Solution to the related variational problems is a key for researches in the direction of control of active systems. Problem-resource approach invoke considering multi-alternativeness and conflictability of controlled by the system's active element situations.

Urgency of the researches. It is always important to find some value or values that remain invariant or constant in the given problem formulation. Conflictability and multi-alternativeness need those values to be discovered and analyzed.

Analysis of the latest researches and publications. We feel the necessity to continue the researches initiated in [1] and are going to add some new theoretical results concerning invariants and conservative values.

Subjective analysis [1-4] and calculus of variations [5] is the basis for the researches. The possible areas of the researches results applications are in the fields of aircraft powerplant [6] and ship propulsion [7, 8] operation, economics [9, 10], and environmental management science [10].

The task setting. The purpose of this paper is to uncover invariants, important dependences, regularities, and identities for a few special cases of the variational problems of control applicable to an active aviation system.

Theoretical considerations of the particulars allow us revealing the important invariants that helps understand the laws of thinking for making the needed controlling decisions.

2.1. The problem formulation. The postulated functional, in the context of the prototype one [1-3]:

\[ \Phi_\pi = \int \left( -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left( \sum_{i=1}^{N} \pi_i(t) - 1 \right) \right) \, dt, \] (1)

where \( t \) – time; \( \sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) \) – entropy of subjective preferences of \( \pi_i(t) \); \( N \) – number of the achievable alternatives; \( \beta, \gamma \) – structural parameters (can be considered in different situations as Lagrange coefficients or weight coefficients, reflecting some peculiarities of the responsible for making decisions individual’s psych through the endogenous parameters of it); \( F_i \) – efficiency function related to the ith reachable alternative; \( \sum_{i=1}^{N} \pi_i(t) - 1 \) – normalizing condition.

In the simplest problem setting we consider \( x(t) \) and \( \dot{x}(t) \) as the efficiency functions of the two achievable alternatives with the corresponding preferences of \( \pi_i(t) \), \( \pi_x(t) \). With respect to particular combinations of \( x(t) \), \( \dot{x}(t) \), \( x(t)\dot{x}(t) \), and \( \frac{\dot{x}(t)}{x(t)} \), we will get eleven variants of the functional (1) with the subjective efficiency functions that include the elementary efficiency functions in the view of the linear combinations in the cases of:
1) \( x(t) \) and \( \dot{x}(t) \);

2) \( x(t) \) and \( x(t) \dot{x}(t) \);

3) \( x(t) \) and \( \frac{\dot{x}(t)}{x(t)} \); 4) \( \dot{x}(t) \) and \( x(t) \dot{x}(t) \);

5) \( \dot{x}(t) \) and \( \frac{\dot{x}(t)}{x(t)} \);

6) \( x(t) \dot{x}(t) \) and \( \frac{\dot{x}(t)}{x(t)} \);

7) \( x(t), \dot{x}(t), \) and \( x(t) \dot{x}(t) \);

8) \( x(t), \dot{x}(t), \) and \( \frac{\dot{x}(t)}{x(t)} \);

9) \( x(t), x(t) \dot{x}(t), \) and \( \frac{\dot{x}(t)}{x(t)} \);

10) \( x(t), x(t) \dot{x}(t), \) and \( \frac{\dot{x}(t)}{x(t)} \);

11) \( x(t), \dot{x}(t), x(t) \dot{x}(t), \) and \( \frac{\dot{x}(t)}{x(t)} \).

This results in the general view functional:

\[
\Phi_{\text{g.v.}} = \int \left[ \sum_{i=1}^{n} \pi_i(t) \ln \pi_i(t) + \beta \left[ \pi_i \pi_i(t) x(t) + \alpha_i \pi_i(t) \dot{x}(t) + \alpha_i \pi_i(t) x(t) \dot{x}(t) + \right. \right.
\]

\[
\left. + \alpha_i \pi_i(t) \frac{\dot{x}(t)}{x(t)} \right] + \gamma \left[ \sum_{i=1}^{n} \pi_i(t) - 1 \right] \right] dt , \tag{2}
\]

where \( \alpha_i \) — coefficients that consider the differences in the measurement units.

It is convenient to consider \( \alpha_i = 1 \).

Functionals derived from (2) differ in:

1. Entropies, though, even when formally their expressions are the same, the preferences functions contain the different efficiency functions, thus, the entropies are not identical;


\[
\beta \left[ \pi_i(t) x(t) + \alpha_i \pi_i(t) \dot{x}(t) + \alpha_i \pi_i(t) x(t) \dot{x}(t) + \alpha_i \pi_i(t) \frac{\dot{x}(t)}{x(t)} \right] ; \tag{3}
\]

and all that because of the diversity in the considered set of the reachable alternatives (the alternatives number and/or their qualities are different).

The last functional (2) and cognitive function (3) are the general ones and each of the particular forms of them derives from those with the corresponding preferences \( \pi_i(t) \) and coefficients \( \alpha_i \).

Solving the functionals obtained from (2) with the application of the necessary conditions for them to have the extremums in the view of the system of the Euler-Lagrange equations, we find:

1. The corresponding expressions for the canonical distributions of the preferences [1] in their general compact view:

\[
\pi_i = \frac{e^{\alpha_i \beta_i}}{\sum_{i=1}^{n} e^{\alpha_i \beta_i}} ; \tag{4}
\]

2. The Euler-Lagrange equation for the sought function in the general case of (2) in the transformed view [1, P. 58, (4)]:

\[
\pi_i = \alpha_i \pi_i + \alpha_i x \pi_i + \frac{\alpha_{i+1} \pi_i}{x} ; \tag{5}
\]

3. The generalized differential equation of the second order from (5) with respect to (4) [1, P. 58, (6)]:

\[
\ddot{x} = \pi_i + A + B + C \frac{D + E + F}{x} . \tag{6}
\]

where

\[
A = \alpha_i^2 \left[ \beta \pi_i \left( \pi_i + \alpha_i x \pi_i - \frac{\alpha_i}{x} \pi_i \right) \right] ,
\]

\[
B = - \alpha_i x \left[ \beta \pi_i \left( \alpha_i x (\pi_i + \pi_i - \pi) - \pi_i + \alpha_i \frac{x}{x} \pi_i \right) \right] ,
\]

\[
C = - \alpha_i \left[ \beta \pi_i \left( \alpha_i x (\pi_i + \pi_i - \pi) - \pi_i - \alpha_i \frac{x}{x} \pi_i \right) \right] ,
\]

\[
D = \alpha_i \left[ \beta \pi_i \left( \alpha_i (\pi_i + \pi_i - \pi) - \alpha_i x \pi_i - \frac{\alpha_i}{x} \pi_i \right) \right] ,
\]

\[
E = \alpha_i x \left[ \beta \pi_i \left( \alpha_i x (\pi_i + \pi_i - \pi) - \alpha_i \pi_i - \alpha_i x \pi_i \right) \right] ,
\]

\[
F = \alpha_i \left[ \beta \pi_i \left( \alpha_i (\pi_i + \pi_i - \pi) - \alpha_i \pi_i - \alpha_i x \pi_i \right) \right] . \tag{7}
\]

It is noticeable that in all cases of the functionals derived from (2) their under-integral functions do not depend explicitly upon the independent variable \( t \), that is upon the time. In accordance with [5], for functionals which do not depend upon the time explicitly, the function of Hamilton (Hamiltonian) represents by itself the first integral of the corresponding canonical system of the equations by Euler-Lagrange (hence, its equivalent system of the differential equations of the first order, and thus, the system of the initial differential equations of the second order, formed from the considered system of the Euler-Lagrange equations) [5].

In the given problem formulation, the Hamiltonians

\[
H = -R' + px . \tag{8}
\]

where \( R' \) — the under-integral function of the corresponding integral obtained from (2); \( p \) — canonical variable.
\[ p_i = \beta f_{\pi_i(t)} \quad p_2 = \beta f_{\pi_2(t)x(t)} \quad p_3 = \beta f_{\alpha(t)x(t)} \quad \text{for} \quad \frac{\partial R}{\partial x} \]  \hspace{1cm} (9)

Here, on conditions of (9)

\[ p_i = \beta f_{\alpha(t)x(t)} \quad p_2 = \beta f_{\alpha(t)x(t)} \quad p_3 = \beta f_{\alpha(t)x(t)} \quad \text{for} \quad \frac{\partial R}{\partial x} \]  \hspace{1cm} (10)

Then, substituting (10) for (8)

\[ H_{am_1} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_2} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_3} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_4} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_5} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_6} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_7} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_8} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_9} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \],

\[ H_{am_{10}} = \sum_{i=1}^{N_2} \pi_i(t)\ln\pi_i(t) - \beta \sum_{i=1}^{N_2} \pi_i(t)x(t) - \gamma \sum_{i=1}^{N_2} \pi_i(t) - 1 \].

\[ \text{Fig. 1. Preferences functions} \]

\[ \text{Fig. 2. Hamiltonian} \]

2.2. The problem solution. The obtained expressions for the desired constant values (11) in some respect look like the same, although, those regularities are just the formal identical symbolizations (and nothing more) of absolutely different, not identical, however, conserved values.

As it is, the expressions for \( H_{am_1} - H_{am_{10}} \) are indicated in their right-hand parts the same, but the corresponding preferences functions connected to the related alternatives are different, and they must be taken in the proper view from (4) for the considered functional of (2) which includes the special cognitive function derived from (3).

On that the same reasons \( H_{am_1} - H_{am_{10}} \) are different as well.

2.3. Practical application of the problem solution. For the first variant of (2) the solution yields the results for the preferences functions:

\[\pi_1 = e^{a\beta x} \quad \pi_2 = e^{a\beta x} \quad \pi_3 = e^{a\beta x}.\]  \hspace{1cm} (12)

The expressions of (12) are the special cases of (4), and they can be found from there as well.

On condition of the Euler-Lagrange equation for the sought function \( \pi_1 = \alpha \pi_2 \).

And the ordinary differential equation of the second order (14), as well as in its turn, is the particular case of the generalized expressions of (6), (7); it derives from them for the related values.

The practice (8)-(10) yields the Hamiltonian \( H_{am_{10}} \) of (11).

2.4. The researches results. Mathematical modeling for the accepted data: \( t_0 = 0; t_1 = 120; \alpha = 95; \beta = 0.1 \) show the calculation experiment results illustrated in fig. 1, 2.
3. Conclusions

Herein we have found the sought after value which remains constant during the controlled by the active element process.

Prospects of further researches.

One of the further researches directions is considering functionals with the under-integral functions which contain the independent variables in the explicit view.

References