1. Introduction

Under current regulations, there are 13 indicators of electric energy quality [1]. Deterioration in these indicators relative to standard values could lead to significant negative consequences in the power supply systems at industrial enterprises, as well as to the energy system in general. Losses of electricity grow, service life of equipment and its reliability are reduced, and dynamic resistance of elements within energy systems deteriorate.

For power generating enterprises, the most important indicator of electric energy quality is a deviation in the electric current frequency. This indicator is ensured at hydroelectric power plants by the effectiveness of work of the system for automated control over frequency and power of hydrogenerator units (a hydro-turbine and a generator). Specifically, indicators for their static and dynamic accuracy, which should provide for the minimum deviation in the rotation frequency of hydrogenerators and their power from the predefined operational values.

The permissible deviations in the frequency for systems equipped with hydraulic or electro-hydraulic controllers, are, respectively, 0.3 and 0.1 % [1]. At the same time, the Western European power systems UCTE ATSOI, BALTSO, ETSO, NORDEL and UKTSOA ensure, through primary and secondary regulation, a better quality of control over the frequency of electrical current. These energy systems are part of ENTSO-E (Integrated Network of Transmission System Operators for Electricity), which is a transnational organization that manages and ensures the harmonized functioning of energy systems in the European Union.
European network of transmission system operators) that provide for a uniform standard of quality. According to the norms of UCTE, frequency control is rated by:

- the magnitude and duration of time to employ reserves;
- a static factor;
- a dead zone in the automated control systems of hydro-generator units at HPP.

Frequency tolerance constitutes no more than 0.06 % [1, 2].

At the same time, the normative-methodical documentation almost lacks universal normative methods for structural-al-parametric synthesis of high-precision automated control systems (ACS) for hydrogenerator units, which could guarantee high indicators of precision in control over frequency rotation and power.

Improvement of efficiency in the automated control over hydrogenerator units represents an important and relevant scientific-technical task. Solving it will make it possible to significantly improve the technical-economic characteristics of hydrogenerator units, as well as electric energy quality, generated by hydraulic power plants.

In this case, the focus of research is the development of systematic and standardized research methods, as well as technical solutions, aimed to substantially improve the precision of control over frequency and power of hydrogenerator units. That, in turn, will lead to the improvement of quality indicators for electric energy generated at hydroelectric plants.

2. Literature review and problem statement

Acting Ukrainian standards and normative documents (GKD 34.20.507-2003, RD 153-34.0-35.519-98, etc.) on the control systems for hydrogenerator units at hydro plants mostly address general technical issues related to maintenance and repair. At the same time, there remain the insufficiently defined scientific and methodical bases for construction of precise control systems that would ensure high indicators of electric energy quality. Existing normative documents miss complete effective solutions that could greatly improve the accuracy and performance speed of electro-hydraulic monitoring systems for automated control over frequency and power of hydrogenerator units. There are also no uniform requirements to methods and techniques for ensuring the required quality parameters for the technological process, reliability, and safety.

Controllers of different modifications are made and successfully operated at the largest hydraulic power plants in post-Soviet countries, as well as at HPP in Canada, Brazil, Yugoslavia, Argentina, Greece, Iceland, India, etc.

Issues of structural features and mechanical components of electrical hydraulic controllers have been investigated in detail. Even though the optimization of processes related to regulating hydro turbines were examined in the 1980s [3], the tasks on enhancing the accuracy of control remain relevant [4].

Special attention is paid to the application and improvement of PID controllers. Parameters of controllers are optimized in order to improve the quality of control [5]. Research is being conducted into various modifications of controllers with subsequent comparative analysis [6]. The objective functions of controllers are adjusted in terms of their use in power units at electric power plants [7]. The principle of robust control is successfully employed, which makes it possible to improve the quality of control [8]. The accuracy of PID controllers is enhanced, based on improve-

ments to the tool base [9]. Modern algorithms are applied in the synthesis of controllers [10]. Self-tuning algorithms are successfully exploited [11]. The latest technology and materials are used [12].

Such a scale of research stems from the fact that about 90–95 % of controllers, which are currently in operation, employ PID algorithms. However, the accuracy of such controllers is not high enough. This is due to the presence of small increments in large magnitudes, and, as a consequence, the emergence of static and dynamic errors in control [13].

Control accuracy also depends on the mathematical models applied for the synthesis of controllers. In order to synthesize controllers over rotation frequency of a hydraulic turbine, scientists constructed mathematical models for basic elements of the control system [14]. The simplified mathematical models have been developed that reduce computational costs and effectively synthesize control algorithms based on modern computer systems [15]. However, existing mathematical models do not take into consideration all the factors affecting accuracy of regulation, which sets the task to improve mathematical models, specifically, accounting for factors that significantly affect accuracy. Such factors include, for example, forces of dry and liquid friction, leaks and fluid overflows in the servomotor, as well as the non-linearity in characteristics of the servo valve [13].

A promising advancement towards improving the accuracy of control over frequency and power is the synthesis of precision (ultra-high precision) automated control systems of hydrogenerator units based on solving the inverse problems of dynamics [13]. In this case, the structural-parametric synthesis and optimization of controllers should be based on the refined mathematical models, modern methods of systems analysis, as well as the theory of automatic control [13].

3. The aim and objectives of the study

The aim of this work is to synthesize automated control systems of rotation frequency in hydro units based on solving inverse problems of dynamics, which would enhance the quality of electric energy by reducing the static and dynamic errors in control.

To accomplish the aim, the following tasks have been set:
- to improve mathematical models of electric hydraulic mechanisms for the automated control systems of hydraulic turbines, which are required for the synthesis of precision controllers;
- to carry out experimental study into automated control systems for hydrogenerator units and to identify the mathematical models developed for controllers;
- to implement a structural-parametric synthesis and the optimization of high-precision controllers over frequency and power of hydraulic turbines based on solving the inverse problems of dynamics.

4. Improvement of mathematical models for the system of automated control over hydrogenerator units

The basic object of research is the electro-hydraulic system of automated control over a hydrogenerator unit, which is designed to maintain the predefined rotation frequency and power [13, 14].
When a controlled parameter (of frequency or power) deviates from the value that is set by a central controlling device, a microprocessor-based controller generates a controlling signal to the corresponding required position of servo motors, which control the position of the guiding unit. Thus, there forms a controlling signal that arrives at the input of an electrohydraulic converter that changes position of a valve and the pressure difference in controlling nozzles, thereby setting a spool into motion. In this case, a rod of the shut-off valve stops in the position corresponding to the required value for pressure differential in the cavities of the servomotor. The rod of the servomotor shifts the regulating ring of blades at a guiding unit and changes the original settings of a hydrogenerator unit. That in turn changes its rotation frequency and power, and, as a consequence, the magnitude and frequency of current, which is supplied into electric network. The presence of non-linearity in the characteristics of elements in a control system, as well as the presence of unwanted leaks and fluid overflows, leads to the emergence of additional errors in control. It is impossible to physically eliminate these phenomena, which sets the task on accounting for these factors in a mathematical model in order to subsequently compensate for their negative impact.

In order to solve the task on constructing precision frequency and power controllers for hydrogenerator units, the refined mathematical models of hydraulic controlling elements in the automated control systems of frequency and power (F&P ACS) can be represented in the following form:

– equation of motion of water flow along a penstock [13]:

\[
\begin{align*}
\frac{\partial \eta}{\partial t} + \frac{\partial P}{\partial t} + k_\eta &= 0; \\
C_\ell \frac{\partial P}{\partial \eta} &= -g \frac{\partial \eta}{\partial t};
\end{align*}
\]

(1)

– equation of dynamics of the rotary motion of a hydraulic turbine’s rotor and a hydraulic generator [13]:

\[
\begin{align*}
J v \ddot{\omega}_v + [M_v] \text{sign} (\omega_v) + k_v \omega_v &= + C_v (\varphi_v - \omega_v) = 0.5 \frac{v}{\omega_v}; \\
J_v \ddot{\omega}_v + [M_v] \text{sign} (\omega_v) + k_v \omega_v + C_v (\varphi_v - \omega_v) &= N_v \omega_v. 
\end{align*}
\]

(2)

Structural mathematical model of electromechanical part in the electrohydraulic converter (EHC) is represented in the form of a differential equation of the second order:

\[
T_{\text{ehc}} \ddot{X}_v + 2 \xi_{\text{ehc}} T_{\text{ehc}} \dot{X}_v + X_v = K_{\text{ehc}} i_v,
\]

(3)

where \(i_v\) is the controlling current at the output from an input summing amplifier; \(X_v\) is the coordinate of EHC valve position; \(K_{\text{ehc}}\), \(T_{\text{ehc}}\), \(\xi_{\text{ehc}}\) is the static transfer coefficient, the time constant, and the coefficient of relative damping of EHC electromechanical part.

The process of change in the pressure in the control line of a cut-off valve is described by the balance equation of a working fluid consumption through the throttle slot of valve of EHC, controlling cavity of the cut-off valve, and a drain line, that is [13]:

\[
Q_{\text{in}} = Q_r + Q_v,
\]

(5)

where

\[
Q_r = \mu_r S_r (X_r) \sqrt{\frac{2(P-P_r)}{\rho}}.
\]

(6)

\[
Q_v = \frac{V}{X_r} P_r + S_r V_v,
\]

(7)

and

\[
Q_{\text{out}} = \mu_s S_o \sqrt{\frac{2(P-P_o)}{\rho}}.
\]

(8)

– the flow rate of a working fluid, respectively, through the throttle slot of valve of EHC, through controlling cavity of the cut-off valve, and through a drain line; \(S_r(X_r), S_r, S_o\) are the areas of the throttle slot of valve of EHC, controlling cavity of the cut-off valve, and a drain line; \(P_r, P_o\) are the pressures in the controlling cavity, in the power and drain lines; \(V_v\) is the reduced (considering the volume of underwater channels) volume of the controlling cavity; \(\rho\) and \(\chi\) is the density and adiabatic module of the volumetric elasticity of a working fluid.

Mathematical notation of dynamic processes in the cut-off valve is based on the equation of motion of the valve’s plunger and the equation of balance of a working fluid flow rate that moves through slots and cavities of the valve.

The equation of motion of the cut-off valve’s plunger is represented in the following form:

\[
m_v \ddot{X}_v + k_v \dot{X}_v + F_r \text{sign} \{\dot{X}_v\} =
\]

\[
= F_l^v + F_u^v + G_v \cos \varphi_v,
\]

(9)

where \(X_v\) is the coordinate of position of the cut-off valve; \(m_v\) is the reduced (taking into consideration the mass of sensors) weight of the cutoff valve’s plunger; \(k_v\) and \(F_r\) is the coefficient of fluid friction and the force of dry friction between a plunger and the valve’s sleeve;

\[
F_l^v = P_l S_d + P_t (S_v - S_d) + P_d S_d
\]

(10)

is the resultant force of hydrostatic pressure on the valve’s plunger; \(P_l\) and \(P_d\) are the pressures in the valve’s respective cavities; \(S_d\) and \(S_v\) are the effective areas of working cavities in the valve; \(F_r\) is the projection onto an axis of the plunger displacement (the \(x\) axis) of the hydrodynamic force that acts on the plunger; \(G_v\) is the weight of the plunger; \(\varphi_v\) is the angular deviation of the plunger from the vertical position.

Hydrodynamic forces that act on the cutoff valve’s plunger are represented as the sum of non-stationary and stationary components. In this case, the generalized ratios for the vectors of hydrodynamic forces that act in the respective cavities in the cut-off valve are represented in the following form [13]:

\[
F_l = \begin{cases}
0, & \text{if } X_v > 0; \\
\rho_l Q_l + \rho_l Q_l^v S_v, & \text{if } X_v < 0;
\end{cases}
\]

(11)

\[
F_u = \rho_l Q_l + \rho_l Q_l^v S_v.
\]

(12)
In formulae (11) to (16):

\[ l_{bx}, l_{by}, l_{cx}, l_{dy} \] — lengths of the respective cavities of the cut-off valve;

\[ Q_{bx}, Q_{by}, Q_{cx}, Q_{dy} \] — flow rates of a working fluid through respective cavities;

\[ S_P = S_C(X_c) \] — areas of flow-through sections of working windows in the cut-off valve, which are the functions of its position \( X_c \) (coordinate);

\[ \varphi_{bx}, \varphi_{by}, \varphi_{cx}, \varphi_{dy} \] — inclination angles of velocity vectors of a working fluid that flows through respective working windows, to the transverse axis of the plunger.

A mathematical model of the servomotor is represented by the equations of balance of a working fluid flow rate through the cavities of the servomotor, the equations of fluid motion along underwater channels, and the equation of motion of the loaded piston. The equation of balance of fluid flow rates through the piston and rod cavities of the servomotor is represented in the form [13]:

\[ Q_{sm} = \frac{dV}{dt} + Q_{sm} + Q_{py} + Q_{mf}, \]  

(17)

where

\[ \frac{dV}{dt} = Q_{sm} + Q_{py} + Q_{mf} - Q_{py}, \]  

(18)

\[ \frac{dV}{dt} = S_{sm}(X_{sm} + \dot{X}_{sm}). \]  

(19)

\[ \frac{dV}{dt} = S_{sm}(X_{sm} + \dot{X}_{sm}) \]  

(20)

– a working fluid flow rates, due to the movement of the piston and body of the servo-motor (the body is displaced due to the elasticity of its supports);

\[ Q_{sm} = \frac{V_{sm}}{\chi} \frac{dP_{sm}}{dt} = \frac{V_{sm}^o + V_{sm}^w + S_{sm}X_{sm}}{\chi} \frac{dP_{sm}}{dt} \]  

(21)

and

\[ Q_{sm} = \frac{V_{sm}}{\chi} \frac{dP_{sm}}{dt} = V_{sm}^o + V_{sm}^w + S_{sm}X_{sm} \frac{dP_{sm}}{dt} \]  

(22)

– components of flow rates arising from pressing a working fluid in the piston and rod cavities of the servomotor;

\[ Q_{py} = k_{py}(P_{sm} - P_{pa}) \]  

(23)

and

\[ Q_{py} = k_{py}(P_{sm} - P_{pa}) \]  

(24)

– flow rates of fluid overflows from the piston and rod cavities of the servomotor;

\[ Q_{mf} = k_{mf}(P_{sm} - P_{ma}) \]  

(25)

– flow rates of fluid overflows between the cavities in the servomotor;

\[ Q_{sm} = Q_{sm}^o \]  

(26)

– the total shift of the servomotor’s body due to the elasticity of its supports;

\[ x_{sm}^o \] and \( x_{sm}^w \) are the components of the total shift of the body caused by the fluid pressure in the piston and rod cavities; \( C_{sm}^o \) and \( C_{sm}^w \) are the rigidities of supports, installed from the side of the piston and rod cavities in the servomotor; \( V_{sm} \) and \( V_{sa} \) are the reduced volumes of the piston and rod cavities of the servomotor; \( V_{sm}^o \) and \( V_{sm}^w \) are the volumes of the piston and rod cavities in the starting (middle) position of the servo motor’s piston; \( V_{sm}^o \) and \( V_{sm}^w \) are the volumes of channels that supply fluid to the piston and rod cavities; \( P_{sm}^o \) and \( P_{sm}^w \) are the pressures in the piston and rod cavities of the servomotor.

The result of mathematical modeling is the obtained closed system of equations, which describes the dynamics of an electrohydraulic controlling element of F&P ACS. The model takes into consideration leaks and fluid overflows in the servomotor, as well as the non-linearity of characteristics of the servo motor. That makes it possible to improve accuracy of the mathematical model and the control system in general. The model forms the basis for theoretical research into the influence of various factors and parameters on frequency and power control accuracy of hydrogenerator units, as well as on the integrated indicators of ACS quality.

\[ 5. \text{Identification of mathematical models} \]

We have identified the constructed mathematical models based on experimental characteristics of hydrogenerator units at actual hydro power plants. Results of the identification for hydraulic turbines in a hydrogenerator unit at HPP Bajtun (Panama) are shown in Fig. 1, 2.

Our study has established that the constructed mathematical model of the system for automated control of fre-
frequency and power is adequate; it takes into consideration the basic factors that affect the accuracy of frequency and power control. Deviations in the values, obtained when modeling, are within a 95%-confidence interval (Fig. 1, 2). The model could be used in order to analyze static and dynamic errors in control, and to work out scientific methods to reduce errors in order to improve quality indicators of electric energy.

Fig. 1. Dependence of relative rotation frequency of turbine on time

Fig. 2. Dependence of relative head of turbine on time

The basic object of research was the vertical, radial-axial hydraulic turbine RO B-195, under operation at HPP Bajtun.

We have conducted experimental study into operation of F&P ACS when resetting the load to idle and setting the load to the rated capacity under the following starting (prior to resetting) operating conditions (Table 1).

Table 1

<table>
<thead>
<tr>
<th>Capacity, Ак, kW</th>
<th>Head, g, m</th>
<th>Reduced rotation frequency, n', min⁻¹</th>
<th>Flow rate, Q', м³/s</th>
<th>Relative opening of the guiding unit, μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>48,600</td>
<td>135.7</td>
<td>66.96</td>
<td>0.88</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The torque and flow rate of water through the turbine were determined based on the model characteristics of the impeller. Introduction of model characteristics is carried out using a tabular technique – by matrices of the reduced flow rate $Q'$ and the efficiency coefficient $\eta$, in the function of two variables: relative opening of the guiding unit $\mu$ and the reduced rotation frequency:

$$n' = \frac{n}{n_{nom}}.$$ 

The equation of change in the water head due to a hydraulic impact is constructed considering the losses of head, wave processes in the water guide, and elasticity of the medium. Law of motion of the guiding unit is defined taking into consideration:

- characteristics of the main turbine controller valve;
- a damping device characteristics, which is enabled near a closed position of the servo motor of the guiding unit and pressure losses in the oil pipes of control system;
- a change in the oil pressure in an oil-pressure unit in the course of control process at air expansion in the hydraulic accumulator;
- modelled force characteristics of the guiding unit's vanes ($c_p$ and $c_w$), assigned by the opening function of the guiding unit.

Calculation results are given below in the form of Table 2.

Table 2

<table>
<thead>
<tr>
<th>t, s</th>
<th>α</th>
<th>μ</th>
<th>q</th>
<th>h</th>
<th>$H_{s.c.}$, m</th>
<th>$H_{s.c.}$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.819</td>
<td>1.02</td>
<td>1</td>
<td>143</td>
<td>5.91</td>
</tr>
<tr>
<td>0.5</td>
<td>1.11</td>
<td>0.804</td>
<td>1</td>
<td>1.04</td>
<td>148</td>
<td>5.87</td>
</tr>
<tr>
<td>1</td>
<td>1.12</td>
<td>0.73</td>
<td>0.959</td>
<td>1.17</td>
<td>165</td>
<td>5.71</td>
</tr>
<tr>
<td>1.5</td>
<td>1.31</td>
<td>0.655</td>
<td>0.868</td>
<td>1.3</td>
<td>183</td>
<td>4.72</td>
</tr>
<tr>
<td>2</td>
<td>1.39</td>
<td>0.581</td>
<td>0.757</td>
<td>1.33</td>
<td>188</td>
<td>2.99</td>
</tr>
<tr>
<td>2.5</td>
<td>1.45</td>
<td>0.509</td>
<td>0.618</td>
<td>1.36</td>
<td>193</td>
<td>1.16</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.441</td>
<td>0.496</td>
<td>1.33</td>
<td>190</td>
<td>-0.427</td>
</tr>
<tr>
<td>3.5</td>
<td>1.52</td>
<td>0.375</td>
<td>0.382</td>
<td>1.24</td>
<td>179</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>1.53</td>
<td>0.311</td>
<td>0.296</td>
<td>1.21</td>
<td>175</td>
<td>-2.83</td>
</tr>
<tr>
<td>4.5</td>
<td>1.53</td>
<td>0.249</td>
<td>0.221</td>
<td>1.19</td>
<td>173</td>
<td>-3.3</td>
</tr>
<tr>
<td>5</td>
<td>1.53</td>
<td>0.189</td>
<td>0.157</td>
<td>1.16</td>
<td>170</td>
<td>-3.73</td>
</tr>
<tr>
<td>5.5</td>
<td>1.51</td>
<td>0.133</td>
<td>0.106</td>
<td>1.11</td>
<td>162</td>
<td>-4.2</td>
</tr>
<tr>
<td>6</td>
<td>1.47</td>
<td>0.0973</td>
<td>0.0768</td>
<td>1.07</td>
<td>157</td>
<td>-4.48</td>
</tr>
<tr>
<td>6.5</td>
<td>1.43</td>
<td>0.0803</td>
<td>0.0626</td>
<td>1.02</td>
<td>151</td>
<td>-4.78</td>
</tr>
<tr>
<td>7</td>
<td>1.39</td>
<td>0.0697</td>
<td>0.0563</td>
<td>1.02</td>
<td>151</td>
<td>-4.81</td>
</tr>
<tr>
<td>7.5</td>
<td>1.34</td>
<td>0.0592</td>
<td>0.0506</td>
<td>1.04</td>
<td>154</td>
<td>-4.7</td>
</tr>
<tr>
<td>8</td>
<td>1.3</td>
<td>0.0488</td>
<td>0.0427</td>
<td>1.04</td>
<td>153</td>
<td>-4.73</td>
</tr>
<tr>
<td>8.5</td>
<td>1.26</td>
<td>0.0385</td>
<td>0.0343</td>
<td>1.03</td>
<td>152</td>
<td>-4.76</td>
</tr>
<tr>
<td>9</td>
<td>1.22</td>
<td>0.0282</td>
<td>0.026</td>
<td>1.04</td>
<td>153</td>
<td>-4.72</td>
</tr>
<tr>
<td>9.5</td>
<td>1.18</td>
<td>0.0181</td>
<td>0.0169</td>
<td>1.04</td>
<td>153</td>
<td>-4.72</td>
</tr>
<tr>
<td>10</td>
<td>1.14</td>
<td>0.008</td>
<td>0.00753</td>
<td>1.03</td>
<td>153</td>
<td>-4.77</td>
</tr>
</tbody>
</table>

In Table 2: $t$ – current time coordinate; $\alpha$ – relative frequency of rotation; $\mu$ – relative opening of the servomotor of the guiding unit; $q$ – relative flow rate of the turbine; $h$ – relative pressure; $H_{s.c.}$ – pressure in a spiral chamber; $H_{s.c.}$ – pressure under the impeller.

Based on the results of experimental research, we have derived characteristics for the transient processes in a hydrogenerator unit, shown in Fig. 3.

Calculations based on experimental data were performed under the following conditions:

Running at idle and at isolated load, when the static circuit is turned off, a PID-controller transfer function takes the form:

$$W_{PID}(s) = \frac{\Delta e}{\Delta C} = \frac{K_p (T_s s + 1)}{T_s s (T_s s + 1)}.$$ 

(27)
A transfer function of the electro-hydraulic monitoring system:

\[
W_{20}(s) = \frac{\Delta \hat{y}}{\Delta x} = \frac{K_1}{s(T_s+1)} \left(1 + \frac{K_s}{(T_s+1)s}\right),
\]  

where \(K_1 = 1/T_{sm}\), \(T_{sm}\) is the servo motor time constant; \(T_s\) is the time constant of the proportional hydraulic distributor.

Charts in Fig. 4 demonstrate that the characteristics of ACS with the proposed controller far exceed the characteristics of ACS with a PID controller. A 50 % decrease is achieved in the time required to enter a permanent regime, the level of overshoot reduces (1.03 for the proposed controller versus 1.17 for the PID-controller). Control quality indicators, when using the proposed controller, are at the level of indicators for the world leader in the field of control over hydrogenerator units (company Emerson).

6. Structural-parametric synthesis of precision frequency and power controller for a hydrogenerator unit

The synthesis of a precision controller employs the principle of multiplication. A general controller circuit consists of three autonomous contours: an electrohydraulic amplifier circuit, a servo motor circuit, and a hydrogenerator unit circuit. That makes it possible to simplify setting the controller.

On order to determine coefficients for the controller, an original system of dynamics equations is represented in the vector-matrix form:

\[
\dot{X} = AX + BU,
\]

where \(\dot{X}\) is the vector of parameters for the state of an object; \(A\) is the matrix of the controlled object’s parameters with elements; \(B\) is the control vector; \(U\) is the vector of controlling signals.

A vector-matrix representation of the mathematical models of autonomous control circuits takes the form:

- a hydrogenerator unit with a pressure conduit:

\[
\dot{X} = \begin{bmatrix} \dot{\omega}_{sm} \\ \dot{Q} \end{bmatrix},
A = \begin{bmatrix} \frac{1}{T_{hg}} & 0 \\ 0 & \frac{1}{T'} \end{bmatrix},
\]

\[
U = (NZ)_{2x2},
B = \begin{bmatrix} \frac{K_{sm}}{T_{hg}} & 0 \\ 0 & \frac{K_{sm}}{T'} \end{bmatrix},
\]

- servo motor:

\[
\dot{X} = \begin{bmatrix} \dot{X}_{sm} \\ \dot{V}_{sm} \end{bmatrix},
A = \begin{bmatrix} 0 & \frac{1}{T_{sm}} & 0 \\ -2\frac{Z_{sm}}{T_{sm}} & \frac{K_{sm}}{T_{sm}} & \frac{1}{T''} \\ 0 & -\frac{K_{sm}}{T_{sm}} & \frac{1}{T''} \end{bmatrix},
\]

\[
U = X_{sm},
B = \begin{bmatrix} 0 \\ 0 \\ K_{sm} ' \end{bmatrix}.
\]
The matrix ratio of the general circuit takes the form:

$$
\tilde{X} = \begin{pmatrix}
X \\
V \\
i
\end{pmatrix},
A = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & \frac{1}{T_{ob}} \\
0 & 0 & -\frac{1}{T_{sc}}
\end{pmatrix},
$$

$$
\tilde{U} = X_{\text{ex}},
B = \begin{pmatrix}
0 \\
K_{m} \\
T_{sc}
\end{pmatrix},
$$

$$
\dot{X} = V; \quad \dot{V} = \frac{1}{T_{m}} X_{\text{ex}}, 2 \frac{2 \nu \nu}{T_{sc}} V_{c}, \frac{K_{c}^{
u}}{T_{sc}} i;
$$

where $\omega_{ng}$ is the rotation frequency of a hydrogenerator unit; $\nu$ is the flow rate; $X_{\text{ex}}$ is the displacement of a servo motor; $V_{c}$ is the speed of a servo motor; $\Delta P$ is the pressure drop in the cavities of a servo motor; $X_{c}$ is the cutoff valve displacement; $V_{c}$ is the speed of the cutoff valve; $i$ is the controlling signal.

The matrix ratio of the general circuit takes the form:

$$
\tilde{X} = \begin{pmatrix}
\omega_{ng} X_{\text{ex}} V_{c}, \Delta P, X_{c}, i
\end{pmatrix}^T,
$$

where $A$ is the assignment program of operation, formed in the controller, passes through an electronic controller, where the control law is formed based on the solution to the inverse problem on the dynamics of control object. The signal is sent to the additional adjustment unit that will ensure the implementation of an alternating gain coefficient of control circuit. Specifically: a high (two-and three-fold from the rated) gain coefficient in the range of small shifts of the object from the nominal position. That compensates for errors caused by static non-linear characteristics. The controlling signal, thus formed, is sent to the electric input of the servo valve, which enables the programmed motion of a hydraulic motor, combined with the control object.

This movement is registered by a feedback sensor, the signal which is sent to the unit that corrects a feedback sensor, and then to the electronic controller (the main negative feedback).

A solution to the inverse problem of dynamics for an equivalent control object is the law of forming a controlling signal in the form of a transfer function [16]:

$$
W_{\nu}(S) = \frac{U_{\nu}(S)}{r(S)} = \frac{r(S)}{S^2 C(S)} = \frac{\sum_{i=1}^{n} r(S)}{\sum_{i=1}^{n} S^2 C(S)}.
$$

where $r$ and $C$ are the controller coefficients, respectively, for setting the influence and for an error.

A principal diagram of the controller that employs the principle of multiplication and which corresponds to control law (34) is shown in Fig. 5.

An electronic controller includes the concatenated adder with five inputs, five integrators, an adder with four inputs, as well as eight proportional elements that enable four negative feedbacks and four positive links.

The assigned program of operation, formed in the controller, passes through an electronic controller, where the control law is formed based on the solution to the inverse problem on the dynamics of control object. The signal is sent to the additional adjustment unit that will ensure the implementation of an alternating gain coefficient of control circuit. Specifically: a high (two-and three-fold from the rated) gain coefficient in the range of small shifts of the object from the nominal position. That compensates for errors caused by static non-linear characteristics. The controlling signal, thus formed, is sent to the electric input of the servo valve, which enables the programmed motion of a hydraulic motor, combined with the control object.

This movement is registered by a feedback sensor, the signal which is sent to the unit that corrects a feedback sensor, and then to the electronic controller (the main negative feedback).
The application of the proposed controller makes it possible, compared to existing analogs, to significantly improve the static accuracy and performance speed of control systems, as well as reduce the cost of electronic controllers [17].

Based on the results of theoretical calculations and an analysis of experimental data, such a controller could ensure an increase in the accuracy of frequency and power control at hydrogenerator units, from 0.3 to 0.1 %, thereby providing for the indicators of electric energy quality at the level of existing international standards.

The system with the proposed controller has the time of first adjustment of 59.5 s, which coincides with the time of control, while the basic object has, respectively, 42.5 s and 109 s (Fig. 4). Thus, the proposed controller ensures a twofold increase in performance speed compared to the basic variant.

7. Discussion of results of studying the construction of precision controllers for hydrogenerator units

This work aims to improve precision systems of automated control over hydrogenerator units at hydraulic power plants by designing the high-precision automated systems to control frequency and power.

Results of the research could be used in hydro energy generation, namely, for the automated control over rotation frequency and power of hydrogenerator units at hydroelectric plants HPP.

The proposed controller has been synthesized based on solving the inverse problems of dynamics. Static and dynamic control errors in the proposed controller are 3–5 times lower than those of existing PID controllers. This is predetermined by the difference in principles underlying the controllers. Specifically: the controller that is built on solving the inverse problems of dynamics includes no direct differentiation, while the transfer function of the controller approaches unity.

Application of the proposed controller would improve the accuracy of control over rotation frequency of hydrogenerator units under stationary (no-emergency) modes of operation. That would make it possible to resolve a relevant task on improving the quality of electric energy, namely to reduce deviations and fluctuations in the frequency of electric current generated at hydro power plants.

The merit of our study relates to that the mathematical models of control system take into consideration those factors that affect accuracy of control. These factors include leakages and fluid overflows from the servo motor, the non-linearity in characteristics, dry friction, etc. Accounting for them, we have verified the constructed mathematical models, in line with the results from experiments, we analyzed the indicators for static and dynamic accuracy of control over frequency and power of hydrogenerator units. Parameters of the proposed controller far exceed the parameters of existing PID controllers. A 50 % decrease is achieved in the time required to enter a permanent regime, the level of overshoot reduces (1.03 for the proposed controller versus 1.17 for the PID-controller). Control quality indicators, when using the proposed controller, are at the level of indicators for the world leader in the field of control over hydrogenerator units.

2. We have conducted a series of experimental studies into the system of automated control over frequency and power of the radial-axial hydro turbine RO V-195; based on them, we have verified the constructed mathematical models of systems for automated control over frequency and power.

3. The structure of the precision controller has been proposed, which ensures the compensation for or a significant decrease in all kinds of errors and the corresponding improvement in control accuracy. We have performed a structural-parametric synthesis and the optimization of parameters for the controller over a primary circuit of the system of automated control over frequency and power of hydrogenerator units. The proposed controller provides an increase in the accuracy of control over frequency and power of hydrogenerator units from 0.3 (classic PID controller) to 0.1 % and makes it possible to provide for the indicators of electric energy quality at the level of acting international standards.

8. Conclusions

1. We have refined mathematical models of control systems for hydrogenerator units. They include those factors that affect the accuracy of control: overflows in a servo motor, the non-linearity in characteristics, dry friction, etc. Based on the developed refined and identified mathematical models, in line with the results from experiments, we analyzed the indicators for static and dynamic accuracy of control over frequency and power of hydrogenerator units. Parameters of the proposed controller far exceed the parameters of existing PID controllers. A 50 % decrease is achieved in the time required to enter a permanent regime, the level of overshoot reduces (1.03 for the proposed controller versus 1.17 for the PID-controller). Control quality indicators, when using the proposed controller, are at the level of indicators for the world leader in the field of control over hydrogenerator units.

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References