1. Introduction

The energy market of Ukraine demands constant operational control to provide consumers with necessary energy resources in the required amount and at affordable price. The study of management model gains particular relevance in the context of European integration of Ukraine. High-energy intensity and low competition in the energy market led to the need to reform the fuel-and-energy complex of Ukraine and to introduce a market approach to its management. Since electricity market liberalization is a strategic direction and to introduce a market approach to its management. The general result of the introduction of a market approach to the management of the fuel-and-energy complex of Ukraine and to introduce a market approach to its management. The general result of the introduction of a market approach to the management of the fuel-and-energy complex will be a detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand. The general result of the introduction of a market approach to the management of the fuel-and-energy complex will be a detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand. The energy market of Ukraine demands constant operational control to provide consumers with necessary energy resources in the required amount and at affordable price. The study of management model gains particular relevance in the context of European integration of Ukraine. High-energy intensity and low competition in the energy market led to the need to reform the fuel-and-energy complex of Ukraine and to introduce a market approach to its management.

The study of management model gains particular relevance in the context of European integration of Ukraine. High-energy intensity and low competition in the energy market led to the need to reform the fuel-and-energy complex of Ukraine and to introduce a market approach to its management. Since electricity market liberalization is a strategic direction adopted by the developed and developing countries, substantiation and transparency are the prerequisites for creation of competition in the energy sector of Ukraine. The model of management of energy market processes in the modern energy space should take into consideration the interconnection of processes of functioning, characteristics, and market parameters. It will help to solve actual issues of energy security in the region in conjunction with the development of a criterion for efficiency of using energy resources, as a component of the criterion for optimal management. The search for optimal management of processes in the energy market requires a reasonable choice of the most suitable methods for the given objects. When arguing about the management of energy market processes, it is necessary to take into consideration several shortcomings and imperfections of its functioning in a market-based approach, which requires detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand. The general result of the introduction of a market approach to the management of the fuel-and-energy complex will be a detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand. The general result of the introduction of a market approach to the management of the fuel-and-energy complex will be a detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand. The general result of the introduction of a market approach to the management of the fuel-and-energy complex will be a detailed study and continuous improvement. In this regard, methodological approaches to constructing a market model of functioning of the Ukrainian energy sector are in demand.

Existing models of centralization and monopoly of the Ukrainian energy market do not take into consideration the market character of process management. This leads to objective difficulties in the management of demand and supply.
processes in the context of European integration. Modeling of energy processes with calculation of the optimal price for energy resources and development of a mechanism for uninterrupted supply of fuel-and-energy resources is a necessary research in order to ensure energy security. However, imperfection of effective management and a lack of a fair market price for fuel-and-energy resources requires continuous improvement of models for assessment of the interconnection of demand and supply processes in the market under conditions of its dynamic development. Recommendations for modeling the processes of functioning of the energy market require an expanded and in-depth study in the context of European integration, therefore, a mathematical apparatus and algorithms for its realization need constant improvement. Under such conditions, the topics of the study into modeling of the Ukrainian energy market management processes are relevant and necessary.

2. Literature review and problem statement

Paper [1] investigates functions of demand and supply of the energy market under conditions of uncertainty. The authors proposed a model of oligopolistic competition in the energy market and investigated a function of the balance of the market through a system of differential equations. However, despite the benefits of such a study, the question about a search for an optimal market price for energy resources remains open.

Authors of work [2] developed approaches to a use of stochastic differential equations for assessment of risks and elimination of shortcomings of the energy market.

Authors of study [3] assessed functioning of the energy market through a prism of a market approach to its management. Differential equations for assessment of local and global stability of the market and stability of its optimal management deserve attention here.

Papers [4, 5] investigated elasticity of supply and demand in the energy market and an algorithm for estimation of elasticity of supply of energy products. Such developments can help to assess a degree of monopoly in a similar market in Ukraine, which is necessary in the context of its reformation.

The system of supply and demand equations can become an important element of the theory of formation of a market model of the Ukrainian energy system. Authors of study [6] built a market model for the energy market and convinced that it is possible to assess the electricity market fully in the context of the main factors of cost, demand, supply, and price management only. Here the demand for electricity is an individual amount of energy, which consumers are ready and able to buy per a certain period. Factors, which influence electricity in the market conditions may be: prices for alternative energy sources, consumer income, level of consumer awareness about possibilities of using alternative energy, and others.

Market supply of goods (electricity) is the amount of goods that individual companies are ready and able to offer for sale over a certain period of time. The main determinants of the market supply of energy can be: a purpose of a company in the industry, a price for a unit of electricity, a price for other related energy sources, a price for factors of production, state of technology, expectations, etc. Major economic processes such as supply, demand and fuel consumption are relatively predictable, but combined with political and regulatory factors, as well as under an influence of financial speculation, energy pricing becomes more difficult to predict.

Paper [7] considers 10 major factors, which influence energy prices. It is possible to include them to development of market models of the energy sector.

Authors of study [8] developed a model for calculation of electricity prices, based on the principle of supply and aimed at establishing of a balance. The model includes latent supply and demand curves, which can change over time, and calculates prices obtained at the intersection of two curves at any given time.

Paper [9] considered a model of competition in the electricity market and an effect on a balance of the supply function. It indicated that continuous balance is unattainable under conditions of an infinite set of goods.

Study [10] presents the methodology for market assessment and market behavior study with a use of models constructed through ordinary differential equations [10].

In the above papers, authors did not consider the energy market as an integral system. They focused on one of the types of energy resources, or if they investigated several resources, they did not study the interaction of these resources at all, although it is obvious that availability of coal, black oil, and gas prices is influencing, that is, on a level of current extraction, production, and on a level of consumption. In addition, they did not consider the energy market as a system, which changes continuously in time. Instead, it is precisely this assumption makes possible to use the approach to modeling of production levels (supply) and consumption levels (demand) through a system of differential equations, which bind these resources into a single whole.

3. The aim and objectives of the study

The objective of the study is to develop a mathematical model for management of the energy sector of Ukraine’s economy based on the principle of market liberalization and, on this basis, to construct a system of differential equations for assessment of processes of production and consumption of fuel-and-energy resources.

We set the following tasks to achieve the objective:

– investigation of the interaction of processes and subjects in the energy market in terms of a market approach to their functioning in the context of European integration;

– development of a system of differential equations, which describe dynamics of states of the energy system throughout a year, such system will give possibility to evaluate processes of the energy market through a prism of a market approach to its management;

– using numerical methods to find a solution to the Cauchy problem (the initial problem) of a constructed system of differential equations that shows dynamics of production or consumption of the corresponding energy resources throughout a year in Ukraine;

– management of the system of the fuel-and-energy complex by a market approach to investigate a ratio of supply-demand processes of the energy market;

– consideration of a variety of possible alternatives for development of a market situation and performing of a structural detailing of components of the energy system of the fuel-and-energy complex for a coherent process management.
4. Materials and methods to study the energy market

The methodological basis for the study is official statistical data [11], reporting analytical data [12, 13]. Methods for processing of the information are abstraction, analysis, synthesis, description, interpretation, and others. We built a normal system of linear differential equations of the first order that describes production and consumption of the corresponding energy resources. We used the Euler method [14] to find an approximate solution of the system. We set the initial condition taking into consideration stocks at the beginning of the year of the respective fuel-and-energy resource and a volume of production or consumption during this period divided by 365. We sought a solution of supply equations (production and import) of fuel-and-energy resources in the form of tabulated values of the production function of the corresponding energy depending on time. We constructed a graph of appropriate dependence based on these solutions. We used the computer algebra system wxMaxima for calculations.

To ensure effective management in the development of the energy market of Ukraine, it is necessary to develop a system of economic and mathematical models and management processes, which includes:

– models of dynamics of the world energy market; sales of energy products in regions of the world; structure of world trade of an energy product; world reserves of energy resources and production, consumption, trade, as well as world prices for fuel-and-energy resources;

– models of dynamics of the structure of export and import of energy in Ukraine in the main categories: gas, coal, atomic energy, coal, uranium, electricity;

– models of dynamics of the fuel-and-energy complex of Ukraine: extraction of fuel resources; structure of production of energy products; estimation of indicators of market concentration by types of an energy product.

5. Application of a systematic approach when studying the energy market

The energy market of Ukraine combines branches for extraction of fuel-and-energy resources (fuel industry), conversion to other types of energy (electricity and petrochemical industry), transportation and distribution of energy resources between consumers (power systems, pipeline systems, provision of heat and electricity to the population).

There are specific management problems to be resolved under conditions of liberalization of the energy market of Ukraine in each of these functional subsystems. Therefore, we must base the methodology for assessment of a state, performance and trends of the Ukrainian energy market on a systematic approach. Such an approach makes it possible to obtain a comprehensive assessment of functioning and development of the energy market as a complex dynamically developing system. The main characteristics of the study are the accounting of systemics of objects and processes of the energy market in terms of integration into the European energy space and formation of a market model of management. Competition should become a basis for its development, high dynamism, and the global nature of functioning of the energy space, which integrates the energy market of Ukraine.

Fig. 1 presents the main areas of systems analysis and assessment of development of the energy market, including forecasting modeling.

We should carry out analysis and evaluation of a state, performance and trends of the Ukrainian energy market development according to the principle of systemics of the investigated object. We can present a systemic study of the energy market as a generalized economic-mathematical model, which is a basic component of its cognitive model. Such a model reflects a conceptual framework of the study, which takes into consideration dynamics over time and multivariate trends in development of a market situation under conditions of globalization.

Based on the systematic approach of organization of processes of management of the energy market, in the following sections, we will formulate methodological provisions of its analysis and evaluation and construct the “supply-demand” model and a system of economic-mathematical models of processes of management of the energy market of Ukraine under conditions of globalization. A level of detailing, intellectual and information capabilities of the study determine a composition and quality of tools used for modeling and forecasting. Fig. 2 shows systems connections and dependences of economic and mathematical models.

The market model must accurately respond to needs of the energy market and expand a structure of alternative energy sources. Consumers can gradually replace the most expensive resource with an alternative source of energy by choosing energy supplies. This, in turn, develops the
structure of the energy market. Competition and alternative supply of fuel and energy resources can provide uninterrupted supply. The market approach to management proposed in this study is fundamentally open in nature and involves possibility of its further development.

6. Study of solutions to the system of differential equations and their graphical interpretation

We described the processes of formation of the “supply-demand” market relation for all types of energy resources by dynamic economic and mathematical models. A base of such models is the system of differential equations of the first order; its left part represents a rate of change of the studied economic indicators.

The proposed approach provides an opportunity to evaluate available supply and demand for fuel-and-energy resources quickly. Such evaluation indicators contribute to calculation of forecast values of supply and demand, which are a basis for making managerial decisions on ensuring uninterrupted supply and differentiation of energy resources.

Let us assume that the demand for fuel-and-energy depends on several consumers $N$ at time $t$. To characterize consumers of the energy market we use coefficients of activity and passivity, $n_a$ and $n_i$, that is, $N = N(n_a, n_i, t)$, as well as $S = (n_a, n_i, n_c, n_g)$ are structural characteristics of consumers, where $n_i(t), i=1,2,...,d$ is a specific weight in the structure of electricity consumption by groups of consumers: industry; agriculture, transport, construction, communal consumers; other non-industrial consumers; population. Demand determines consumption, and supply — production of fuel-and-energy resources. All variables and parameters of the equations below are dependent on time $t$ — the number of days per year in the study period. We denoted a volume of production of energy products as $O_i$, for

$$O_i = (i = r + 1, r + 2, \ldots, m),$$

the following production function takes place:

$$O_i = f(a, Q_i, \alpha_1 Q_1, \ldots, \alpha_s Q_s; \alpha_1, \ldots, \alpha_s; t).$$ (1)

where $Q_i, i=1,2, \ldots, s$ are the volumes of production of fuel-and-energy resources $Q_i$ in Ukraine; $Q_j, j=s+1, s+2, \ldots, g$ are the volumes of production of fuel-and-energy resources $Q_j$ abroad; $0 \leq \alpha_i \leq 1$ is a share of the volume of fuel-and-energy resources $Q_j, k=1,2, \ldots, g$ consumed for production of a final energy product $O_k, k=r+1, r+2, \ldots, m$; $f$ is a production function for energy product $O_k$ which expresses the constituent aspects of production of energy products.

Some of energy products go as exports from Ukraine and imports fully meets the rest of the domestic demand.

In the vector form, we use the following designations:

$$O = (O_1, O_2, \ldots, O_m), \quad E = (E_1, E_2, \ldots, E_m),$$

$$I = (I_1, I_2, \ldots, I_m), \quad a = (a_1, a_2, \ldots, a_m),$$

— volumes of production, exports, imports and consumption standards of all types of fuel-and-energy products in Ukraine.

Then, we will also apply functional expressions for production of fuel-and-energy resources in Ukraine

$$O_i = O_i(N, a, E, i, t).$$ (2)

where $i=1,2, \ldots, m$,

$$Q_i = Q_i \left( I_i, I_0, t \right).$$ (3)

where $j=1,2, \ldots, s$ and $I_Q$ is the import of the corresponding fuel-and-energy resource.

The proposed approach implies the principled openness of a model, which means that we can change several parameters in equations (1) to (3) for any other set of fuel-and-energy resources in the market.

All these parameters depend on time $t$ in general. In addition, a use of any other market indicators that reflect both internal and external processes and connections of the market of energy products: economic, social, environmental, etc., is possible.

Expressions (1) to (3) represent decomposition, analytical and formalization components of modeling and are a basis of a synthetic model of the energy market.

We can describe the dynamics of change in the volumes of production of $O_i$ product in Ukraine by the differential equation:

$$\frac{dO_i}{dt} = k \left( a_i N + (\delta_i - \Delta_i - b) O_i \right),$$ (4)

with initial conditions (output of $O_i$ product at the beginning of the study period):

$$O_{i,0} = O_{i,0}.$$ (5)

where $E_i = \delta_i O_i$, $I_i = \Delta_i O_i$ are the volumes of export-import of $O_i$ product. $\delta_i$, $\Delta_i$ are, respectively, the shares of export-import in relation to the volume of $O_i$ product produced in the country, which depends on both $O_i$ and time $t$. A ratio of values $\delta_i$, $\Delta_i$ characterizes market position of Ukraine in the world in relation to $O_i$ product; $k$ is a coefficient, which...
reflects dynamics of the market balance for $O_1$ product; $b_i$ is a coefficient of $O_i$ product’s overrun of supply over demand close to one (supply, in our opinion, has to exceed a demand a little due to the social orientation of the energy market).

Components on the right part of equation (5) with the plus sign represent demand for $O_i$ product (consumption and export), with the minus sign – supply (production by the fuel-and-energy complex of Ukraine’s and a volume of imports).

In the general case, we can relate values of parameters $\delta_i$, $\Delta_i$, $k_i$ and $b_i$ to regulated parameters of a model of functioning of the energy market of the country.

Consumption of fuel-and-energy resources that is not produced in Ukraine can be provided only at the expense of imports. If production of $O_i$ product stops in Ukraine, including for export, from the time $t_{i0}$, then we assume that all parameters in an equation (5) are equal to zero, including a value of $O_{i0}$.

Let us describe a structure and consumption of electricity and other energy resources by groups of consumers (industry, agriculture, transportation, construction, communal consumers, other non-industrial consumers, and the population) in differential equations:

Let us assume that $VVP_{r}$ is the share of GDP of Ukraine in the $r$-th economy sector. Accordingly, we denote $Q(t)$ – a volume of energy production in Ukraine: $Q_1$ – coal, $Q_2$ – oil, $Q_3$ – gas, $Q_4$ – uranium; $Q_5$ – black oil, $Q_6$ – electricity, $Q_7$ – thermal energy. We determine a volume of production per year as $Q(365) - Q(0)$.

Equation of the supply of coal will take the form:

$$\frac{dQ_1}{dt} = q_1 \cdot Q_1 \left( \alpha_{16} \cdot Q_4 + \alpha_{17} \cdot Q_2 + \gamma_{1r} \cdot VVP_r - \Delta_{1} - \beta_{1} \right),$$

where $\gamma_{1r}$ is the share of coal consumed for $r$ sector, $\alpha_{16}$ is the share of coal consumed for electricity, $\alpha_{17}$ is the share of coal consumed for thermal energy, $\Delta_{1} = \frac{I_{1}}{Q_{1}}$ is the share of coal imports in the volume of its supply by the fuel-and-energy complex of Ukraine; $I_{1}$ is the volume of coal imports; $\beta_{1}$ is a coefficient of advance of the supply of coal $Q_1$ over demand close to one, $\beta_{1} \geq 1$, $\beta_{1} \rightarrow 1$.

The following equation represents volumes of oil supply:

$$\frac{dQ_2}{dt} = q_2 \cdot Q_2 \left( \alpha_{25} \cdot Q_3 + \sum_{r=1}^{7} \gamma_{2r} \cdot VVP_r - \Delta_{2} - \beta_{2} \right),$$

where $\gamma_{2r}$ is the share of oil consumed for $r$ sector, $\alpha_{25}$ is the share of oil spent for black oil and other types of fuels, $\Delta_{2} = \frac{I_{2}}{Q_{2}}$ is the share of oil volume of imports in a volume of its supply by the fuel-and-energy complex of Ukraine; $I_{2}$ is the volume of oil imports; $\beta_{2}$ is a coefficient of advance of supply of oil $Q_2$ over demand close to one, $\beta_{2} \geq 1$, $\beta_{2} \rightarrow 1$.

The gas supply takes the form:

$$\frac{dQ_3}{dt} = q_3 \cdot Q_3 \left( \alpha_{36} \cdot Q_4 + \alpha_{37} \cdot Q_2 + \sum_{r=1}^{7} \gamma_{3r} \cdot VVP_r - \Delta_{3} - \beta_{3} \right),$$

where $\gamma_{3r}$ is the share of gas consumed for $r$ sector, $\alpha_{36}$ is the share of gas consumed for electricity production, $\alpha_{37}$ is the share of gas consumed for thermal energy production, $\Delta_{3} = \frac{I_{3}}{Q_{3}}$ is the share of oil imports in a volume of its extraction by the fuel-and-energy complex of Ukraine, $I_{3}$ is the volume of gas imports; $\beta_{3}$ is the coefficient of advance of supply of gas $Q_3$ over demand close to one, $\beta_{3} \geq 1$, $\beta_{3} \rightarrow 1$.

The equation of uranium supply is as follows:

$$\frac{dQ_4}{dt} = q_4 \cdot Q_4 \left( \alpha_{46} \cdot Q_4 + \sum_{r=1}^{7} \gamma_{4r} \cdot VVP_r - \Delta_{4} - \beta_{4} \right),$$

where $\gamma_{4r}$ is the share of uranium consumed for $r$ sector, $\alpha_{46}$ is the share of uranium spent for electricity production, $\Delta_{4} = \frac{I_{4}}{Q_{4}}$ is the share of uranium import volume in the volume of its extraction by the fuel-and-energy complex of Ukraine, $I_{4}$ is the volume of uranium import; $\beta_{4}$ is a coefficient of advance of the supply of uranium $Q_4$ over demand close to one, $\beta_{4} \geq 1$, $\beta_{4} \rightarrow 1$.

Equation of supply of black oil takes the form:

$$\frac{dQ_5}{dt} = q_5 \cdot Q_5 \left( \alpha_{56} \cdot Q_4 + \alpha_{57} \cdot Q_2 + \sum_{r=1}^{7} \gamma_{5r} \cdot VVP_r - \Delta_{5} - \beta_{5} \right),$$

where $\gamma_{5r}$ is the share of black oil consumed for $r$ sector, $\alpha_{56}$ is the share of black oil consumed for electricity production, $\alpha_{57}$ is the share of black oil consumed for thermal energy production, $\Delta_{5} = \frac{I_{5}}{Q_{5}}$ is the share of black oil import volume in a volume of its extraction by the fuel-and-energy complex of Ukraine, $I_{5}$ is the volume of import of black oil; $\beta_{5}$ is a coefficient of advance of supply of black oil $Q_5$ over demand close to one, $\beta_{5} \geq 1$, $\beta_{5} \rightarrow 1$.

The equation of the supply of electricity is as follows:

$$\frac{dQ_6}{dt} = q_6 \cdot Q_6 \left( \alpha_{66} \cdot Q_4 + \alpha_{67} \cdot Q_2 + \sum_{r=1}^{7} \gamma_{6r} \cdot VVP_r - \Delta_{6} - \beta_{6} \right),$$

where $\gamma_{6r}$ is the share of electricity consumed for $r$ sector, $\alpha_{66}$ is the share of electricity consumed for heat production, $\alpha_{67}$ is the share of electricity consumed for electricity production; $\Delta_{6} = \frac{I_{6}}{Q_{6}}$ is the share of electricity import volume in the volume of its extraction by the fuel-and-energy complex of Ukraine, $I_{6}$ is the volume of import of electricity; $\beta_{6}$ is a coefficient of advance of supply of electricity $Q_6$ over demand close to one, $\beta_{6} \geq 1$, $\beta_{6} \rightarrow 1$.

The equation of the supply of thermal energy is as follows:

$$\frac{dQ_7}{dt} = q_7 \cdot Q_7 \left( \sum_{r=1}^{7} \gamma_{7r} \cdot VVP_r - \Delta_{7} - \beta_{7} \right),$$

where $\gamma_{7r}$ is the share of thermal energy consumed for $r$ sector, $\Delta_{7} = \frac{I_{7}}{Q_{7}}$ is the share of thermal energy import volume in the volume of its extraction by the fuel-and-energy complex of Ukraine, $I_{7}$ is the volume of import of thermal energy; $\beta_{7}$ is the coefficient of advance of supply of thermal energy $Q_7$ over demand close to one, $\beta_{7} \geq 1$, $\beta_{7} \rightarrow 1$. For thermal energy $\Delta_{7} = 0$.

We model an equation of demand for fuel-and-energy resources. We assume $D_1(t)$ to be the volume of fuel-and-energy consumption,

$$\frac{dD_1}{dt} = k_1 \cdot \left( \sum_{r=1}^{7} \lambda_{r} \cdot VVP_r + (-\delta_{i} + \Delta_{i} - b_{i}) D_1 \right),$$

where $\delta_{i}$ is the share of exports relative to a volume of fuel-and-energy product produced (extracted) in Ukraine,
\( \delta = \frac{E}{Q} \) is the share of exports relative to the volume of product \( i \) produced (extracted) in Ukraine. \( Q(0) \) are the fuel-and-energy reserves at the beginning of a year, \( D(0) = 0 \), \( k \) is a coefficient that reflects dynamics of a market balance for a fuel-and-energy product \( i \), \( \lambda_{ii} \) is the share of fuel-and-energy resource \( i \) consumed by \( r \) sector.

\[
E_i(t) = \alpha_{ii} \left( RE_i(t) \right)^{\gamma_i} \cdot Y_{vt} \cdot E_i(t)^{\beta_i}, \quad (14)
\]

\[
I_i(t) = \alpha_{ii} \left( RE_i(t) \right)^{\gamma_i} \cdot Y_{vt} \cdot I_i(t)^{\beta_i}, \quad (15)
\]

where \( RE_i \) is the relative price for an imported fuel-and-energy product, that is

\[
P_{rel}(t) = \frac{P_{im}(t)}{P_{n}(t)}.
\]

\( P_{rel}(t) \) is the price of imported fuel-and-energy resources; \( P_{n}(t) \) is the price of the national fuel and energy resource; \( Y_{vt} \) is the real GDP abroad; \( Y_{vt} \) is the real GDP in Ukraine; \( a_{ii} \) and \( \alpha_{ii} \) are coefficients of proportionality; \( E_i(t) \) and \( I_i(t) \) are the volumes of export and import of the corresponding energy resource for the previous period.

We obtain the following system of equations:

\[
\begin{align*}
\frac{dQ_i}{dt} &= q_i \left( \alpha_{ii} \cdot Q_i + \alpha_{ij} \cdot Q_j + \sum_{r=1}^{7} \gamma_{ir} \cdot V_{rp} \cdot -\Delta_i - \beta_i \right), \\
\frac{dQ_j}{dt} &= q_j \left( \alpha_{ij} \cdot Q_i + \alpha_{jj} \cdot Q_j + \sum_{r=1}^{7} \gamma_{jr} \cdot V_{rp} \cdot -\Delta_j - \beta_j \right), \\
\frac{dQ_k}{dt} &= q_k \left( \alpha_{ik} \cdot Q_i + \alpha_{jj} \cdot Q_j + \sum_{r=1}^{7} \gamma_{kr} \cdot V_{rp} \cdot -\Delta_k - \beta_k \right), \\
\frac{dQ_l}{dt} &= q_l \left( \alpha_{il} \cdot Q_i + \alpha_{ij} \cdot Q_j + \sum_{r=1}^{7} \gamma_{lr} \cdot V_{rp} \cdot -\Delta_l - \beta_l \right), \\
\frac{dQ_m}{dt} &= q_m \left( \alpha_{im} \cdot Q_i + \alpha_{jj} \cdot Q_j + \sum_{r=1}^{7} \gamma_{mr} \cdot V_{rp} \cdot -\Delta_m - \beta_m \right), \\
\frac{dQ_n}{dt} &= q_n \left( \alpha_{in} \cdot Q_i + \alpha_{ij} \cdot Q_j + \sum_{r=1}^{7} \gamma_{nr} \cdot V_{rp} \cdot -\Delta_n - \beta_n \right), \\
\frac{dD_i}{dt} &= k \left( \sum_{r=1}^{7} \lambda_{ir} \cdot V_{rp} \cdot \left( -\delta_i + \Delta_i - \beta_i \right) D_i \right),
\end{align*}
\]

where \( i \in \{1, \ldots, 7\} \).

All the parameters included in the system change over time.

The components on the right side of an equation, which have a plus sign, reflect the demand \( Q_i \) product (domestic energy production and exports), with the minus sign – the supply (domestic production and imports).

Products \( Q_i, i=1, 2, \ldots, s \) include also \( O_i, i=1, 2, \ldots, r \), therefore \( s \geq r \).

If we stop to produce \( Q_i \) product Ukraine, including for export supplies, from the time \( t_0 \), we take all parameters in equation (6) to be equal to zero, including a value of \( Q_0 \).

Thus, the constructed system of differential equations (16) makes it possible to represent formally the dynamics of the supply-demand ratio for energy resources over a certain period. Each equation reflects a certain aspect of decomposition during the system analysis of the Ukrainian energy market, and the system of equations represents a synthetic component of the study. It is possible to set a variation of parameters of equations by expert way, or normative way, or by approximation of revealed trends and changes.

Different combinations of the approaches are possible, which give possibility to carry out scenario forecasting of the development of the energy market of Ukraine. If \( Q_i \) product or \( Q_j \) fuel-and-energy product started to be produced only from the time \( t_0 \geq 0 \), then the corresponding initial conditions would be \( Q_{i(0)} = 0 \) and \( Q_{j(0)} = 0 \).

If \( O_i \) or \( Q_i \) fuel-and-energy resources went for export from the time \( t_0 \geq 0 \), then all parameters will be equal to zero, including \( O_{i0} \) and \( Q_{i0} \), which means termination of its production in Ukraine.

The obtained solutions to modeling the supply of fuel-and-energy resources from the beginning of year \( t_0 \) to the corresponding day \( t_0 \) demonstrate accuracy and adequacy of the equations proposed above. In calculations by the Euler method, we selected levels of alignment and proportionality of the system of differential equations so that simulated production or extraction of the corresponding energy resources coincided with the 2016 statistical energy balance data.

If we denote right sides of system equations (16) as \( dQ_i \) and \( dP_i \), respectively, then we obtain the following formulas:

\[
\begin{align*}
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
Q_i(t+1) &= Q_i(t) + h \cdot dQ_i(Q_i(t), Q_j(t), Q_k(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t), \\
P_i(t+1) &= P_i(t) + h \cdot dP_i(Q_i(t), P_j(t), t).
\end{align*}
\]

The formulas calculated by the Euler method (17) provide accurate and adequate calculations of processes of functioning of the energy market.

In order to ensure accuracy and adequacy of the modeling, in the equation of calculation of a volume of thermal energy supply, we took into consideration seasonality of thermal energy production. A piecewise-stable function represents the corresponding leveling factor \( q_1 \) in the equation for \( Q_i \). It is equal to zero for the spring-summer period of a year. Fig. 3 reflects the seasonality \( 3(d) \) in the form of a broken line.
Similarly, we can build charts for demand for fuel-and-energy resources for a year. Integration of the data obtained from modeling of supply of fuel-and-energy resources from the beginning of a year \( t_0 \) to the corresponding day \( t_n \) is consistent with a graphical abstraction with absolute indices of the energy balance of Ukraine’s production and consumption of fuel-and-energy resources [11]. The obtained solutions offer additional opportunities in the management of the end state, namely forecasting taking into consideration stochastic phenomena of price and temperature. It is clear that we can also take into consideration seasonality for consumption, due to the dependence of leveling factors, proportionality, and the rest on \( t \) variable, which specifies time.

7. Modeling the mechanism for maintaining the dynamic balance of the energy market

A key position of the conceptual market approach to managing processes in the energy sector of the Ukrainian economy is a study of the supply-demand ratio. Fig. 4 presents a mechanism of maintenance of its dynamic balance in
the form of a system archetype of a Cognitive map. Such a mechanism includes the ratio of demand and supply of fuel-and-energy resources. Development of the energy market of Ukraine requires an increase in the supply of fuel-and-energy resources. Extraction of own fuel-and-energy resources and development of renewable energy resources are of prime importance in the context of ensuring of the country’s energy security.

One of the revealed patterns of development of the world energy market under conditions of globalization is a growth of a role of its management subsystem.

For a more detailed cognitive study of the energy system in the context of European integration, we propose a system for management of the energy market, which is formalized based on a use of conceptual management models. In our opinion, insufficient attention to ontological and epistemological aspects of the market reduces efficiency of management and energy business in general.

The sign “+” means that an increase in one component entails an increase in the other one associated with it, and the sign “−” indicates that an increase leads to a decrease.

We propose the following formalized model of the energy market management system to manage processes of the fuel-and-energy complex system. We can describe it in terms of the theory of sets, mathematical logic and optimization problems.

We propose modeling of energy market processes as an object of theoretical research by the system approach of conceptual models of management.

We can represent the energy market management system in the form of an appropriate composition of subsystems $S_k^M$, that are formed of the following functional areas: extraction of fuel-and-energy resources $S_{ex}^M (i = 1, 2, ..., I)$, import of fuel-and-energy resources $S_{imp}^M (r = 1, 2, ..., R)$, informational provision $S_{inf}^M (j = 1, 2, ..., J)$, supply of energy resources $S_{sup}^M (k = 1, 2, ..., K)$, processing of fuel-and-energy resources $S_{proc}^M (l = 1, 2, ..., L)$, energy consumption by individuals $S_{mer}^M (m = 1, 2, ..., M)$ and organizations $S_{org}^M (n = 1, 2, ..., N)$.

We performed structural detailing of the components in dependence with a stage of development of the energy market, its hierarchical level (world, regional, etc.) and research tasks. Together they form the system of the energy market presented in the form of sets:

$$ S^M = \bigcup_{i=1}^{I} S_{ex}^M \bigcup_{r=1}^{R} S_{imp}^M \bigcup_{j=1}^{J} S_{inf}^M \bigcup_{k=1}^{K} S_{sup}^M \bigcup_{l=1}^{L} S_{proc}^M \bigcup_{m=1}^{M} S_{mer}^M \bigcup_{n=1}^{N} S_{org}^M. \quad (18) $$

The system $S^M$ unites the subsystems $S_k^M$.

$$ g \in G = I \times R \times J \times K \times L \times M \times N $$

formed of the above components of the energy market in accordance with its structure. Such an approach makes it possible to consider energy management in terms of achievement of synergies between elements of the system.

An emergent effect of system management is a component of competitive advantages of all parties in the energy market, the optimal combination of which can provide an increase in the overall effect of the system. The sum of effects of individual independent elements of the system does not provide adaptability and flexibility of system’s operation. Therefore, for a criterial assessment of effectiveness of processes to achieve the mentioned effect, each of $S_k^M$ subsystems of $S^M$ energy market may be given some target function $f_e$.

Integration of $S^M$ into a holistic system takes the form of unifying functions of management of the energy market (economic FE, organizational FO, regulatory FR and informational FI)

$$ S^M \supseteq \{ < FE, S^M > \cup < FO, S^M > \cup < FR, S^M > \cup < FI, S^M > \}. \quad (19) $$

In addition, the process involves formulation of a single objective functioning of the public energy market management system - ensuring a high level of energy security of a state. The target function (or set of target functions) $S^M$, in particular, can look as some combination of target functions forming its subsystems $S_k^M$, for example, with a help of target functions $f_e$. Specifically, for the sphere of extraction of fuel-and-energy resources $S_{ex}^M (i = 1, 2, ..., I)$, two objective functions are definitely important:

1) maximization of extraction;
2) minimization of expenses per unit of the extracted resource.

With respect to the import of energy products and fuel-and-energy resources $S_{imp}^M (r = 1, 2, ..., R)$, the target functions will be:

1) minimization of a price of FER on the Ukrainian border;
2) maximization of the number of suppliers.

In the sector of information provision $S_{inf}^M (j = 1, 2, ..., J)$, the target function will be:

1) minimization of a monopoly influence of the energy supplier on the market.

The sector of supply of energy resources $S_{sup}^M (k = 1, 2, ..., K)$ can have the following target functions:

1) minimization of expenses for supply of energy resources;
2) minimization of time for delivery of fuel-and-energy resources from the place of production and storage to a consumer;
3) minimization of a total time of recovery of a supply chain after accidents.

For the sphere of processing of fuel-and-energy resources $S_{proc}^M (l = 1, 2, ..., L)$, the following may serve as the target function:

1) minimization of expenditures of fuel-and-energy resources per unit of finished products.

For the sphere of consumption by individuals $S_{mer}^M (m = 1, 2, ..., M)$ and organizations $S_{org}^M (n = 1, 2, ..., N)$, the target function is:

$$ (*) \text{ for personal use, } \sum_{l=1}^{L} S_{proc}^M \bigcup_{m=1}^{M} S_{mer}^M \bigcup_{n=1}^{N} S_{org}^M. $$
1) minimization of prices for an energy product;
2) maximization of a share of renewable and alternative energy sources in the overall energy balance;
3) maximization of energy efficiency.

The corresponding systemic and coordinated restrictions of all subsystems of \( S \) system determine a set of possible alternatives for development of a market situation. We can define target system solutions in the direction of maximization of the criterion function \( P \). Optimization of activity of the entire management system requires consistency of restrictions on inputs and outputs of all subsystems. Thus, an agreed transformation takes place in determination of their local extrema.

We can apply the proposed model of management of energy market subsystems at any level of management, whether global, national, or regional.

### 8. Discussion of results of the implementation of the market model of management of the energy market

Each energy carrier – gas, electricity, oil, coal, thermal energy, has its own specifics of use and mode of delivery. First, the market model of management implies a need for institutional changes in the management of production processes and consumption of fuel-and-energy resources under an influence of demand and supply on energy markets. Such changes should ensure high-energy efficiency, energy saving, openness and transparency of markets, life safety and energy supply, synergy and innovative functioning, and development of competitive relations.

The liberal energy market involves producers, traders and investors both nationally and foreign ones based on free and clear market access. Transparent energy market should eventually ensure uninterrupted power supply, optimum satisfaction of domestic demand and stimulation of national energy production. Consequently, Ukraine should move toward an open market where producers operate in the market freely, carrying out export operations for profit.

The market approach to management of the energy sector of the Ukrainian economy, considered in this study, is useful because the proposed economic and mathematical models specify processes of demand and supply of fuel-and-energy resources. The developed system modeling helps to assess functioning and development of the energy market adequately.

The proposed system of economic and mathematical models of management of the energy market on the principle of its liberalization gives an opportunity to get an accurate assessment of supply and demand of fuel-and-energy resources.

The results obtained using a given approach, taking into consideration corrective factors, are fully consistent with the absolute indices of production and consumption of fuel-and-energy resources of the Energy Balance of Ukraine [11]. The system of differential equations is an integral system of interconnection of parameters of production, import, export, final consumption, and external and internal prices for gas, oil, black oil, coal, uranium, electricity, and thermal energy.

We described processes of formation of the market “Demand-supply” ratio for all types of fuel-and-energy resources by a conceptual dynamic economic-mathematical model. A base of the model is the system of differential equations of the first order, the left part of which represents a rate of change of the studied economic indicators. The market approach takes into consideration qualitative and quantitative characteristics of supply and demand, which reflect consumption and production of fuel-and-energy resources, exports and imports under an influence of globalization.

The practical significance of the results obtained is that the proposed econometric and mathematical models ensure accuracy and efficiency in making managerial decisions by managing systems. The market approach provides a principle of liberalization in management of processes of the energy market of Ukraine. The proposed approach involves the principle of openness of the energy system.

Further scientific necessity arises for carrying out a prognostic analysis of trends in development of the energy market in the context of European integration, taking into consideration such random factors as air temperature and fluctuations of world prices for energy resources. This may lead to consideration of a system of stochastic managerial equations, which in turn requires additional assumptions regarding probabilistic laws of temperature distribution and price fluctuations.

### 9. Conclusions

1. We performed an analysis of the state of the energy market in Ukraine based on the systemics of the investigated object. We presented the main directions of system analysis and evaluation of development of energy market processes. The structure of functioning of the energy market reflects a conceptual framework of the study in the form of a generalized system of processes of functioning of the energy market of Ukraine. We developed a model of system connections of subjects of the energy market based on the systematic approach of organization of management of energy market processes. Unlike the existing ones, the market model must accurately respond to the needs of the energy market and expand a structure of alternative energy sources.

2. We developed economic-mathematical modeling of energy market processes based on the principle of a market approach. The proposed system of differential equations makes possible to assess volumes of supply and demand for fuel-and-energy resources accurately. A closed system of 14 linear differential equations describes dynamics of the energy system. The system of equations allows us to investigate behavior of the dependence of the main macroeconomic parameters on a change in a price of imported fuel-and-energy resources accurately, and to build possible forecasts of economic development.

3. We proposed formulas based on the Euler method to provide accurate calculations of functioning of energy markets. We established that models are adequate because the simulated production or extraction of the corresponding energy resources coincides with the 2016 statistical data of the Energy Balance. Graphical interpretation of the results of economic and mathematical modeling shows possibility of assessment of Ukraine’s daily supply of oil, coal, black oil, thermal energy, electricity, gas, and uranium. Taking into consideration corrective factors, calculation of annual supply volumes of fuel and energy resources by the end of 2016 reaches the following values: oil – 1,623 thousand tons, coal – 43,000 thousand tons, black oil – 390 thousand tons, heat energy – 447,526 TJ, electric power – 164,573 million kWh, gas – 705,926 TJ, uranium – 1,100 tons. The presented system
of differential equations is an integral system of interconnection of parameters of production, import, export, final consumption and external and internal prices for gas, oil, black oil, uranium, electricity, and thermal energy.

4. We developed a mechanism for maintenance of the dynamic balance of the energy market. It includes the main components of market functioning: consumers, fuel-and-energy complex, import and export. Such a mechanism demonstrates the unity of its quantitative and qualitative characteristics in formation of the demand for fuel-and-energy resources and their supply in the context of ensuring of the country's energy security. We proved that the development of national production of fuel-and-energy resources has a significant influence on formation of supply of energy products.

5. The corresponding system-coordinated restrictions of all subsystems of the energy system determined a set of possible alternatives to development of a market situation. We proposed the model of management of subsystems of the energy market in the form of an appropriate combination of subsystems for extraction of fuel-and-energy resources, import of fuel-and-energy resources, informational provision, supply of energy resources, processing of fuel-and-energy resources, energy consumption by individuals and organizations. Together they build the energy market system presented in the form of sets. Such approach makes possible to consider energy management in terms of achievement of synergies between elements of the system. We described possible target functions of management of subsystems of the energy market. We established that optimization of activity of the whole management system requires consistency of restrictions at inputs and outputs of all subsystems of the fuel-and-energy complex. We proved that liberalization of the energy market will ensure equality of access to fuel-and-energy resources and services of electricity networks, as well as purchase and sale of fuel and electricity at market prices under conditions of market competition.

6. The system of differential equations suggests that its application to the market model of management of the energy sector of the economy makes possible to estimate a level of demand and supply in such models. Taking into consideration factors of influence on processes of demand and supply contributes to the accurate assessment of a state of the energy market.

References