1. Introduction

The importance of solving the problems of identification of operations in terms of their effectiveness is related to the need to maximize the pace of an enterprise development and investment capabilities of its owners [1]. In turn, the use of the effectiveness formula as an optimization criterion [2] makes it possible to formalize and...
fully automate processes of adaptation and optimization of functional systems of an enterprise that have high resource capacity [3–5].

Attractiveness of the idea of maximization of financial capabilities has been obvious for many decades and still attracts researchers trying to determine parametric and structural features of the effectiveness formula [6, 7].

In this regard, a large number of indicators, proposed to be used as decision making criteria or optimization criteria, have been developed and are being developed [8]. At the same time, it is possible to note that there is a small number of studies that make attempts to verify indicators for optimization problems.

Complexity of this task is caused by the fact that verification of compliance of an estimation indicator with the original effectiveness formula can be carried out exclusively with the use of the indirect method. This is due to the fact that effectiveness is determined rather than measured.

On the other hand, the use of an estimation indicator that has not been verified as an effectiveness criterion, in the best case, leads to a decrease in competitiveness of an enterprise.

That is why the development of a method for verification of estimation indicators that is supposed to be used as an effectiveness criterion is an important scientific problem.

To enhance reliability of the verification method, it is necessary to expand the number of limited classes of test models of operations. A special place within a set of these classes is occupied by the models of operations with distributed parameters of different duration. Reliable identification of rating effectiveness of such operations is the most difficult and relevant problem.

2. Literature review and problem statement

Numerous debates, publications and research, related to development of an effectiveness indicator, demonstrate the relevance of the topic of verification of the indicators, which are positioned as an effectiveness criterion. This is largely due to the lack of a commonly accepted definition of the category of “effectiveness” [9, 10]. In turn, this leads to the use of, for example, various technical indicators. Using these indicators, the attempts to make judgments on effectiveness of studied operations and even to give quantitative assessment were made.

Thus, in paper [11], the attempt to express judgments about effectiveness of operations with the use of the category “critical load” was made. The study [12] establishes conformity of the category “effectiveness” with the category “reliability”. In article [13], the “filling criterion” is used as a decision-making criterion, while in paper [14], it is “sampling error energy”. In research [15], the magnitude of minimal deviation from the assigned trajectory is proposed to be used as an effectiveness criterion, whereas paper [16] proposes “productivity” criterion and paper [17] emphasizes “technical efficiency”.

In recent years, there have been a great number of works, related to the definition of energy effectiveness [18]. Traditionally, there are many studies, in which effectiveness is associated with minimization of costs [19] or aggregated criteria, obtained by scaling of basic indicators [20].

Paper [21] proposed to use a generalized indicator, defined as “effectiveness criterion”, which is formed according to results of processing the values of extremes of integral indicators.

The use of this approach also eliminates the possibility of verification of the obtained indicator, because the used indicators are not quantitatively comparable among themselves.

In article [22], optimization is carried out using technical-economic indicator. As a result, evaluation criteria do not have a consistent structure, within which non-comparable measurement units are used.

In research [23], it was found that it is possible to make a judgment about effectiveness of an operation if the input objects of an operation (input function) and results (output function) are represented in the form of comparable functions of time. Transition to integral pulse magnitudes of input and output functions, as well as an explicit definition of the operation time makes it possible to represent an operation as a threesome of parameters characterizing input, time of operation and output.

In [24], the method for verification of the effectiveness formula for evaluation of models of simple operations was developed, and in [23], the method for a class of models of synchronized operations with distributed parameters was developed.

Thus, at present, within the framework of the developed verification method, there is no limited class of models of operations, designed for testing operations with distributed parameters of different duration. An increase in the level of automation of system processes, in turn, leads to the requirement of an increase in reliability of the method for verification of the effectiveness formula. Since an increase in the number of limited classes of identified models of operation in terms of their rating efficiency enhances verification reliability, development of such class of models is an important scientific problem.

3. The aim and objectives of the study

The aim of present research is to develop a limited class of global models of operations with distributed parameters of different duration with predetermined identification of their rating effectiveness for enhancing verification accuracy of the effectiveness formula.

To accomplish the set goal, the following tasks were set:

– to determine the rules for formation of a limited class of global models of operations with distributed input parameters and predetermined rating efficiency;

– to develop a method for formation of a limited class of global models of operations with distributed parameters of output of different duration and predetermined rating efficiency;

– to conduct test studies of the estimation indicator that underwent verification on previously developed classes of models of operations with the use of the developed class of models.

4. Designation system

Let us determine the main concepts that are used in this research. To write down the statements, we will use the predicate logic language. Let us determine the basic sets. We will introduce predicate variables for designation of the main objects and properties of these objects or relations between
them: $M$ is the set of operations; $\tilde{M}$ is the set of transformation system processes; $R$ is the set of input products of an operation; $Z$ is the set of output products of an operation; $\tilde{x}$ is the transformation system process; $x$ is the operation; $r$ is the input products of an operation; $p$ is the output products of an operation; $\tilde{v}$ is the expert evaluations of input products of an operation; $v$ is the expert evaluations of output products of an operation; $A(x)$ – transformation process $\tilde{x}$ is operation $x$; $S(x)$ – operation $x$ is a simple operation; $B(x)$ – the moments of beginning of operation $x$ is determined; $C(x)$ – the moment of the end of operation $x$ is determined; $R(x, y)$ – product $r$ is the input product of operation $x$; $P(x, p)$ – product $p$ is the output product of operation $x$; $D(x, r)$ – product $r$ simultaneously arrives at the input of operation $x$; $K(x, p)$ – product $p$ is simultaneously formed at the output of operation $x$; $A'(x)$ – operation $x$ is an operation with distributed parameters by output; $T'(x)$ – operation $x$ is an operation with discrete-portion character of distribution of accounted parameter of the product by output; $P'(p, x, t)$ – product $p$ is formed at the output of operation $x$ at moment of time $t$; $R'(r, x, t)$ – product $r$ is sent to input of operation $x$ at moment of time $t$; $S(x)$ is the global model of a simple operation; $SN(x)$ – operation $x$ is constructed with the use of the first set of parameters; $SN_{p}(x)$ – operation $x$ is constructed with the use of the second set of parameters; $W(\tilde{y}, x_{1}, x_{2}, \ldots, x_{n})$ – a global model of operation $y$ is a composition of a finite set of simple operations $(x_{1}, x_{2}, \ldots, x_{n})$.

4.1. Determining the operations with discrete-portion character of distribution of its products

From the perspective of cybernetics, any operation is a process of directed transformation of input products into output products, having the moment of the start and the moment of completion.

$$\forall x \in \tilde{M}[R(\tilde{x}, r) \land P(\tilde{x}, r) \land B(\tilde{x}) \land C(\tilde{x}) \rightarrow A(\tilde{x})].$$

From the mathematical point of view, an operation is a model of a transformation process. In general case, this model operates with input data sets. As a result of a modeling operation, as well in general case, the result in the form of an output data set is obtained.

For example, for a formula, modeling the process of fluid heating to the desired temperature, in order to give a “reasonable” result, it is necessary to send a certain data set to the input. This set should characterize the properties of an object for heating (fluid volume, initial temperature of the fluid volume, assigned value of heating, ambient temperature). It is also necessary to assign the flow rate of an energy product. As a result, at the output, it will be possible to obtain a data set, which allows determining volume of heated fluid, taking into account evaporation, volume of consumed power, heating time and wear and tear of heating mechanisms [25, 26].

Since the result of a heating operation varies depending on the flow rate of the energy product, there is a need to study the set of alternative heating operations. Such a study is carried out by comparing the input and output objects of an operation and is separated in a separate direction, defined by the concept “operations research”. To solve the problem of operations research, it does not matter whether an actual or a mathematical object is studied, because the source data of the operations research problem are solely input and output objects of the studied operation. The output object of research operation must be a result that allows making evaluating judgment on the studied operation.

In order to solve the problem of research, input and output data sets for each operation should be properly prepared before the following operation transformation stage.

In the general case, quantitative parameters of signals of registration of input and output operation’s products are distributed in time and can be represented as a set of functions $rq(t)$ and $pq(t)$. Here $i$ is the set of input products of an operation, $j$ is the set of output products of an operation.

Since physical quantitative parameters of input and output operation’s products are not comparable between themselves, it is necessary to establish the correspondent expert evaluation on operation input ($rs$) and operation output ($ps$) for each of them. As a result, many non-comparable functions of input and output can be represented as comparable functions

$$re(t) = \sum_{i \in r} (rs, rq(t)), \quad pe(t) = \sum_{j \in p} (ps, pq(t)).$$

(1)

Thus, the model of an operation in the form of $(re(t), pe(t))$ is a global cyber model of an operation with distributed parameters.

In cases, when distributed nature of functions $re(t)$ and $pe(t)$ can be neglected, the object of studying of an operation is the threesome $RE, TO, PE$, where

$$RE = \int_{t_{s}}^{t_{f}} re(t)dt, \quad PE = \int_{t_{s}}^{t_{f}} pe(t)dt, \quad TO = t_{f} - t_{s}.$$ (2)

Here $t_{f}$ is the moment of the start of an operation; $t_{s}$ is the moment of completion of an operation.

A transformation process, the input products of which simultaneously arrive at its input and output products are simultaneously formed at its output, will be called a simple operation

$$\forall x \in M \left( \forall r \in R \left( \forall p \in Z \left[ A(x) \land RE(x) \land PE \land \left[ A(x) \land S(x) \right] \right] \right) \right).$$ (Fig. 1).

Fig. 1. Graphic model of a simple operation

The main objective of the study of operations is to assess their effectiveness. However, to solve this problem, it is first necessary to verify the estimation indicator, with the help of which data set $(re(t), pe(t))$ in $(RE, TO, PE)$ is processed to ensure its compliance with the original effectiveness formula.

The verification process, in turn, is accompanied by development and research of operations within its limited class for identification of their rating effectiveness.

Seven classes of operations, presented in format $(RE, TO, PE)$ [24, 27] and two classes of operations, presented in format $(re(t), pe(t))$ [23], have recently been developed.
In this research, the operations of \((re(t), pe(t))\) format are treated as operations with discrete-portion character of distribution of input and output products of the researched operation of \((re[n], pe[n])\) form.

Among the class of operations model of \((re[n], pe[n])\) format, we will determine operations with distributed parameters by output. We will assume that in such models, input products arrive at the input of an operation simultaneously, and output products are formed at the output of an operation at discrete moments not simultaneously (Fig. 2)

\[
(\forall x \in M)(\exists r \in R)\left[ A(x) \land R(x,r) \land D(x,r) \land \right.
\]
\[
\exists p_1, p_2, ..., p_n \in Z\left\{ P(x, p_1) \land P(x, p_2) \land ... \land P(x, p_n) \right\} \land
\]
\[
\exists n_1 < n_2 < ... < n_n \left( P\left\{ p_1, n_1, p_2, n_2, ..., p_n, n_n \right\} \right) \rightarrow T'(x) \right].
\]

Fig. 2. Model of a discrete operation with discrete portion character of distribution of its products by output

The advantage of representation of an operation in the discrete-portion form opens up the possibility of its decomposition into a set of models of simple operations and, respectively, for implementation of a composition operation.

Since subsequently we are talking about the models with discrete-portion character of products' distribution, determining of moments of time in the form of \(t_r\) and \(n_r\) will be considered equivalent.

4. 2. Determining the principle of composition of the global model of an operation with distributed parameters by operations’ output

We will determine an operation with the distributed character of output products by the composition of global models of simple operation of different duration, synchronized by the moment of their beginning. In this case, expert assessment of the input product of an operation is equal to the sum of expert assessments of models of simple composition operations, and moments of time and magnitudes of expert assessments of output products correspond to moments of time and magnitudes of corresponding output products of models of simple composition operations

\[
(\forall x_1, x_2, ..., x_n \in M) \land S(x_1, x_2, r_1, r_2, ..., r_n \in R) \land R'(t_1, x_1, t_2) \land
\]
\[
\land R'(t_2, x_2, r_3) \land ... \land R'(t_n, x_n, r_n) \land \exists p_1, p_2, ..., p_n \in Z\left\{ P\left\{ p_1, x_1, t_1, p_2, ..., p_n, x_n, r_n \right\} \right\} \rightarrow W'(y) \right].
\]

Fig. 3. Designing a global model of an operation with distributed parameters by output in the form of composition with the use of two original simple models of operations with different duration

The decomposition of the global model of an operation is breaking of this model of an operation into a finite set of simple models of operations (Fig. 4).

We will take the following statement as a postulate.

If the number of simple operations, obtained as a result of decomposition of operation \(A\), is equal to the number of simple operations, obtained as a result of decomposition of operation \(B\), and for each simple operation of decomposition of operation \(A\), it possible to establish in correspondence a single simple operation of decomposition of operation \(B\) with less effectiveness, operation \(A\) is more effective than operation \(B\).

4. 3. Decomposition of the model of an operation with discretely distributed parameters of output products into two simple models of operations of different duration

The following data set will be accepted as original data

\[
(RE, k, t_r, TO).
\]

(3)

Let us determine data set (3) as original data set.

We will determine a short simple model of operation \(A\) with the help of the first data set with the use of the original data set \((RE_A=RE, k_A=k, t_A=t, TO_A=TO, PE_A=kRE)\).

We will determine the second dataset of a longer simple model of operation \(B\) with the use of the original data set \((RE_B=RE, k_B=k, t_B=t, TO_B=2TO, PE_B=k^2RE)\).

The condition of formation of source models \(A\) and \(B\) is the inequality of their duration and a higher value of the output product in relation to the value of the input product

\[
((t_{A} - t) > (t_{B} - t)) \land ((k_B > 1) \lor (PE_A > RE_A \land PE_B > RE_B)).
\]

Thus, operation \(A\), obtained as a result of composition of a finite number of simple operations models, constructed with the use of the first data set, can be identified as an operation with reduced effectiveness.

Operation \(B\), obtained as a result of the composition of a finite number of simple operations models, constructed with the use the second data set, can be identified as more effective operation in relation to operation \(A\) (Table 1).
\[ W(h, a, c) \land SN_1(a) \land SN_2(c) \land W(f, b, d) \land \\
\land SN_1(b) \land SN_2(d) \land E(a) < E(b) \land \\
\land E(c) < E(d) \rightarrow E(h) < E(f). \]

**Table 1**

<table>
<thead>
<tr>
<th>Original data</th>
<th>Class of operations</th>
<th>Rating effect</th>
<th>Rule</th>
</tr>
</thead>
</table>
| \( RE, TO, k, t \) | OPDE                | 2             | \( RE_A=2RE, PE_A=kRE, \\
| \( k \in \{1; 2\} \)    |                     |               | \( t_{A1}=t_r, \\
|                         |                     |               | \( t_{A1}+2TO, t_{A2}=t_r+2TO \) |
|                         |                     | 1             | \( RE_B=2RE, PE_B=kRE, \\
|                         |                     |               | \( t_{B1}=t_r, \\
|                         |                     |               | \( t_{B1}+2TO, t_{B2}=t_r+4TO \) |

The obtained results make it possible to proceed to determining the method for the formation of the class of operations with distributed parameters by output of different duration and predetermined effectiveness.

4. 3. Development of the method for determining a limited class of discrete-portion global models of operations

To develop the method, we will determine original parameters for construction of models of operations of a limited class with predetermined rating effectiveness in the form of data set \( (RE, k, TO) \).

Using the rule of formation of a less efficient operation [27], we will obtain a model of operation \( A \).

For example, let us assume that \( R_E=1; k=1.1; TO=1. \) Then, \( R_E=1; PE_A=kRE=1.1; TO_A=TO=1 \) (Fig. 5).

![Fig. 5. Simple model of operation](image)

Let us determine the global model in a simple operation \( B \) with predetermined higher rating effectiveness in accordance with rule [27] \( R_E=RE=1; PE_B=k^2RE=1.21, TO_B=4TO=4 \) (Fig. 6).

![Fig. 6. Simple model of operation B, effectiveness of which is higher than the one of a simple model of operation A](image)

Operation \( A \) is less effective in relation to operation \( B \) due to the fact that by using of output product \( A \) in the following operation \( A^* \) with the same parameters, we will obtain \( RE_A=PE_A=1.1; PE_A=kRE_A=1.21, TO_A=TO=1 \) (Fig. 7).

![Fig. 7. On the explanation of principle of formation of operations with predetermined effectiveness](image)

Then, for operation \( C \) with less effectiveness, we will obtain \( R_E=RE, PE_C=kRE=1.1, TO_C=2TO=2 \) (Fig. 8).

![Fig. 8. Model of a developing operational process on operations of C type](image)

For operation \( D \) with a higher effectiveness, we will obtain \( R_E=RE, PE_D=k^2RE=1.21, TO_D=4TO=4 \) (Fig. 9).

![Fig. 9. Operation D, effectiveness of which is higher than that of operation of C type, by definition](image)

We will carry out composition of operations \( A \) and \( C \), as well as operations \( B \) and \( D \) (Fig. 10) according to rule (Table 1).

![Fig. 10. Graphical representation of global models of operations with distributed parameters of different duration with predetermined effectiveness](image)

The operation, presented in Fig. 10, belong to the class of global models of operations, limited by rule (Table 1). These models have different duration. Parameters of these models are represented in the portion-discrete form, and parameters that characterize the output product of an operation are portionally distributed in time.

4. 4. Testing of the estimation indicator with the use of operations of the developed class

To illustrate capabilities of the developed class, we will verify possibilities of assessment of effectiveness of operations (Fig. 10) with the use of the indicator, having the form \( Q_{\text{eff}}(re(t), pe(t)) \), which was verified on the class of synchronized operations with distributed parameters [23].
Mathematics and cybernetics – applied aspects

\[
E = \int_{t_a}^{t_b} \left( \int_{t_u}^{t_d} p(t) dt - \int_{t_u}^{t_d} r(t) dt \right) dt
\]

where \( t_a \) is the moment of logical completion of operation, which is determined as
the moment of time by the condition of functions’ equality

\[
\int_{t_u}^{t_d} r(t) dt = \int_{t_u}^{t_d} p(t) dt;
\]

\( t_a = t_a + 1 \) [28].

To determine the value of indicator \( E \), we will use specially developed software [29].

To this end, it is necessary to introduce value \( t_a = 1 \) to the Time section of module Expenses/Costs, and value \( RE_E = 2 \) to the section Sum of the same module and click Insert Record. The data, associated with output of products \( (t_1E = 1; RE_1 = 2) \) and \( (t_2E = 3; RE_2 = 2) \), are sequentially introduced to the correspondent sections of module Results (Fig. 11).

In section Indicators, the model of operation \( E \) in a tabular form and results of calculation of EE, Res and E of the researched operation are displayed.

Fig. 12 shows the interface of the program input and output data for operation \( F \).

It is seen that value \( E_F = 0.004 \) is lower than value \( E_1 = 0.00412 \). It means that indicator \( E \) successfully completed the testing stage.

Fig. 11. Interface of the program, displaying results of input and output of data of operation \( E \)

Fig. 12. Interface of the program that displays results of data input and output of operation \( F \)

5. Discussion of results of research, related to development of the method for verification of the effectiveness formula

It is clear that the method for verification of formula effectiveness is an important scientific problem. The use of a non-verified estimation indicator in the automatic functional systems can quickly lead to undesirable consequences, such as the loss of competitive advantage or even bankruptcy. Fearing this, at operating enterprises, it is preferred to establish control modes manually, relying mostly on common sense. Of course, this control technique also inevitably leads to losses. In this case, it is next to impossible to estimate the level of these losses.

The advantage of using the verified effectiveness formula is obvious. This is a possibility to fully automate management processes. At the same time, when relying on maximal resource efficiency in the search mode, the functional system will automatically change the control mode depending on a change in external factors. It can be weather conditions, level of demand, a change in prices for input and output products of an operation.

The problem of verification of effectiveness formula is that effectiveness cannot be directly measured. That is why the verification method can be based only on the use of indirect methods.

The limited class of models of standard operations, developed in this research, is unique because local criteria of effectiveness are not used for identification of rating effectiveness. This is due to the fact that the method of rating identification
of operations’ effectiveness is based on the law of conservation of energy. In this case, it manifests itself in the fact that output products of a previous operation cannot be transferred to the input of the subsequent operation without losses.

The model of an operation, constructed without taking into account additional transfer resource, is inadequate. It is the point that is considered when making a judgment about an operation with predetermined less high effectiveness relative to the operational process, in which there is no values transfer within the examined interval or there are few transfers of this kind.

Advantage of this class of operations, in addition to the fact that these are operations with distributed parameters and different duration, is a small difference in effectiveness. On the other hand, this study did not address the issue of coefficient of added value of short operations, which obviously cannot be less than a certain magnitude, after which reliability of the data of a modeling process will be lost. This problem is the subject of subsequent research.

6. Conclusions

1. The rules of formation of a limited class of global models of operations with distributed parameters by output and predetermined rating effectiveness were determined. This makes it possible to use formal methods for automatic formation of a class of models and their rating identification without using a local effectiveness criterion.

2. The method of formation of a limited class of global model operations was developed. The specific feature of the method is different duration of the models of operations with distributed parameters by output and predetermined value of their ratings effectiveness. This makes it possible to use this set of operations as standards for testing of estimation indicators of a particular structure. The testing result is positive if rating estimation of operations, obtained with the use of the verified expression, coincides with predetermined rating of testing operations.

3. Testing of the estimation indicator, which passed verification on all previously developed limited classes of global models of operations, showed adequacy in evaluation of models of operations of the developed class. In this case, as a result of evaluation of a short operation, a lower value of the indicator, relative to results of evaluation of a longer operation, was obtained. This indicates that the tested indicator correctly assessed the operations, relative to predetermined rating evaluation of effectiveness.

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