes to an increase. This is proved by the curve in Fig. 4. Magnitude ΔF here is the difference between calculated traction power for a given mode and readings of traction power of the diagnosis system. ΔF is positive, i.e. at an increase in non-uniformity of current distribution, the implemented locomotive traction power decreases.

Thus, the results of study of the non-uniformity of current distribution indicate that there is a relationship between the magnitude of deviation of current of traction motors and...
traction effort and energy efficiency of a locomotive. In addition, the difference of powers, which are implemented by traction motors, leads to non-uniformed wear of wheel-motor units and a decrease in serviceability of rolling stock.

4.2. Determining the parameters of wheel-motor units that affect the non-uniformity of current distribution

**Difference of magnetic characteristics.** In [15], it is indicated that the deviation of speed characteristic of the traction motor from the calculated one, like the difference between characteristics of motors between one another, is determined by a number of factors. These include asymmetry of magnetic system of motors and the position of brushes relatively to the neutral.

This component, in turn, depends on magnetizing longitudinal force and degree of saturation of the magnetic system.

With displacement of brushes from the neutral in the direction of rotation of the armature of the traction motor, magnetic flux increases and rotation frequency of the armature of the traction motor decreases. Displacement of brushes opposite the direction of rotation of the armature, in contrast, leads to a decrease in the resulting flow of the motor, and rotation frequency of the armature increases. Experience shows that the 1 mm displacement of brushes, for example, of the traction motor NB-406 B, at the hourly mode causes a change in rotation frequency of armature on average by 4.3 rpm. This proportion remains until the brush is displaced approximately by 10 mm.

For motors of series excitation, at relative rigidity of speed characteristics of 2–2.5 load deviation reaches 15–20%. It significantly increases at more strict characteristics of motors of mixed and independent excitation.

Boundary deviations of parameters of traction motors with different operation modes are regulated by the standard on their production. Tolerance of the armature rotation frequency when operating at nominal power and full field is ±3%; when operating on a weakened field of up to 50%, it is ±3.5%; larger than 50%, it is ±6%.

Tolerances on rotation frequencies when operating with minimal and maximal speeds are respectively ±3.0; ±5.0; ±7.0% on the full field, on weak minimal and maximal speeds are respectively ±3.0; ±3.0; ±3.5%; larger than 50%, it is ±6%.

Non-uniformity of current distribution on divergence of magnetic characteristics of traction motors is determined by the graphic-analytical method in the following way.

**Speed characteristics of the traction motor applying dependences [16] are calculated**

\[ n = \frac{0.5I_{em} - IR_g}{c_e \Phi}, \]  

where \( U_{em} \) is the current of the contact network, W; \( I \) is the current of motor loading, A; \( c_e \) is the constant of the motor by emf; \( R_g \) is the total resistance of the hot traction electric motor, Ohm; \( \Phi \) is the main magnetic flux, Wb;

Total active resistance of a hot traction motor is determined from expression

\[ R = (R_{20} + R_{mp20} + R_{mp20}) \beta, \]  

where \( R_{20} \) is the resistance of winding of the armature of the traction electric motor at 20 °C, Ohm; \( R_{mp20} \) is the resistance of winding of main poles and shunting resistors at 20 °C, Ohm; \( \beta \) is the temperature coefficient of resistance of copper of winding of electric motor.

In this case

\[ R_{mp20} = \frac{R_{mp} - R_{mp} \cdot R_{min} \cdot R_{mp} + R_{mp} \cdot R_{mp} \cdot R_{min}}{1 + \alpha}, \]  

where \( R_{mp} \) is the resistance of winding of main poles of traction electric motors at 20 °C, Ohm; \( R_{mp} \) is the resistance of the shunting resistor, Ohm; \( \alpha \) is the coefficient of field weakening.

\[ \Phi = \Phi_v - \Phi_{ar}, \]  

where \( \Phi_v, \Phi_{ar} \) are the magnetic fluxes of idle running and the reaction of the armature of the traction electric motor, Wb.

**Deviation of resistances of armature circuits.** A branch of the power circuit of modern direct current locomotives consists of many elements. Considering non-uniformity of current distribution at a parallel connection of traction motors, it may be noted that it depends on two groups of factors:

- structural, caused by the location of traction electric motors, and hence the difference in lengths of connecting wires and magnitudes of resistance of resistors;
- technological, dependent on tolerances on the elements of the armature circuit, resistances in electric contacts.

Calculation of non-uniformity of current distribution on divergence in resistances groups of armature circuits of traction electric motors is performed in the graphic-analytic way. A number of values of resistances of the armature circuit, equal to nominal and increased by 5, 10, 15 and 20 %, are accepted. For each selected value of current and resistance, rotation frequencies, magnetic fluxes are calculated from formulas and characteristics are constructed.

The value of non-uniformity of current distribution in characteristic points is determined and dependences of maximum possible values of non-uniformity of current distribution \( \Delta I \) on deviations of resistances \( \Delta R_g \) of armature circuit are constructed.

On the full field of excitation, minimum value of resistance of one power circuit that affects non-uniformity of current distribution is calculated by the formula (for DE1)

\[ R_{minFF} = R_{Cmin} + 2R_g + 9R_{kmin} + R_{cont}, \]  

where \( R_{minFF} \) is the minimal resistance of the power circuit of the full field, Ohm; \( R_{Cmin} \) is the resistance of cables, Ohm; \( R_{kmin} \) is the minimal resistance of contacts of the armature field, Ohm; \( R_g \) is the resistance of the armature circuit of the traction motor, Ohm; \( R_{cont} \) is the resistance of elements of control and automation (contacts of the linear contactor, load current sensors, shunts of amperemeters, contacts of braking switch, etc.)

For other series of direct current locomotives, calculation is similar, it differs only by the coefficient at \( R_{kmin} \), depending on the layout of the scheme.

Maximum resistance \( R_{maxFF} \) of the power circuit on full field of excitation is found in a similar way.

Boundary minimum resistance of circuit \( R_{minWF} \) at operation of the electric motor on a weakened field of excitation is
Control processes

\[
R_{\text{minWF}} = (R_{\text{maxWF}} - 2R_{\text{WE}}) - \frac{(R_{\text{min}} + R_{\text{min}} + R_{\text{min}} + 2R_{\text{WE}})(R_{\text{min}} + R_{\text{min}} + R_{\text{min}} + 2R_{\text{WE}})}{R_{\text{min}} + R_{\text{min}} + R_{\text{min}} + 2R_{\text{WE}}} \tag{6}
\]

where \( R_{\text{WE}} \) is the resistance of winding of excitation of a traction electric motor, Ohm; \( R_{\text{min}} \) is the minimum resistance of wires, Ohm; \( R_{\text{min}} \) is the minimal resistance of shunting resistor, Ohm; \( R_{\text{min}} \) is the minimal resistance of contactor of shunting, Ohm.

Maximal value of resistance of armature circuit at weakening of field \( R_{\text{maxWF}} \) is determined in a similar way.

Deviation of resistances of excitation circuits. The circuit of excitation of the traction electric motor consists of a winding of excitation of a traction electric motor, connecting wires, inductive shunt, resistors of field weakening, contactors, shunting.

Non-uniformity of current distribution for different coefficients of shunting is determined by the graphical-analytical method. Values of resistances of shunting resistors that provide regulated magnitudes of coefficients of field weakening are calculated. Operation characteristics of a motor are determined for five selected values of currents and coefficients of shunting. After assigning deviations of magnitudes of shunting resistors, that cause deviations of field weakening by 10, 20, 30 % of the calculated, working characteristics for these deviations are determined. In this case, the characteristic that corresponds to calculated coefficient of shunting is accepted as basic.

In coordinates of the difference of currents \( \Delta I \) and percentage deviation of value of resistance of shunting resistors, we construct dependences of non-uniformity of current distribution for different coefficients of shunting. Operation characteristics of a motor are determined for five selected values of currents and coefficients of field weakening by 10, 20, 30 % of the calculated, working characteristics for these deviations are determined. In this case, the characteristic that corresponds to calculated coefficient of shunting is accepted as basic.

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\[ n_x = n_i - \Delta n. \]  

Resistance of windings of motor 2

\[ R_{ckv} = \Delta R_{ckv} + R_{ckv}. \]

Then (11) will take the form

\[ \Delta I = \left( \frac{U - c_n \Phi_1 - n_i - U - c_n (\Phi_1 + \Delta \Phi) (n_i + \Delta n)}{R_{ckv} + \Delta R_{ckv}} \right) = \frac{U \cdot \Delta R_{ckv} - \Delta R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta R_{ckv} \cdot n_i}{R_{ckv} + \Delta R_{ckv}} + \frac{\Delta n \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1}{R_{ckv} + \Delta R_{ckv}}. \]  

Let us represent magnetic flux through load current. For motors of series excitation at full field

\[ \Phi_x = k_1 I_1, \quad \Phi_x = k_1 I_2. \]  

Here \( k_1 \) and \( k_2 \) are the variable coefficients, the values of which vary depending on currents \( I_1 \) and \( I_2 \).

From formula (12)

\[ \Delta \Phi = k_1 I_1 - k_1 I_2. \]  

Then expression (15) will take the form

\[ \Delta I = \frac{U \cdot \Delta R_{ckv} - \Delta R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta R_{ckv} \cdot n_i + \Delta n \cdot c_n \cdot k_1 \cdot I_1 + n_i \cdot R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1}{R_{ckv} + \Delta R_{ckv}}. \]  

Let us express \( I_2 \) through \( I_1 \):

\[ I_2 = I_1 + \Delta I. \]

Then

\[ \Delta I = \frac{U \cdot \Delta R_{ckv} - \Delta R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta R_{ckv} \cdot n_i + \Delta n \cdot c_n \cdot k_1 \cdot I_1 + n_i \cdot R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1 + \Delta n \cdot c_n \cdot k_1 \cdot I_1}{R_{ckv} + \Delta R_{ckv}}. \]

We decompose the first fraction into two, open the brackets in the numerator of the second fraction and separate the fractions, which contain in numerator \( I_1 \) from those that contain \( \Delta I \).

\[ \Delta I = \frac{U \cdot \Delta R_{ckv} - \Delta R_{ckv} \cdot c_n \cdot k_1 \cdot I_1 + \Delta R_{ckv} \cdot n_i + \Delta n \cdot c_n \cdot k_1 \cdot I_1}{R_{ckv} + \Delta R_{ckv}} + \frac{I_1 \cdot c_n \cdot 2 \Delta n \cdot k_1 + n_i \cdot k_1 - k_1 \cdot n_i + \Delta n \cdot k_1 + \Delta I \cdot c_n \cdot k_1 \cdot I_1 - k_1 \cdot I_1 + I_1 \cdot c_n}{R_{ckv} + \Delta R_{ckv}}. \]

We solve the equation relative to \( \Delta I \):

\[ \Delta I \left( 1 + c_n \cdot k_1 \right) = \frac{U \cdot \Delta R_{ckv}}{R_{ckv} + \Delta R_{ckv}} + \frac{I_1 \cdot c_n \cdot \left( \frac{\Delta R_{ckv} \cdot k_1 \cdot n_i}{R_{ckv} + \Delta R_{ckv}} + \frac{\Delta R_{ckv} \cdot k_1 \cdot n_i + \Delta n \cdot k_1 + 2 \Delta n \cdot k_1 + n_i \cdot k_1 - k_1 \cdot n_i + \Delta n \cdot k_1}{R_{ckv} + \Delta R_{ckv}} \right)}{R_{ckv} + \Delta R_{ckv}}. \]  

Dependence of magnitude of magnetic flux on current of the motor will be presented in the analytical form. As a result of breaking the curve of dependences \( \Phi(I) \) into three sections, approximations of the middle curvilinear part, we can write down the formula of magnetic flux of the motor. For the motor of series NB-406B, it would be in the following form:

\[ \Phi = \begin{cases} 
0,000588I \quad \text{if} \quad 0 \leq I \leq 200, \\
1,54321 \times 10^3 I^2 - 1,74537 \times 10^2 I - 0,000768241I + 0,021421 \quad \text{if} \quad 200 < I < 380, \\
0,00008955 \times I + 0,11197 \quad \text{if} \quad 380 \leq I \leq 600.
\end{cases} \]  

For the motors of series ED141AU1 (locomotive DE1), dependence of magnetic flux on load current is determined in the similar way based on its characteristics.

Formula (19) describes the standard (reference) traction electric motor that has no deviations from the passport data. However, in actual motors, there are deviations in characteristics, which have different causes. It is possible to judge about these deviations, based on the bench test data, from the passports of motors.

Let us assume that bench measurements revealed that rotation frequency of a tested TEE is different from the standard. Assuming all other parameters being equal, we will find magnetic flux for \( n_i \).

\[ \Phi = \frac{U - IR_{ckv}}{n_i c_n}. \]

where \( n_i \) is the frequency, measured during bench tests.

It is not difficult to see that

\[ \Phi = k_n \Phi_s. \]

Magnitude

\[ k_n = \frac{n_i}{n_s}. \]

will be called coefficient of deviation of magnetic flux, which shows by how many times the magnetic flux of the reference motor differs from the magnetic flux of the actual motor. Then

\[ \Phi = k_n \Phi_s. \]

In accordance with these considerations, the formula of rotation frequency for each actual motor is

\[ n = \frac{U - R_{ckv} \cdot I}{c_n \cdot k_n \cdot \Phi_s}. \]
Returning to formulas (16), magnetic flux can be represented in the form of

\[
\Phi_i = k_{et} \Phi_{et} I_i,
\]

\[
\Phi_{et} = k_{et} \Phi_{et} I_{et},
\]

(25)

where \( k_{et} \) and \( k_{et} \) are the coefficients of deviations of fluxes of motor 1 and motor 2; \( k_{et} \) is the coefficient of proportionality between magnetic flux and load current of the reference motor.

In formula (18), we give the values of coefficients \( k_1 \) and \( k_2 \) to \( k_{et} \) and \( k_{et} \).

\[
\Delta I = \frac{U \cdot \Delta R_{et}}{R_{et}^2 + R_{et} \cdot \Delta R_{et} + R_{et} \cdot c_i \cdot k_{et} \cdot k_{et} \cdot V \cdot i \cdot 60 \left( \frac{2}{D_1} - \frac{1}{D_2} \right) +} + I_i \cdot V \cdot i \cdot 60 \left( \frac{2}{D_1} - \frac{1}{D_2} \right)
\]

Expression (28) demonstrates the structure of occurrence of non-uniformity of current distribution in the power circuit of locomotives. It takes into account the basic factors influencing deviation of current – the difference of electrical resistances, diameter of tires and magnetic fluxes.

5. Results of research into current distribution in a power circuit

Summing up the above calculation order, it is possible to represent deviations of currents in a general form as

\[
\Delta I = \rho + I_i \cdot \varepsilon,
\]

(29)

where

\[
\rho = \frac{U \cdot \Delta R_{et}}{R_{et}^2 + R_{et} \cdot \Delta R_{et} + R_{et} \cdot c_i \cdot k_{et} \cdot k_{et} \cdot V \cdot i \cdot 60 \left( \frac{2}{D_1} - \frac{1}{D_2} \right) +}
\]

\[
\varepsilon = V \cdot i \cdot 60 \left( \frac{2}{D_1} - \frac{1}{D_2} \right)
\]

(30)

Coefficients \( \rho \) and \( \varepsilon \) in the general case depend on connected voltage, speed of motion, difference of ohmic resistances of motors’ circuits, design of a machine, coefficients of flux deviations and diameter of threads of motor’s wheelsets. That is:

\[
\rho = f(U, \Delta R_{et}, V, c_i, k_{et}, k_{et}, k_{et}, i, D_1)
\]

(31)
At $\Delta R_{\text{dc}}=0$, expression (29) takes the form
\[
\Delta I = I_1 \cdot V \frac{i \cdot 60 \cdot c_\epsilon \cdot k_\text{st}}{\pi} \times \\
2 \left( \frac{1}{D_1} - \frac{1}{D_2} \right) k_{s1} + 1 \cdot \frac{D_1}{D_2} k_{s1} - k_{s2} \left( \frac{1}{D_1} \left( \frac{1}{D_2} - \frac{1}{D_1} \right) \right) k_{s2}.
\]
\[
(32)
\]
At $D_1=D_2$, expression (29) takes the form
\[
\Delta I = \frac{U \cdot \Delta R_{\text{dc}}}{R_{\text{kvr}1} + R_{\text{kvr}2} \cdot \Delta R_{\text{dc}} + R_{\text{kvr}1} \cdot c_\epsilon \cdot k_{\text{st}} \cdot k_{s2} \cdot V \frac{i \cdot 60 \cdot c_\epsilon \cdot k_{\text{st}}}{\pi \cdot D_1}} + 1 \cdot \frac{D_1}{D_2} k_{\text{st}} \times \\
\begin{bmatrix}
\Delta R_{\text{dc}} \cdot k_{\text{st}} \cdot \frac{1}{D_1} \\
R_{\text{kvr}1} + R_{\text{kvr}2} \cdot \Delta R_{\text{dc}} + R_{\text{kvr}1} \cdot c_\epsilon \cdot k_{\text{st}} \cdot k_{s2} \cdot V \frac{i \cdot 60 \cdot c_\epsilon \cdot k_{\text{st}}}{\pi \cdot D_2} \\
+ \frac{1}{D_1} k_{\text{st}} \\
R_{\text{kvr}1} + \Delta R_{\text{dc}} \cdot c_\epsilon \cdot k_{\text{st}} \cdot k_{s2} \cdot V \frac{i \cdot 60 \cdot c_\epsilon \cdot k_{\text{st}}}{\pi \cdot D_2}
\end{bmatrix}
\]
\[
(33)
\]
Thus, the approach to identification and assessment of separate factors influencing non-uniformity of current distribution in the power circuit, was developed.

**6. Discussion of results of the study of power circuit of traction rolling stock**

Results of the research make it possible to argue that to eliminate the magnitude of deviation of currents, it is necessary that $\rho=0$ and $\varepsilon=0$, i.e., numerators of fractions turned into 0. Let us analyze conditions of their being equal to zero.

\[
U \cdot \Delta R_{\text{dc}} = 0,
\]
\[
(34)
\]
\[
\Delta R_{\text{dc}} \cdot k_{\text{st}} \cdot \frac{1}{D_1} = 0,
\]
\[
(35)
\]
\[
2 \left( \frac{1}{D_1} - \frac{1}{D_2} \right) k_{s1} + 1 \cdot \frac{D_1}{D_2} k_{s1} = -k_{s2} \left( \frac{1}{D_1} \left( \frac{1}{D_2} - \frac{1}{D_1} \right) \right) k_{s2} = 0.
\]
\[
(36)
\]

To satisfy conditions (34) and (35), it is sufficient that there should be no deviation of ohmic resistances in circuits of motors. Under operation conditions, magnitude $\Delta R_{\text{dc}}$ is insignificant and can be neglected. As a result of simple transformations, it is seen that to satisfy condition (36), it is necessary that the equality (37) should be satisfied
\[
\frac{D_1}{D_2} = \frac{-k_{s2} + 2k_{s1}}{3k_{s1} - 2k_{s2}}.
\]
\[
(37)
\]
Thus, during selection of wheel-motor units in the depot, it is necessary to try to satisfy the equation (37). If it is necessary to select a wheelset, it is required to use formula (38)
\[
D_1 = D_2 \frac{k_{s2} + 2k_{s1}}{3k_{s1} - 2k_{s2}}.
\]
\[
(38)
\]
To select a motor, it is necessary to use formula (39)
\[
k_{s1} = k_{\text{st}} \frac{3D_2 + 2D_1}{2D_2 - D_1}.
\]
\[
(39)
\]
If we perform calculations $\Delta I$ for a particular locomotive, magnitudes $c_\epsilon$, $k_{\text{st}}$, $k_{s1}$, $k_{s2}$, $i$ will completely determine values, constant in operation process. Though $D_1$, $D_2$ change, they do it so slowly that for the purposes of this calculation they can be accepted as constant. $\Delta R_{\text{dc}}$ is subject to changes under the influence of temperature of heating, i.e., $\Delta R_{\text{dc}} = f(T)$.

In this connection, expressions (30) and (31) take the form
\[
\rho = f(U, \Delta \Sigma R, V),
\]
\[
\varepsilon = f(\Delta \Sigma R, V).
\]

Determining of a condition of the absence of difference of currents.

The difference of currents of parallel branches of the power circuit depends on current strength and coefficients $\rho$ and $\varepsilon$ in accordance with formulas (29).

If we assume that the $\Delta I=0$, we will obtain
\[
0 = \rho + I_1 \cdot \varepsilon.
\]

Hence,
\[
I_1 = -\frac{\rho}{\varepsilon}.
\]
\[
(52)
\]

Expression (52) can be considered a condition for the absence of difference of currents, i.e., magnitude of current in traction electric motor must be equal to a negative value of the ratio of $\rho$ to $\varepsilon$ for all modes of operation.

Analyzing the above results, it should be noted that they were obtained in order to provide a strictly uniform distribution of currents by parallel branches of the power circuit. It is not difficult to introduce the expression (52) into the algorithm of functioning of automated regulation systems. However, with regard to the phenomena of dynamic redistribution of loadings from the locomotive's wheelsets during implementation of traction effort [18], there is a possibility of a more perfect control of currents. It is based on estimation of the current value of traction mass of separate axes: a decrease in current of a motor of unloaded wheelset and an increase in current of the motor of additionally loaded axis. However, this approach requires subsequent research. Thus, presented results so far can be recommended to use only during assembly of wheel-motor units under conditions of depot or factories.

Based on the presented models, for obtaining a dynamic condition for maximizing of coefficient of traction mass, it is necessary to carry out further research. The result can be a refined algorithm of automatic control of traction transmission, implementation of which will make it possible to increase operation efficiency of traction rolling stock even more.
7. Conclusions

1. A negative impact of the non-uniformity of current distribution in the power circuit on operation efficiency of locomotives was established. Measurement results show that overall power, implemented by a locomotive, decreases by 30–37 kW (approximately up to 1 %). Traction force also decreases by magnitude of up to 6 kN (up to 1.5 %). That is why it is possible to conclude that there are certain reserves for enhancing operation efficiency of the direct current traction rolling stock (DE1, VL8, VL10 and similar) due to alignment of currents on the motors.

2. The influence of different factors on uniformity of current distribution was detected and assessed. Under operation conditions, such factors include the difference of magnetic characteristics of traction motors (up to 5 %), deviations of resistance of armature circuits and excitation circuits of motors (up to 9 %), difference of diameters of wheels sets’ threads within one carriage (up to 1 %) and within a locomotive (up to 2 %).

3. By applying the developed calculation method, it was possible to analyze the impact of various factors on current distribution and relationship between parameters of electric motors, switched on in parallel. They clearly show that deviation of currents is affected not only by certain parameters of wheel-motor units, but also by their connection. The benefit of this method is a possibility of estimation of non-uniformity of current distribution depending on the design of a machine and on connection of design parameters of a wheel-motor unit.

References


