1. Introduction

Ukraine occupies one of the leading positions in the world for the volume of production and processing of oils and fats. Modernization of the equipment used at oil-fat-producing enterprises, the employment of modern resource-saving technologies, a wide range of products, the high level of competitiveness, are the attributes that characterize a contribution of this sector to the local economy. Energy crisis and an increase in consumer demand for more high-quality products, specifically vegetable oil, stimulated more research aimed at finding effective technologies of extraction.

Extraction of valuable components from vegetable (oil) raw materials refers to the important processes that are characterized by high energy intensity. Improving the efficiency of extraction significantly affects the technical-economic indicators of processing industry in agricultural production. At the present stage of development of equipment and technology for extracting the oil raw materials, the potential of microwave technology should be recognized: the application of electromagnetic field (EMF) of ultra-high frequency (UHF). Employing the specified technology, in our opinion, will lead to the intensification and improvement of efficiency of the traditional manufacture of vegetable oils to
obtain products with the new, better consumer qualities. The theoretical postulates and experimental data acquired earlier have demonstrated the effectiveness of the application of microwave technology for extraction.

Microwave technologies relate to those energy-saving because of their natural specifics. However, in order to design actual equipment, it is necessary to conduct comprehensive studies, the purpose of which is to determine the rational conditions for the process under which it would be possible to achieve the uniformity of a microwave field, failure-safe operation, as well as labor safety.

In order to deeper study the influence of UHF EMF on the processes of mass and heat transfer, it is necessary to have mathematical models that would describe the process under consideration in the simplified yet adequate form. Undertaking a theoretical research into mathematical modeling of the process of extraction will make it possible to specify experimental studies and will enable the construction of an algorithm for improving the technologies of oilseed crops processing.

The scientific literature provides many models for the extraction processes regarding the conditions for periodicity or continuity of their course at various intensification techniques (increasing pressure and temperature, imposing vibrations, mechanical agitation, etc.) [1–5]. The basic theoretical concepts that underlie the extraction processes are given in fundamental studies [4, 5]. The theoretical references, substantiation of appropriateness and effectiveness of UHF EMF application, as well as a review of existing modeling methods, are described in detail in works [3, 5]. Specifically, it was noted that in terms of the physical effect of EMF on solid materials that are extracted, the leading role in the intensifying action belongs to barodiffusion processes.

It is a relevant task to undertake a research into the further development and improvement of the mathematical apparatus that characterizes the heat-and-mass exchange processes in the specified extraction units.

### 2. Literature review and problem statement

One of the principal stages in the design of extraction units is mathematical modeling. A procedure for calculating extraction units is based on taking into consideration the three basic aspects of the process: equilibrium in the system, kinetic patterns at all stages of mass transfer, and the structure of phase flows in the device.

Research into the process of extraction of oil-containing crops is mostly based on conducting experimental studies into extraction kinetics. Thus, authors of [6] acquired experimental data on the extraction kinetics of soybean oil and free fatty acids (FFA) for systems containing soy and ethanol with different levels of hydration (0 and 5.98% by weight of water) at temperatures of 40, 50 and 60 °C. The obtained experimental data reveal that an increase in the level of ethanol hydration inhibits the extraction of soybean oil but increases the extraction of FFA while the temperature promotes the solubility of both fatty compounds. The experimental data were compared to models [7, 8], which make it possible to determine coefficients of mass transfer at the stages of diffusion and estimate the coefficients of diffusion. The derived values for diffusion coefficients show that the applied models were suitable to describe the kinetics of oil extraction, as well as other compounds present in soy. A value for the diffusion coefficient for soybean oil increased with an increase in temperature and a decrease in the level of hydration in the solvent. However, the authors failed to propose any mathematical models that would describe the process of extraction in general; the calculation procedure is not given either.

Paper [9] reports results of research into kinetics of the solid-phase extraction from Fumaria officinalis, in order to derive diffusivity and explain the mass transfer. It is shown that the extraction was carried out by changing the following operating conditions: temperature, hydromodule “solid body – solvent” and the percentage of ethanol content in solvent. A simple method is described, which is based on the Fick’s laws for predicting the effective coefficient of diffusion and speed intensity from the experimental kinetics. The Bio number revealed that the diffusion inside particles is the degree of control over the process of extraction. It is shown that it is desirable to operate with a moderate weight ratio “solid substance–liquid” at the maximally possible temperature and at a moderate content of ethanol in water [9].

However, there are remaining issues, not addressed by authors in [9], related to forecasting an increase in the concentration of the substance that is extracted depending on the supplied power of the ultra-high frequency electromagnetic field.

Regular extraction methods are described using the models of derivatives from the Fick’s laws since parameters of the model include physical values that can be applied for further interpretations. For example; the Bio number (Bi) expresses the relative significance of the internal and external mass transfer resistance [10]. Theoretical kinetic models are also applicable for extending the modeling of the extraction process. On the other hand, when simulating the process of extraction “solid body–solvent”, associated with the supply of microwave energy, ultrasound, electrical charge, simplified (empirical) models from the modified Fick’s law are applicable. Empirical models, in such cases of energy supply, are the best and indicate a much more complicated process of mass transfer. Such models are more suitable for extraction processes using the ancillary methods, since they cannot be adequately described in theory [11]. However, it is probably needed to conduct a theoretical research into the process of modeling in a microwave field and compare theoretical curves to experimental data.

A model of the process of extraction kinetics should define a process duration (exposure) under a periodic mode, as well as the estimation of the mass transfer intensity.

During the process of extraction (removal of an extractive substance from solid components of vegetable raw materials) in the UHF electromagnetic field, the intensity of mass transfer is defined by: the mass conductivity, convective diffusion, as well as the effect of a microwave radiation.

When considering in detail the effect of the latter factor on the process, it was established in papers [3, 12–14] that energy of the microwave field, by concentrating in the liquid phase of capillary volume of a solid particle, induces the formation of “steam bubbles” of the extracted substance. The result of the action of electromagnetic energy is the emergence of a pressure gradient with the liquid (periodically) pushed out of the capillary into the flow of solution.

In contrast to the classical determination of a barodiffusion mass transfer, which defines the filtration motion of a steam phase under the influence of pressure gradient $D$, effect of barodiffusion of an electromagnetic field implies...
the removal of the liquid phase (its part). The intensity of barodiffusion is defined by the magnitude of pressure that grows in a capillary at an increase in the intensity of electromagnetic influence.

The mechanism of the combined process of mass transfer of the extracted substance from a capillary-porous structure into extractant is explained in accordance with [4, 15] by the electrodiffusion model representation: diffusion in a solid phase is determined from the first Fick’s law:

\[ I = -D \frac{dc_n}{dm} = -D \frac{dc_n}{dx} \]

Next, the dissolved substance moves to the outer surface (along capillaries) where concentration is determined by the diffusion resistance: the same flow is transferred by the convective diffusion to the “core” of the extractant flow because the substance does not stay at the interphase surface. Thus, it is possible to record:

\[ I_1 = \beta (C_e - C_n) = \beta (C_e - \bar{C}). \]

where \( C_e, C_n \) is the concentration at the interphase surface and in an extractant; \( \bar{C} \) is the mean concentration of an extractant; \( \beta \) is the effective coefficient of mass release and a mass exchange coefficient.

In parallel to flow \( I_1 \), the flow \( I_2 \) emerges in the particles of the solid-phase in a microwave field due to an increase in pressure in capillaries (barodiffusion). The flow of mass \( I_2 \) is defined by a “specific” effective coefficient of mass release \( \beta_p \) and a difference in pressure in the capillary:

\[ I_2 = \beta_p (P_p - P_s). \]

where \( P_p, P_s \) is the pressure in a capillary and solvent (extractant).

Flow \( I_2 \) must turbulize a boundary layer and increase, accordingly, the intensity of mass transfer \( \beta_p \).

The total flow of mass is the sum of flows:

\[ I = I_1 + I_2 = -Ddc_n/dx. \]

An analysis of prospects to solve the problem on mass transfer during extraction in a microwave field analytically (models of mass transfer from a plate to a flow of liquid; diffusion at flow motion in a channel, diffusion from a point source), given in [4, 5], revealed that even under very serious simplifications of the problem it is almost impossible to derive analytical dependences for a quantitative analysis of the process in an actual device. Thus, a promising way to study the process of extraction in an UHF electromagnetic field would be to model it at the level of micro kinetics with the identification of the derived models based on experimental data. In the practice of calculating parameters and operational modes of units for the extraction of a substance from a solid body, the key issue is to devise a simple (simplified) mathematical notation of the process. A mathematical model would reliably take into consideration, while limiting and intensifying, the physical phenomena, and the assumptions accepted in this case would be neutralized by the experimentally derived values for effective coefficients. Such approaches to investigating complex phenomena of transfer have been employed in papers [1, 2].

Given the impossibility to optimally run a material extraction process in the system “solid body–liquid” with MW energy supply based on the criteria of energy efficiency and quality of the resulting product without an adequate mathematical model.

3. The aim and objectives of the study

The aim of this study is to construct a mathematical model for the extraction process, which would take into consideration the effect of UHF EMF on intensity of the transfer processes.

To accomplish the aim, the following tasks have been set:
- to substantiate the physical parameters that are included in the structure of a mathematical model for the heat-and-mass processes in the extraction of oil raw materials in an ultra-high frequency electromagnetic field, which would take into consideration all interacting objects, and to derive an approximated solution to it;
- to study experimentally the process of extraction of oil (rapeseed) raw materials in an ultra-high frequency electromagnetic field and to verify the theoretical and experimental data obtained.

4. Construction of a mathematical model for the process of extraction of oil-containing raw materials with a microwave energy supply

Under conditions of the electromagnetic field, there is the interaction between polar molecules of the system “product–extractant” and electromagnetic energy, the result of which is the formation of a steam phase in a capillary-porous structure of the raw material with the intensification of the hybrid flow, which can significantly reduce the internal diffusion resistance of oilseed raw materials, intensify the mass exchange process, shorten the duration of the process, and improve the yield of extraction components. The power of such a flow is defined by the characteristics of the raw materials and parameters of the electromagnetic field. A complete model of mass transfer processes during extraction in a microwave field in the differential form would make it possible to establish the conditions for comprehensive experimental study, which would fully define the extraction process of oil raw materials.

In order to construct a mathematical notation of the extraction process (under a periodic mode) under the action of UHF EMF, it is required to formalize the physical processes (phenomena), which predetermine certain forms of a substance transfer under the action of a microwave field. In the general case, the effect of UHF EMF implies the intensive selective (in terms of volume) heating of a liquid substance contained in a solid body and produces a thermobarodiffusion effect [5, 15].

In order to formalize the phenomenon of a barodiffusion transfer of a substance’s component that is extracted under the action of a microwave field, we shall employ the general theory of heat and mass transfer in a capillary-porous body [14, 16, 17]. It follows from the heat-and-mass transfer that the intensifying effect of a microwave field is largely predetermined by an intensive volumetric heating of the liquid phase in a solid skeleton of the substance. Volumetric heating decreases fluid viscosity, gives rise to vaporization, increases pressure of the steam phase and the liquid phase filtration transfer (“release” of the fluid from a capillary into...
the external environment). Thus, both at thermal dehydration and extraction there occur the same, mentioned above, flows of mass: diffusion, capillary, barodiffusion [14, 16, 18]. Based on the law of preservation of a substance mass, a local derivative from the volumetric concentration of the i-th component of the substance for time equals the sum of divergence of the flows of mass and the source of the substance, predetermined by the phase transition (liquid to steam) [16]:

$$\frac{\partial (U \cdot S)}{\partial t} = \text{div} \left( J_{\text{diff}} + J_{\text{cap}} \right) + I_b,$$

(5)

where $J_b$ is the density of the diffusion flux; $J_b = -\rho_i \lambda \text{grad} U$; $J_{\text{cap}} = \Pi \rho \delta \delta$ is the density of the flow of capillary moisture; $\Pi$ is the porosity of the body; $\rho$ is density; $b$ is the saturation of pores, $\delta$ is the average linear speed of the molar motion of the i-th substance; $I_b = \varepsilon \rho_i \lambda \frac{\partial U}{\partial t}$ is the intensity of the source of the substance predetermined by the phase transition (steam formation) and by pushing the liquid phase by the pressure of steam formed under the influence of an electromagnetic field; $D$ is the diffusion coefficient; $\varepsilon$ is the criterion of a phase transition; $\varepsilon = (1 - \varepsilon^2)$ is the equivalent coefficient of a phase transition and thermomechanical moisture removal, $\varepsilon$ is the coefficient of the “thermomechanical” pushing of a substance [18] (effect of the thermomechanical pushing of moisture out of the pores of a solid body when drying using a pressure drop is estimated by the magnitude $\varepsilon = 0.05 - 0.4$ ([18])).

Given that the flow of a substance is predetermined by a thermodiffusion transfer in a monoporous structure of the body, which is defined by the uneven temperature field of the body, almost absent at volumetric heating, we shall rerecord equation (5) substituting $J_b$, $I_b$ with appropriate expressions:

$$\frac{\partial U}{\partial t} = \rho_i \lambda \nabla^2 U + \varepsilon \frac{\partial U}{\partial t} \rho_i,$$

(6)

or, for a one-dimensional body, for the concentrations of the extracted substance:

$$\frac{\partial c}{\partial t} = \frac{D_i}{1 - \varepsilon'(\Theta)} \frac{\partial^2 c}{\partial x^2} = D_i \left( \Theta \frac{\partial^2 c}{\partial x^2} \right),$$

(7)

where $D_i$ is the effective diffusion coefficient that characterizes all possible types of mass transfer of the target component in the form of a certain unified mass [1].

Thus, we have derived a differential equation that describes the non-stationary field of concentration of the component inside the capillary-porous materials. In a differential equation, the physical effects of a transfer, related to the movement of the liquid and steam-gas phase in a material, are integrally accounted for by a unified quasi-diffusion transfer with effective coefficient $D_i$. In practical calculations, one must have a dependence (acquired from experiments) of the magnitude of a diffusion coefficient on temperature and power of internal heat release due to the influence of UHF EMF.

To solve equation (7), when implementing the process of extraction of the dispersed solid phase, the boundary conditions of third kind are applied. According to the boundary conditions of third kind, the exchange of target component between the surface of a capillary-porous body and the surrounding liquid environment can be recorded in the form of equation for an external mass release:

$$j_f = -\beta (c_f - c_p),$$

(8)

where $j_f$, $c_f$ is the flow and concentration of the target component at the surface of solid particles; $c_p$ is the concentration of the component in the external environment (extractant); $\beta$ is the mass release coefficient.

A flow that is guided to the surface from the depth of a porous structure through effective diffusion (in the absence of a source at the interphase surface, it is equal to the flow that is guided away from the surface) because the substance at the surface does not accumulate and there is the following equality:

$$D_i \frac{\partial c}{\partial x} = \beta (c_p - c_f),$$

(9)

which is the boundary condition; the initial condition, at $x = 0$; $c = c_{0|0}$. For a body (a particle) that has the shape of a one-dimensional plate of thickness $2R$, a compatible solution to equations (7), (9) is known [19] and, according to the mean-volumetric value for a change in the concentration of a component, is derived from equation:

$$\bar{c}(x) - c_p = \sum_{n=1}^{\infty} B_n e^{-\frac{n^2 \pi^2}{4R^2}},$$

(10)

where

$$B_n = \frac{2 \sin^2 \mu_n}{\mu_n (2 \mu_n + \sin \mu_n \cos \mu_n)},$$

where $\mu_n$ are the roots of the characteristic equation:

$$\mu = B_i \varepsilon \tan \mu; \quad B_i = \frac{\beta R}{D_i},$$

is the mass-exchange Bio criterion.

Equation (10) describes the kinetics of extraction – the removal of a target component from the solid phase in a time function $\tau$, at constant value for the concentration of solution $c_p$=const and diffusion coefficient $D_i$.

It is known from [19, 20] that at large values for the Fourier criterion $F = \frac{D_i R^2}{\pi^2}$ (practically at $F > 0.3$), a series in equation (10) quickly converges and one can neglect all the terms in a series except for the first one and use the one-term approximate solution:

$$\bar{c}(x) = c_p + (c_0 - c_p) B_1 e^{-\frac{x^2}{4R^2}}.$$  

(11)

Differentiate equation (11) for time:

$$\frac{\partial \bar{c}(x)}{\partial t} = -\Delta \overline{\varepsilon} \frac{1}{\pi} \frac{D_i}{R^2} e^{-\frac{x^2}{4R^2}} B_1,$$

(12)

where $\Delta c = c_0 - c_p$.

We determine from equation (12) the magnitude $B_1 e^{-\frac{x^2}{4R^2}} . \Delta c$, substitute into equation (11); we obtain after transforms:
\[
\frac{d}{dt}(\bar{c}(t)) = K_{\text{hyd}}(\bar{c} - c_p(t)) = K_e(\bar{c}(t) - c_p(t)),
\]

where, respectively, \([20]\):

\[
K_{\text{hyd}} = \mu_i \frac{D_{ij}}{R^2}; \quad \mu_i = \frac{1}{\pi} \frac{1}{B_i}
\]

The magnitude \(K_{\text{hyd}}\) is termed an extraction coefficient (similar to the drying coefficient by O. V. Lykov); we determine it experimentally, depending on the operational parameters of the process. The magnitude \(c_p\) can accept a value for the equilibrium concentration, which is also determined experimentally.

Theoretical methods of the diffusion coefficient calculation, as known from the scientific literature \([1, 2]\), are not applicable in real systems. The only way to obtain data on the magnitude of diffusion coefficients \(D\) and \(D_{ij}\) are the experimental measurements of process parameters in the system “solid fraction – liquid (a solvent)”. In order to verify experimental data on the kinetics of extraction using equation (13), it is necessary to determine the constants of the process: \(D, B, B_i, D_{ij}, K_e\). The simplest experimental procedure for determining the specified magnitudes is the periodic process of extraction \([2]\). During experiments, one measures the quantity (concentration) of a target component transferred from the solid phase to the liquid phase at certain points of time in the non-stationary process.

To solve equation (13), it is necessary to establish relations between the systems (solid and liquid), essential magnitudes in the extracted material and an external solution. Such relation can be obtained from the equation of material balance in the differential form:

\[
-V_m \frac{dc}{d\tau} = V_p \frac{dc_p}{d\tau},
\]

where \(V_m, V_p\) are the volumes of interacting phases.

The ratio of magnitudes of the mass of a solid particle to the mass of a solvent is used as a parameter of the process: \(m_m = V_m \rho_m; m_p = V_p \rho_p\), where \(\rho_m, \rho_p\) is the density of the solid phase and solution.

\[
\frac{dc_p}{d\tau} = \frac{K_{\text{hyd}}}{\rho_p} \frac{dc}{d\tau}
\]

where \(R = \frac{m_m}{m_p}\) is the hydromodule.

The kinetics of a mass transfer, a change in the concentration of solution over time, is determined from equation:

\[
\bar{c} = \beta \cdot F_i (\bar{c} - c_p),
\]

where \(\bar{c}, c_p\) are the mean-volumetric values for concentrations; \(F_i\) is the specific volumetric interphase surface; \(\beta\) is the mass exchange coefficient.

Thus, the kinetics of extraction is described by a system of three differential equations \((13), (14)\) and \((16)\). In the specified differential equations the effect of a microwave field is accounted for through coefficients \(K_{\text{hyd}}\) and \(\beta\) (since the release of a substance from surface under the action of steam pressure, predetermined by the influence of UHF EMF, turbulizes the surface layer of solution).

For the parametric identification of the structured mathematical model (differential equations \((13), (14), (16)\)), we must have an analytical dependence of solvent concentration (extractant) on the process duration, that is, a description of the kinetic curve.

Deduce equation (16) from equation (13):

\[
\frac{d}{dt}(\bar{c} - c_p) = -(K_e + \beta F_i)(\bar{c} - c_p).
\]

Upon division of variables, we obtain:

\[
\frac{d}{dt}(\bar{c} - c_p) = -K d\tau,
\]

where \(K = K_e + \beta F_i\).

The solution to equation (18) under initial conditions: \(\tau = 0; c = c_0; c_p = c_{p0}\) will be written in the following form:

\[
c(\tau) = c(\tau) = (c_p - c_0)e^{-K\tau}.
\]

Hence, we obtain:

\[
c(\tau) = c_0 + \Delta c e^{-K\tau};
\]

\[
c_p(\tau) = c(\tau) - \Delta c e^{-K\tau};
\]

where \(\Delta c = c_0 - c_{p0}\).

Substituting the value of \(c(\tau)\) from (19) into equation (16), and the value of \(c_p(\tau)\) from (21) into equation (13), upon transforms, we obtain:

\[
\frac{c(\tau)}{c_p(\tau)} = \beta F_i \Delta c e^{-K\tau},
\]

\[
\frac{dc(\tau)}{d\tau} = -K \Delta c e^{-K\tau}.
\]

By integrating equations (22) and (23) under the predefined initial conditions, we obtain:

\[
c(\tau) = c_0 + \frac{\beta F_i}{K} \Delta c (1 - e^{-K\tau}),
\]

\[
c(\tau) = c_0 \frac{K}{\Delta c} (1 - e^{-K\tau}).
\]

By integrating equation (14), we obtain

\[
c_0(\tau) = \bar{c} - \frac{V_0}{V_p} c(\tau),
\]

where

\[
\bar{c} = \frac{c_0}{\bar{c}} + \frac{V_0}{V_p} c(\tau).
\]

By substituting values of \(c(\tau)\) from (25) into equation (26), we obtain the equation of extraction kinetics for the liquid phase.
A change in the concentration of an extractant over time:

\[ c_p(t) = \frac{K_v \Delta c_p}{(K_v + \beta F) \tau_p} (1 - e^{-\delta \tau}) + c_{p0}. \]  

(27)

Equation (27) includes three unknowns \( K_v, \beta F, \Delta c_p \), which can be derived from the experimental curve of change in the solution concentration over time, obtained for different magnitudes of power of the source of UHF EMF.

To account for the impact of UHF EMF on the extraction process in the mathematical notation in an imaginary form, we shall consider the thermal balance of a material’s particle. The heat that is released in the particle (by the absorption of electromagnetic waves) \( P_d \) is spent on heating the material (a solid body plus a liquid) \( m \) and a partial conversion of a liquid substance into steam \( m_r \) (a solid body plus a liquid) and solution \( m_p \), following the transforms, we obtain:

\[ P = mc \frac{d\Theta}{dt} + m_r c_r \frac{dU}{dt} \]  

(28)

Using the definition for the Kosovitch criterion:

\[ K_v = \frac{rdU \epsilon'}{cd\Theta} \]  

(the ratio of heat on evaporation to the heat on heating a material) we write:

\[ \frac{rdU}{d\tau} = \frac{K_v \epsilon' \theta}{d\tau} \]  

(29)

Substituting the value of \( \frac{d\Theta}{d\tau} \) into equation (6), taking into consideration \( U = \frac{c}{\rho_0} \), following the transforms, we obtain:

\[ \frac{dU}{d\tau} = \left( \frac{m}{K_v} + m_\epsilon \right) \tau \frac{P}{r} \]  

\[ \frac{dc}{d\tau} = D \frac{dc}{d\tau} + \epsilon' \left( \frac{m}{K_v} + m_\epsilon \right) \]  

(30)

The second term in the right part of equality (30) has the dimensionality of kg/m’s and describes the intensity of the internal source of a substance released under the action of a source of microwave radiation of power \( P = \eta N \) (\( N \) is the power of the generator; \( \eta \) is its efficiency).

Equation (30) is similar to the equation of thermal conductivity of a plate (infinite) with an internal source of heat, the solution to which is known [19], hence, by analogy [19], we write down the solution to equation (30) under boundary conditions (9) for a mean-volumetric concentration of the substance:

\[ \frac{c(t) - c_p}{c_0 - c_p} = \frac{P_0}{m} \left( 1 + \frac{3}{B_m} \right) - \sum_i \left( \frac{P_0}{\mu_i} \right) B_{ri} e^{-\delta \tau} \]  

(31)

where

\[ P_0 = \frac{q_m R^2}{D(c_0 - c_p)} \]

is the mass-transfer equivalent of the Pomerantsev criterion; \( q_m \) is the intensity of the internal source:

\[ q_m = \frac{\epsilon' P \rho_0}{r} \]  

(32)

Confined to the first term in the series in equation (31):

\[ \frac{d\tilde{c}(t)}{dt} = c_p(\tau) + \Delta c \left( 1 + \frac{3}{B_m} \right) - \Delta c \left( 1 + \frac{P_0}{\mu_i} \right) B_{ri} e^{-\delta \tau} \]  

(32)

(33)

By determining from equation (33) the magnitude:

\[ \Delta c = \frac{B_{ri} e^{-\delta \tau} \epsilon'}{m \epsilon'} \]  

(34)

and by substituting it into equation (32) after the disclosure of value for criterion \( P_0 \) and the respective transforms:

\[ \frac{\tilde{c}(t) - c_p(\tau)}{c_0 - c_p} = \frac{q_m R^2}{(c_0 - c_p)} D \left( 1 + \frac{3}{B_m} \right) \]  

(35)

\[ \frac{\tilde{c}(t) - c_p(\tau)}{c_0 - c_p} = \frac{q_m R^2}{(c_0 - c_p)D} \]  

(36)

In order to determine the kinetic dependences of change in the concentration of solid phase \( \tilde{c}(t) \) and solution \( \tilde{c}_p(\tau) \) equation (22):
under initial conditions:

\[
\tau = 0, \quad \tau = \tau_0, \quad \varepsilon_p = \varepsilon_{pw}, \quad \Delta \varepsilon_0 = \varepsilon_0 - \varepsilon_{pw}.
\]

The solution to equation (37) relative to variables \(c(\tau)\) and \(c_p(\tau)\) will be written down in the form:

\[
c(\tau) = c_0 - \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau},
\]

\[
c_p(\tau) = c_p(0) - \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau}.
\]

By substituting the derived values for variables \(c(\tau)\) and \(c_p(\tau)\) in the first and second equation of system (36), we obtain:

\[
\frac{dc}{d\lambda} = \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau},
\]

\[
\frac{dc_p}{d\lambda} = \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau}.
\]

By integrating equations (40) and (41) under initial conditions: \(\tau = 0, c = c_0, c_p = c_{pw},\) we obtain dependences of concentration of the target component on extraction duration:

\[
c(\tau) = c_0 - \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau},
\]

\[
c_p(\tau) = c_p(0) - \frac{K_o}{K} Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} e^{-\beta \tau}.
\]

Substituting values \((c_0 - c(\tau))\) from equation (42) in the third equation in system (36), we obtain:

\[
c_p(\tau) = c_p(0) - \frac{\rho_m K_o}{\rho_m K_o + M} \left[ Q_o \frac{Q_o + \Delta \varepsilon_0}{Q_o + \Delta \varepsilon_0} (1 - e^{-\beta \tau}) \right] \left( 1 + \frac{Q_o}{Q_o + \Delta \varepsilon_0} \right).
\]

The result equation (44) describes a change in the concentration of solution over time under the influence of UHF electromagnetic field.

Equation (44) contains three unknown constants \(K, K_o, Q_o,\) which can be calculated from equation (44) if one has data on measurement of concentration \(c_p(\tau)\) for various time intervals \(\tau\) of process duration. And, by deriving these values based on ratios:

\[
K_o = \frac{R^2}{D} \left( \frac{4}{\pi^2 + 2} + \frac{1}{B_1} \right) e, \quad K = K_o + B F, \quad Q_o = \frac{B_1}{\pi^2 + 4 B_1}.
\]

It is possible to calculate the kinetic coefficients of the original equations.

The considered model representations of the kinetics of extraction of vegetable raw materials do not explicitly take into consideration the influence of a temperature regime of interaction between a solid and liquid phase, therefore, we shall consider a possibility of taking into consideration the effect of temperature (of the solid phase and environment).

As was shown above, the intensive steam formation under the action of a microwave field (boiling liquid in the microvolumes of a product [4, 5, 13]) creates excess pressure in the solid phase, which, according to experiments [2], changes little in terms of volume. The excess pressure is determined by the intensity of the phase transformation of liquid-vapor, which is quantitatively determined from equation [16]:

\[
\frac{\partial p}{\partial \tau} = -\frac{e^*}{c_s} \frac{\partial u}{\partial \tau},
\]

where \(c_s = M M_B / \rho_0 R T\) is the specific capacity of the fluid's steam (a capillary body capacity relative to the moist air), \(M_B\) is the molecular weight of the substance (formed by steam), \(\rho_0\) is the density of a dry solid body, \(R\) is the universal gas constant, \(T\) is the absolute temperature, \(\eta\) is porosity.

The magnitude of pressure of steam depends on its concentration and temperature according to ratio:

\[
P = \rho_0 R T (\Theta + 273),
\]

we shall obtain by differentiating it:

\[
\frac{\partial p}{\partial \tau} = \rho_0 R T \frac{\partial \Theta}{\partial \tau}.
\]

We obtain the following equation from equations (45) and (47):

\[
\frac{\partial p}{\partial \tau} + \epsilon \frac{\partial u}{\partial \tau} = \epsilon \rho_0 R T \frac{\partial \Theta}{\partial \tau}.
\]

Substituting values of \(\epsilon \frac{\partial u}{\partial \tau}\) (48) in equation (6) and passing over to the volumetric concentration, we shall write equation (7) in the form:

\[
\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + c \rho_0 R T \frac{\partial \Theta}{\partial \tau}.
\]

In equation (49), the last term accounts for the flow of a substance under the influence of an increase in temperature predetermined by UHF heating.

The kinetics of heating a solid phase in a microwave field can be derived from the solution to the equation of thermal conductivity of an infinite plate with an internal source of heat:

\[
\frac{\partial \Theta(x, \tau)}{\partial \tau} = a \frac{\partial^2 \Theta(x, \tau)}{\partial x^2} + \frac{q_v}{c_p},
\]

under boundary conditions of third kind:

\[
\frac{\partial \Theta(R, \tau)}{\partial x} + \alpha \left( \Theta(R, \tau) - \Theta_0 \right) = 0.
\]

for a mean-volumetric temperature (at small size of particles), it takes the form [19]:

\[
\frac{\partial (\Theta(\tau) - \Theta_0)}{\partial \tau} = 1 + B \left( \Theta(\tau) - \Theta_0 \right) \left( \sum_{i=1}^{n} \left( \frac{P}{\mu_2} \right) B \right) e^{-\mu_2 \Delta E_0}.
\]
where \( t_p \) is the temperature of solution:

\[
P_0 = \frac{q_0 R^2}{\lambda(t_p - \theta_0)}
\]

is the Pomerantsev criterion;

\[
F_0 = \frac{D}{R^2} \tau
\]

is the Fourier criterion; \( 2R \) is the thickness of the plate; \( \lambda_a \) are the coefficients of thermal conductivity and thermal diffusivity; \( \mu_n \) are the roots of the characteristic equation.

Equation (52) is similar to equation (31), therefore, leaving the intermediate transforms (employed above), we write the final equation of the kinetics of heating.

\[
d\theta(\tau) = K_n(t_p - \theta(t)) + Q_n,
\]

where

\[
K_n = \frac{R^2}{\pi^2 \left( 1 + \frac{1}{B} \right)}
\]

\[
B_n = \frac{\alpha R}{\lambda}
\]

\[
Q_n = \frac{q_0 R^2}{\lambda} \left( 1 + \frac{1}{B} \right)
\]

At a preliminary heating of the solution to boiling point temperature \( t_p = t_k \), equation (53) can be integrated:

\[
\theta(t) = (t_k + Q_n) - \left[ (t_k + Q_n) - \theta_0 \right] e^{-\alpha t},
\]

where \( \theta_0 \) is the initial temperature of the material; \( t_k \) is the boiling temperature of solution.

Relation between the temperature of solution and the power of heat release in its volume under the action of UHF magnetic field and the temperature of the solid phase is determined from the equation of thermal balance for a solution:

\[
Q_2 = \alpha F_0 (\theta - t) = k_f (t - t_w),
\]

where \( Q_2 = P_2/V_0 \) is the specific heat of heat release in solution under the influence of UHF EMF; \( P_2 = N \eta \) is the power of a solution heating source in UHF field; \( F \) is the interphase surface; \( t_w \) is the surface of the container with a solution (external); \( a, k \) are the coefficients of heat exchange between a solid-phase and solution and heat transfer from the solution into the environment; \( t_w \) is the ambient temperature.

We obtain from equation (54):

\[
t_w = a + b \theta,
\]

where

\[
a = \frac{P_1 + k_f t_p}{k_f + \alpha F}, \quad b = \frac{\alpha F}{k_f + \alpha F}.
\]

By substituting values for \( t_w \) into equation (56):

\[
T_w = \frac{d \theta(t)}{d \tau} = A - B \theta(t),
\]

where

\[
A = \frac{P + k_f t_p}{\alpha F + k_f}, \quad B = \frac{k_f}{\alpha F + k_f}
\]

We shall obtain a solution to equation (58) under initial conditions: \( \tau = 0, \theta = 0 \) in the following form:

\[
\theta(t) = \frac{A}{B} - \left( A/B - \theta_0 \right) e^{-\alpha \tau},
\]

where \( k_n = \frac{1}{T_n} \).

The resulting equation defines the kinetics of warming (heating) of a material in solution under the influence of UHF EMF. Temperature of the solution is determined from equation (58) by substituting \( \theta(t) \) from equation (59), that is:

\[
t_w = a + b \theta(t).
\]

By differentiating equation (59) for time:

\[
\frac{d \theta}{d \tau} = D_0 \frac{\partial^2 \theta}{\partial t^2} + \omega_e e^{-\alpha \tau},
\]

and by substituting the derived value in equation (49):

\[
\frac{\partial \theta}{\partial \tau} = D_0 \frac{\partial^2 \theta}{\partial x^2} + \omega_e e^{-\alpha \tau},
\]

where

\[
\omega_e = c_0 P_2 R K_n \left( A/B - \theta_0 \right)
\]

is the equivalent value of the maximum intensity of the component flow that is released under the action of UHF EMF.

The resulting equation in terms of its structure is similar to the equation of thermal conductivity of a plate with an internal source of heat release, whose power decreases in line with the exponential law over time. A solution to equation (62) under boundary conditions of third kind, by analogy to [19], is recorded in the following form:

\[
\frac{c(x, \tau) - c_0(\tau)}{c_0 - c_0(\tau)} = \frac{P_0}{Pd} \left[ 1 - \frac{\cos Pd \frac{x}{R}}{\cos Pd \frac{x}{R} + \sin Pd \sqrt{Pd}} \right] \exp(-Pd F_0) - \sum_{n=1}^{a} \left[ \frac{P_0}{Pd (\mu - \mu^*)} A_n \cos \mu \frac{x}{R} \exp(-\mu^* F_0) \right],
\]

where \( F_0 = \frac{D}{R^2} \) is the Fourier criterion; \( Pd = \frac{k}{a} \) is the Predvoditelev criterion.
A change in time of the mean-volumetric value for the concentration of a target component \( \tau(\tau) \) will be derived from the explicit formula:

\[
\tau(\tau) = \frac{1}{\tau} \int_0^\tau c(x, \tau)dx;
\]

\[
\tau(\tau) - c_\tau(\tau) = \frac{P_{\text{in}}}{2Pd} \left[ 2 - \frac{A_1 \exp(-Fo)^(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right] = (64)
\]

where

\[
\mu_i = \sqrt{\frac{4}{\pi^2 + B_i^2}}.
\]

Equation (64) was obtained when taking into consideration only the first term in the series in equation (63). We obtain from equation (64):

\[
\tau(\tau) = c_\tau(\tau) + \frac{c_\tau(\tau)}{P_{\text{in}}} \times \left[ 2 - \frac{A_1 \exp(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right].
\]

(65)

By differentiating equation (65) for time, we obtain:

\[
\frac{d\tau}{d\tau} = \frac{P_{\text{in}}}{2Pd} \left[ 2 - \frac{A_1 \exp(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right] \times \frac{1}{2Pd \cdot R^2}
\]

\[
\times \left[ A_1 \frac{(P_{\text{in}} - Pd + \mu_i^2)}{\mu_i^2 - Pd} \cos\mu_i \exp(-\mu_i D_R^{\tau}) + c_\tau(\tau) \right]
\]

\[
\times \left[ 1 - \frac{P_{\text{in}}}{2Pd} \frac{2 - \frac{A_1 \exp(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right].
\]

(66)

The equation of material balance for a solid phase and solution will be applied in a differential form (14). Substituting the value for derivative (66) in equation (14), we obtain:

\[
\frac{dc_p}{d\tau} = \frac{V_P V_{\tau}}{V_P V_{\tau}} \left[ A_1 \frac{(P_{\text{in}} - Pd + \mu_i^2)}{\mu_i^2 - Pd} \cos\mu_i \exp(-\mu_i D_R^{\tau}) \right] + c_\tau(\tau)
\]

\[
\times \left[ 1 - \frac{P_{\text{in}}}{2Pd} \frac{2 - \frac{A_1 \exp(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right].
\]

(67)

Solution of equation (67) under initial conditions: \( \tau = 0, c_p = c_{p0} \)

\[
c_p(\tau) = \left[ c_{p0} \left( A_1 \frac{(P_{\text{in}} - Pd + \mu_i^2)}{\mu_i^2 - Pd} \cos\mu_i \exp(-\mu_i D_R^{\tau}) \right) \right]
\]

\[
+ \frac{P_{\text{in}}}{2Pd} \exp \left[ \frac{D_i \cdot (P_{\text{in}} - Pd + \mu_i^2)}{\mu_i^2 - Pd} \cos\mu_i \exp(-\mu_i D_R^{\tau}) \right] \times \left[ 1 - \frac{P_{\text{in}}}{2Pd} \frac{2 - \frac{A_1 \exp(-\mu_i D_R^{\tau})(P_{\text{in}} - Pd + \mu_i^2)\cos\mu_i}{\mu_i^2 - Pd} - \frac{B_i \cos\sqrt{Pd} \cdot \exp(-Pd \cdot Fo)}{B_i \cos Pd - \sqrt{Pd} \sin Pd} \right].
\]

(68)

where

\[
A_1 = A_1 \frac{(P_{\text{in}} - Pd + \mu_i^2)}{\mu_i^2 - Pd}
\]

\[
B_i = \frac{B_i \cos\sqrt{Pd}}{B_i \cos Pd - \sqrt{Pd} \sin Pd}.
\]

According to equation (68), the concentration of the extracted raw material depends on the criteria by Pominov, Predvoditelev, Bio, the coefficient of diffusion, and a hydromodule. Respective parameters largely depend on the characteristics of the substance extracted, the solvent, and the conditions for the course of the process. Therefore, the verification of mathematical models for the extraction process necessitates conducting an experimental research.

5. Results of experimental research

The main elements of the experimental microwave bench (Fig. 1) was a chamber in which, by using a magnetron, a microwave field was induced, as well as a container, which actually hosted the process of extraction of the examined ob-
jects: soybean of the varieties “Vinnychanka” and the winter rapeseed variety “Champion”.

Operating principle of the experimental bench is as follows: the process of extraction occurs in the container with a product under the action of a microwave field in chamber 1. The power mode is set using regulator 3. Steam from the extractant enters inverse refrigerator 2, where it is condensed and drained back into the reaction container with the studied sample and a solvent. A micelle is taken by a syringe for the further examination of the concentration of oil.

The main factors that affect the process of extraction: a size fraction of raw materials, the existence and magnitude of power of the pulsed microwave field, a hydromodule of the extract, temperature, time of extraction, solvents, ethyl alcohol C₂H₅OH, hexane C₆H₁₄.

Classic technologies for extraction of oil employ the non-polar aliphatic hydrocarbons due to their greatest effectiveness among solvents. Given the use of the action of a microwave field as an intensifying factor, we conducted research involving a polar ethyl alcohol, the intensity of whose removal under MW irradiation grew to match the efficiency of hexane (Fig. 2). When studying the process of extraction without exposure to a microwave field, only under the action of temperature, the solvent hexane proves significantly more effective than ethyl alcohol (Fig. 3).

Verification of the constructed mathematical model (68) can be based on data from experiments, by comparing the analytical and the experimentally obtained dependences. To be determined are the resulting parameters of the solution concentration under the influence of temperature, size of fraction, solvent, and the influence of a microwave field. The experimental data were approximated by second-order polynomials; graphical interpretation of the experimental data and the calculated curves for dependences of change in the concentration in the process of extraction are shown in Fig. 2, 3.

By employing the methods of analytical modeling based on equations of energy and material balances, we have constructed a mathematical model of the heat and mass transfer processes during extraction of oil raw materials in an electromagnetic field of ultra-high frequency. A special feature and the significance of the proposed model are in the fact that it establishes patterns in the heat and material interaction between all objects engaged in the heat- and mass exchange inside an extraction unit. This means that the obtained scientific result in the form of the developed mathematical model for the processes of heat and mass exchange during extraction of oil raw materials in an electromagnetic field of ultra-high frequency, the equations of material balance for a solid phase and solution, as well as energy balance, is interesting from a theoretical point of view.

The given approximated analytical solution to the presented model makes it possible to:
- to approximately determine the concentration of an extracted material over time;
- to determine and refine the parametric complexes of the model based on the empirical dependences of the kinetics of extraction of a plant-derived raw material (oil);
- to further use a given model for the synthesis of the system of optimal control over the process.

Based on the constructed empirical mathematical model (68) of the kinetics of extraction in a microwave field, we parametrically identified the approximate solutions to the analytical mathematical model. The research results reported here could prove useful for the improvement of installations and technologies for extraction in UHF EMF.

The approximate solution to the system, presented in the framework of this study, makes it possible to predict an
increase in the concentration of an extracted substance depending on the supplied power of an electromagnetic field of ultra-high frequency in order to calculate energy efficiency of the extraction process and to synthesize a system for optimal control over the process. Therefore, the applied aspect of the application of the obtained result is the possibility of using it to identify the parameters of a mathematical model after conducting experimental research into the kinetics of extraction of oil raw materials. That creates prerequisites for the transfer of the obtained solutions when constructing a parametric series of the extraction unit designs.

7. Conclusions

1. Analytical research into a mathematical model of the process of raw materials extraction during application of the addressed delivery of UHF electromagnetic energy directly to the polar molecules will initiate a powerful, specific hydrodynamic flow. A given flow occurs in the interaction between the electromagnetic field and the polar molecules of fluid inside the capillaries. This would, in our opinion, significantly improve the intensity of mass transfer due to a sharp decrease in the internal diffusion resistance, reduce energy costs and duration of the process.

2. The reported approximated solution to the system makes it possible to predict an increase in the concentration of an extracted substance depending on the supplied power of an electromagnetic field of ultra-high frequency.

3. The experimental study that we conducted established that under the action of microwave irradiation a value for the coefficient of mass release during extraction of oil-containing raw materials grows by an order of magnitude ($\beta = 1 \cdot 10^{-6}$), compared to extraction without an effect of a MW field ($\beta = 1 \cdot 10^{-6}$). Extracting the oil under the action of a microwave field increases to 30%, while electricity consumption decreases by 93–97%.

4. The constructed mathematical model is employed for engineering calculation for determining the main structural elements of a device with the predefined productivity at the recommended operational parameters. Structural parameters include the geometrical dimensions of the unit and location of MW emitters. Operating parameters include a hydromodule and power of the emitter.

References