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Розглядається технологія моделювання/дослідження явищ теплотворення, тепловіддачі, тепловикористання у двигуні швидкого внутрішнього згоряння, в основу якої покладено принципи прaxeологічності. Визнано, що подальший розвиток класичних підходів до моделювання робочих процесів у двигуні, спираючись суто чи здебільшого на аналітико-алгоритмічні описи, є практично неможливим. Тож запропоновано залучити в модель також і реальний робочий простір двигуна, системно приєднуючи його до віртуального, втіленого в програмно-алгоритмічному середовищі, і тим самим впроваджуючи частину реальності в модель цієї ж реальності. В рамках дослідження за натурний робочий простір використовувався циліндр дослідницького двигуна BRIGGS&STRATTON, змонтованого на спеціальному випробувальному стенді.

При цьому з'являється можливість суттєво спростити аналітичну складову модельного відображення робочих процесів в двигуні, вибудовуючи її на основі класичних аналітичних співвідношень, що відображають закон збереження речовини, закон збереження енергії, закон тепловіддачі, рівняння термодинамічного стану робочого тіла. Модель набуває конкретності не за рахунок спеціальних емпіричних описів, а завдяки чертанню поточної інформації з реального інформаційного простору на засадах теорії подібності.

Потрібної ефективності моделі надає імітація в програмному середовищі взаємодії між собою і довкіллям двох зон, на які поділено модельний робочий простір двигуна. Двобонна модель протиставлена так званому багатозонному, у рамках яких завжди існує високий ризик виникнення майже не контрольованих помилок і похибок – моделям, які потребують складного й трудомісткого інформаційного супроводу й обслуговування. Саме у разі двобонного трактування модельного робочого простору стає можливим відмовитись від аналітичного контролю за хімічною рівновагою в робочому середовищі і не існує причин, які б зумовлювали речовинний обмін між зонами. А тому тепловіддачу у стінки робочого простору можна визначати за прикладом однозонної моделі.

З проведеного дослідження випливає доцільність застосування Вібе-функції для віртуального симулювання явища теплотворення. Якість симулювання суттєво зростає завдяки залученню інформації, отримуваної у процесі, так би мовити, «on-line-спілкування» віртуальної (у формі комп'ютерної програми) та реальної (у формі натурального робочого простору) частин модельного середовища.

Виклад матеріалу супроводжується ілюстративним матеріалом, який відображає таку отриману засобами моделювання інформацію про перебіг: робочого тиску в робочому просторі двигуна, температури робочого тіла, коефіцієнта надміру повітря, коефіцієнта тепловіддачі. Наводяться також приклади зміни інтенсивності теплотворення та інтенсивності тепловіддачі у поверхні: робочого простору загалом, гільзи циліндра, кришки циліндра, головки поршня. Серед ілюстрацій – характеристики внутрішнього (міжзонного) теплообміну

**Ключові слова:** двигун швидкого внутрішнього згоряння, теплотворення, теплоспоживання, прaxeологічні засади моделювання

# DEVELOPMENT OF PRAXEOLOGICAL PRINCIPLES TO MODEL/STUDY HEAT GENERATION AND HEAT CONSUMPTION PROCESSES IN THE ENGINE OF RAPID INTERNAL COMBUSTION

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## 1. Introduction

Heat formation and energy consumption are the most complicated phenomena/processes that occur/take place in a working space of the internal combustion engine and which largely affect energy efficiency, effectiveness, and environmental friendliness of a given heat machine. The specified processes are not fully subject to any existing theories. Therefore, in the internal combustion engines, one is forced to investigate them, relying largely on empiricism. That gave rise to the idea to introduce a working space of the engine to

the modeling environment, thereby combining the naturalness and virtuality. Thus, a research within the framework of this old problem could be undertaken by means of a hard/soft-technology [1, 2]. This technology combines in a single system a testing bench and a computer, thereby enabling the “communication” between an actual examined engine and a virtual engine in the form of a computer model.

There are quite objective reasons why the examined object in this case implies only a rapid internal combustion engine (a gasoline engine, Otto-engine, engine on light fuel, engine with spark-ignition), disregarding the diesel engine.

The world has been refusing the services of the diesel engine while losing hope to bring it to acceptable standards in terms of the environment requirements. That is done both by automobile companies that produce “mass-market” cars and companies that offer automobiles of “premium class”. Sometimes this is achieved under a direct pressure from organizations that fight for a cleaner future (an example is initiatives by the environmental organization Deutsche Umwelthilfe (DUH)), sometimes it is the decision of the authorities – urged by legal means or motivated by their own understanding of the problem. Some companies (specifically Toyota), while flatly refusing the promotion of any new “diesel” technologies, are focused on the technologies of hybrids, in the framework of which, however, the internal combustion engine (gasoline, of course) will take one of the leading places. At the same time, it is appropriate to emphasize that the “achievements of the diesel” were etched in the Otto-engine as well.

Internal combustion engines generate considerable chemical (biological) and thermal pressure on the environment, burn a huge amount of fuel and air at a relatively low productivity.

Therefore, improving the internal combustion engine makes sense in terms of better efficiency of energy conversion. Energy efficiency is among the most important attributes of perfection of any mobile technology [3, 4]. Thus, the only way to solve a general problem on energy efficiency of the internal combustion engine is to improve the quality of burning a working mixture and the efficiency of heat consumption in its workspace. To completely understand the process of heat formation/heat consumption, it is necessary to be able to reproduce it many times in a controlled manner. Because of the limited capabilities to apply measuring equipment (imperfection of measuring equipment, measurement errors, etc.) it is required to explore thermal processes that take place in the cylinder of engines, and by analytical means at that. Thus, the subject of this paper is highly relevant.

Research is conducted in different ways. Thus, for example, by using the equations of chemical reactions of interaction between fuel and air one could derive the rate of combustion, or calculate the rate of energy conversion considering the size, the laws of motion, the evaporation of fuel drops, etc. However, these models of routine type have a major drawback: they need significant simplifications that provoke fundamental errors. It is much more accurate to model thermal processes based on the measured pressure and temperature in the engine cylinder.

An indicator chart is the most laconic means to render information on the progress of a working process in the internal combustion engine. Moreover, a procedure for measurement and recording of alternating pressure is much more efficient than a procedure for the identification of other thermodynamic quantities. Studying indicator charts makes it possible to comprehensively and objectively evaluate the kinetics of a fuel combustion-burning process, the dynamics of the processes of heat formation and heat consumption. The chart can also be supplemented with the first derivative from it based an angle of rotation of the cranked shaft [5], which also provides useful information about the heat formation process. Measurement of temperatures, although it is a much more difficult thing, is feasible for an advanced technology anyway. It is the manipulation of charts of change in pressure and temperature that makes it possible to achieve a high praxeology level of modeling the processes in the rapid internal combustion engine.

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## 2. Literature review and problem statement

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Modern gasoline engines are powered both by a fuel injection technology, before inlet valves (to the intake tract) and by the technology of direct fuel injection to the workspace. Increasingly common is the so-called HCCI-technology for the organization of a working cycle. The phenomenon of combustion in the gasoline engine of direct injection and in HCCI-engine has become an object or subject of thorough studied relatively recently [6, 7], which is why the phenomenon of heat transfer through the walls of cylinder in such engines has been paid very little attention. And if one is to rely on the research methods that directly combine experiment and calculation, the difference between these engines in the methodological sense of elucidating the features of heat transfer is blurred. Therefore, a task related to modelling the processes of heat transfer/ heat consumption, which is touched upon in this paper, is quite general, and the methodology to solve it has every reason to become universal.

A working process of piston internal combustion engine is an extremely complex phenomenon that refers to an artificially provoked and, to a certain degree, controlled consistent conversion of various forms of energy. The laws of energy forms conversion are in general quite clear [8–10], but when applied to engineering tasks, saturated in concreteness, this transparency is lost almost entirely.

Modeling a work of the thermal engine in general is to accurately compare the phenomena of heat formation in a workspace, heat transfer beyond this space, heat exchange inside the space, heat consumption, and efficiency of the engine.

The hardest thing to model is the heat transfer inside the cylinder and the heat exchange through the walls of the cylinder to the means that cool the engine [11]. The heat, which in this case is removed from a working gas, inevitably causes energy loss. Therefore, the evaluation of heat transfer/heat exchange, especially during combustion of fuel, is extremely important and inexhaustible task, in terms of the completeness of solution, arising each time when a research into properties of thermal engines is undertaken. The degree of thermal energy utilization defines the level of environmental impact [12, 13].

The processes of heat transfer are modeled based typically on the equations of the similarity of processes related to forced convective heat exchange  $F(\text{Nu}, \text{Re}, \text{Pr})=0$ , where Nu, Re, Pr are the criteria of similarity by Nusselt, Reynolds, Prandtl, respectively.

Instead, the description of heat generation in the internal combustion engine typically employs a very convenient I. Wiebe function [14]. The description that applies a Wiebe function is quite simple both in content and form.

For example, in the framework of the so-called two-zone model of combustion, paper [15] reported a detailed study into the efficiency of ethanol with different content of water as a fuel for the rapid internal combustion engine. In this case, heat generation was rather reliably modeled by using exactly a Wiebe function, and the heat transfer through the wall of the cylinder was investigated with a simultaneous comparative analysis of the adequacy of its four different descriptions.

The application of a two-zone pattern is a positive feature of the research. However, the conclusion about the benefits of the description of Hohenberg is too categorical.

A source of valuable information is such studies into processes of heat formation when the authors manage to

compare at the same time the efficiency of work of the internal combustion engine using different fuels – gasoline, ethanol, natural gas, busine-ethanol, busine-ethanol-hydrogen mixture, etc. [16, 17]. It is in this case that it is possible to achieve a high level of theoretical generalizations. Work [16], in particular, focuses on acquiring mostly experimental information and needs more meaningful interpretation. Paper [17] addresses the development of a zero-dimensional (Zero-D) computational model of fuel combustion that does not require solving differential equations. However, the study is based on experimental data and engage at once two I. Wiebe functions into the model space. Actually, work [17] is a kind of model of praxeological orientation of research methodology. However, a zero-dimensional interpretation of the engine working space too much simplifies the physical content of the model.

It is appropriate to note that the so-called zero-dimensional models are the most attractive kind of thermodynamic models that, due to its simplicity, has found widest application. Paper [18] used the means of zero-dimensional modeling to examine the sensitivity of model for heat formation to a change in the parameters for a Wiebe function. Among these parameters is the factor of combustion efficiency  $a$ , the so-called form-factor  $m$ , angle  $\varphi_0$  of the crankshaft position, corresponding to the onset of combustion process, the angular duration of combustion process  $\Delta\varphi$ . It was emphasized that parameters  $m$  and  $\varphi_0$  require careful assessment, whereas parameters  $a$  and  $\Delta\varphi$  can be evaluate not too meticulously. It was also highlighted that the initial and final stages of heat generation are advisable to ignore, focusing on the accuracy of reproducing a point of maximal intensity of heat transfer – both in the case of ordinary and in the case of double Wiebe functions. Subsequently, they managed to develop an appropriate algorithm for the automated calibration of a zero-dimensional model of heat formation based on the Wiebe function [19]. The algorithm, so to speak, is online-capable of diagnosing and controlling the process of combustion.

Data set forth in [18, 19] apply to diesel engines and, in general, can be used for engines with forced spark ignition or HCCI-engines. What distinguishes these works is that heat formation and heat removal within the model are seen as interrelated and interdependent processes. However, it would be more appropriate to apply the optimization of a combustion engine model within the improved, the so-called two-zone model of the working space.

One often resorts to using the so-called double (dual) Wiebe functions (refer to [17, 18]). In paper [20], within the framework of a single-dimensional toolset for simulating the work of engine with spark ignition, this particular function proved to be suitable for modeling the intensity of heat formation for the case of ethanol-gasoline blends burning at different values for the degree of compression of the fuel mixture and the level of exhaust gases recycling. Parameters for the Wiebe function were determined using the method of least squares based on experimental information. In this case, quality of the approximation was estimated based on comparing the progress of pressure in the engine cylinder, reproduced by the model, to that registered experimentally.

Benefits of the dual Wiebe function were also confirmed in [21], whose authors, in the framework of a zero-dimensional model, investigated efficiency of work of the engine with spark ignition on methane and a mixture of methane and hydrogen. Quality of simulation using an approximating least square methodology was assessed based on the level of adequacy of representation of a change in pressure in the en-

gine cylinder, as well as based on the accuracy of predicting values for an average effective pressure.

In fact, the superposition of two Wiebe functions, applied in papers [17, 18, 20, 21], almost did not affect the general complexity of the model for heat formation/heat consumption in the rapid internal combustion engine. However, there is a proposal from [22] to modify the Wiebe function so that it contains only one constant – a dimensionless parameter  $C_i$  (a factor or a coefficient of combustion). The modified version of the Wiebe equation makes it possible to consider a change in the intensity of combustion (heat formation rate) as being dependent only on a single parameter  $C_i$ , thereby facilitating the process to accurately determine the parameters for form  $m$  and combustion efficiency  $a$ . Thus, a possibility of superposition or/and modification of the Wiebe functions is easy to predict within any model if there is any need in it.

The absolute appropriateness of applying the Wiebe function to virtually simulate a phenomenon of heat formation follows from the above. The mentioned modification version of the Wiebe equation is also a useful tool to analyze the influence of different factors on the engine efficiency. And yet the information would be more valuable acquired from the process of “on-line-communication” between the virtual, in the form of software, and the actual, in the form of a natural working space, parts of the model environment. That gives rise to the task on rational correlation and combination of both the virtual and actual in a single model.

Heat transfer is often investigated by the tools of direct measurement (where it is absolutely impossible to proceed without abstract modeling representations). In addition, they resort to model calculations (where it is impossible to navigate without experimental information). Thus, it is only natural to combine directly the means of experiment and the tools for model calculations. It is in this case that there appears a possibility to significantly simplify and visualize the analytic part of model representation and reproduction. Specifically, it becomes possible, by using relatively simple means, to represent internal fluxes of heat in the model.

Internal heat transfer is the notion that motivates or moves to distinguish at least two parts of the workspace, one of which serves a source of heat, the other acts as its receiver. A two-zone interpretation of the engine workspace is a motivated denial of the eligibility of a zero-dimensional model.

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### 3. The aim and objectives of the study

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The aim of this study is to develop, based on the so-called two-zone interpretation of a working space, the praxeological fundamentals of synthesis, and to evaluate the potential effectiveness of the model for heat formation–heat consumption in the rapid internal combustion engine. That would improve the technology to construct a model of the working process in the thermal engine, by rationally differentiating between adequacy and complexity of modeling representations and by harmoniously combining, into an integrated system, the means of experimental information support, classic analytical descriptions and computer algorithms for analysis.

To achieve the set aim, the following tasks have been defined:

- to assess a possibility of using a two-zone model to study processes inside engines;

- to construct a mathematical model of thermodynamic processes in the engine working space with praxeological attributes;
- to devise an analytical description of the process of heat formation;
- to analyze, based on the information acquired by modeling-experimental means, the patterns in the course of processes of internal and external heat transfer-heat exchange.

#### 4. Methods and results of studying the processes of heat formation and heat transfer

##### 4.1. A two-zone model of workspace

Immediately after the injection of fuel directly into the engine's cylinder workspace, such processes are activated that it does make sense to split a given space, in the first approximation (conventionally, formally), into parts with specific characteristics, Fig. 1. The same zones can be distinguished also in the case when fuel is injected into the inlet tract and in the case of exclusively external mixture formation. The concept of zones within a working space is key for the methodology of synthesis of the model of heat formation/heat transfer/heat consumption in the internal combustion engine.

Of course, the zone model can be interpreted very differently, Fig. 2.

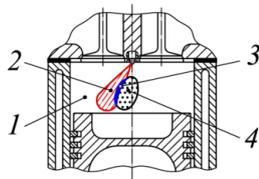


Fig. 1. Primitive schematic of zones in the working space of the engine cylinder: 1 – a zone with the air that was captured by the engine cylinder at inlet stroke; 2 – a fuel jet; 3 – a zone of fuel oxidation, the flame front; 4 – a zone of combusted, burnt mixture

For example, a single zero-dimensional model (Fig. 2, a) implies the complete uniformity of gas in the engine cylinder throughout the entire working process, while the temperature and pressure (and all other parameters as well) are the same at all points of the working space. This model is characterized by simplicity, but it is considered perfectly acceptable only for the case of simulating a gas exchange in the engine or in the case when the engine's accelerating operational modes without fuel consumption are reproduced (these include the so-called external braking modes of the engine).

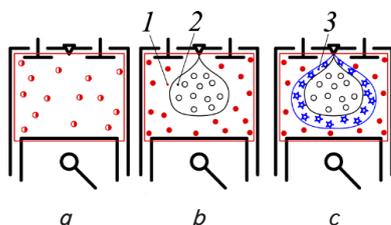


Fig. 2. Schematics of model working space of the engine: a – a single-zone zero-dimensional model; b – a two-zone model; c – a multi-zone model: 1 – a zone of the unburnt; 2 – a zone of the burnt; 3 – a zone of combustion; 4 – fresh fuel mixture; 5 – exhaust gases; 6 – fuel mixture in the state of combustion

The essence of a two-zone model, Fig. 2, b, implies that the workspace is conventionally divided into two zones – a zone of the combusted (burnt) fuel mixture and a zone of the non-combusted (non-burnt) mixture. The first zone contains exhaust gases, residues of unburned fuel. The second zone contains a mixture of air and fuel and the residues of exhaust gases from a preceding stroke. The first of these is behind the front of the flame, and the second one is ahead. In the process of combustion, a zone with non-burnt fuel is continuously narrowing, whereas a zone with exhaust gases is expanding in the same way. A layer of flame separating the zones, fundamentally different in terms of chemical composition and parameters of the working body, is considered to be vanishingly thin and such that it does not have the properties of real substance (mass, volume, etc.). Each zone is formed by a completely homogenic mixture-gas, so the temperature and parameters for the working body at all points of the zone are the same. The pressure, since it propagates at the speed of sound, is considered the same in both areas. Two-zone models, due to their adequacy and relative simplicity, are the most useful for studying thermal processes predetermined by fuel combustion.

Multi-zone models imply the existence of several (many) zones, for instance, a zone (zones) of non-combusted mixture, a zone (zones) of combusted mixture, and a zone (zones) of combustion (Fig. 2, c). These models are rarely used due to the significant risk of barely controlled bugs and errors. Thus, there is a need for a complex and time-consuming information support and maintenance.

Thus, in the case of a two-zone model (Fig. 3), somebody must control pressure  $p$  of gases (same in both zones), mass  $m_{zh}$ , volume  $v_{zh}$ , temperature  $T_{zh}$ , etc. of the working body in the zone of the burnt, as well as parameters  $m_{nh}$ ,  $v_{nh}$ ,  $T_{nh}$ , etc. (same in content) for the working body in the zone of the non-combusted.

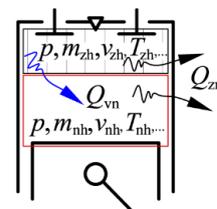


Fig. 3. Abstract schematics of the engine model workspace

In the case of a two-zone model, we cannot, of course, leave unattended both the external  $Q_{zn}$  and, certainly, the internal (intra-zone)  $Q_{vn}$  heat fluxes.

##### 4.2. Working pressure and temperature

Because the fundamental laws of thermodynamics involve one way or another such quantities as pressure and temperature of a thermodynamic body, a change in these particular magnitudes over time would provide a very desirable information for the estimation analysis of the process of heat transfer. A procedure for measuring an alternating pressure in the fleeting processes is the most perfect. Therefore, a key role in the thermodynamic model could belong exactly to the experimentally measured dependence of change in pressure in the workspace on the angle of rotation of the engine shaft.

Fig. 4 shows the experimentally derived charts of change in pressure  $p'$  in a cylinder of the specialized single-cylinder test engine BRIGGS & STRATTON (USA); Fig. 5 demonstrates the charts with the same content, reproduced in a

specially designed programming environment based on the above-mentioned two-zone model ( $\varphi$  – rotation angle of the crankshaft;  $\Delta\varphi$  – an offset of the chart relative to the upper dead point;  $\Delta p$  – an offset of the chart relative to the line of zero pressure; S and E – the beginning and end of a region of high pressure; SC and EC – the beginning and end of combustion region). In this case, somebody introduced respective amendments to the experimental indicator charts using calculations in line with a single-zone model. The course of the mean temperature of working gas  $T$  over a working stroke is shown in Fig. 6 ( $T_0$  – ambient temperature). All these charts relate to the unchanged rotation frequency of the engine crankshaft  $n_e=2,400 \text{ min}^{-1}$  and different loads  $p_e=0; 0.10; 0.18; 0.30; 0.45; 0.62 \text{ MPa}$  ( $p_e$  – average effective pressure;  $p_0$  and  $T_0$  – ambient pressure and temperature).

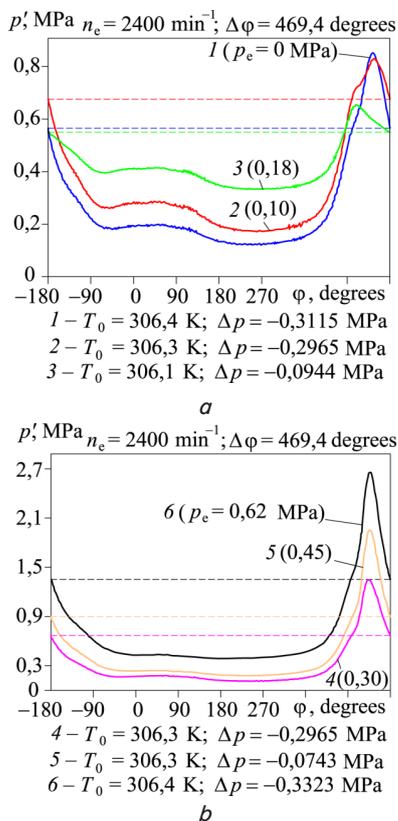


Fig. 4. Indicator diagrams deployed based on the rotation angle of the engine crankshaft: *a* – under small loads; *b* – under large loads

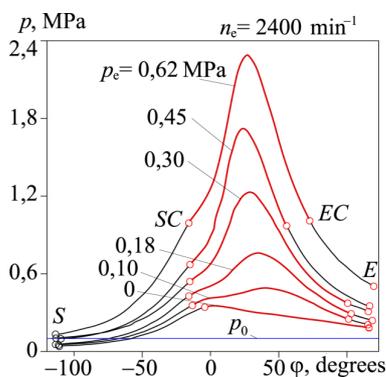


Fig. 5. Fragments of indicator diagrams reproduced by computer based on experimental information

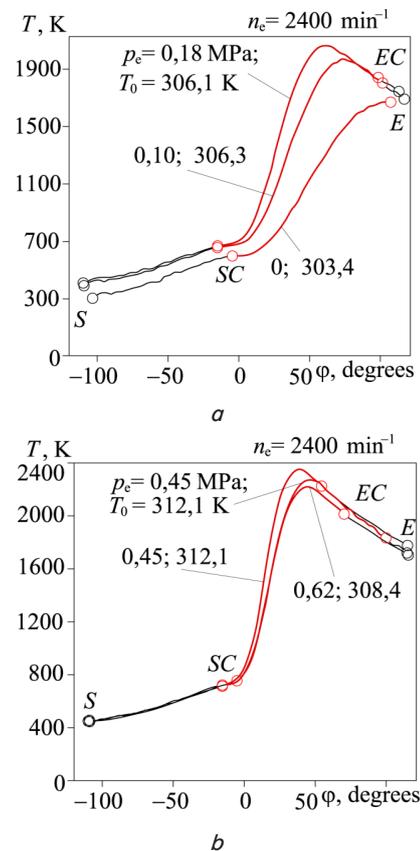


Fig. 6. Charts of change in the working body temperature averaged in the cylinder space: *a* – under small loads; *b* – under large loads

The pressure in the engine cylinder in Fig. 4 is characterized by magnitude  $p'$  that satisfies equation  $dp' = dp$  ( $p$  – absolute pressure). Counting the angle of rotation of the engine crankshaft is not adjusted to the dead points. In essence, the dependences shown in Fig. 5, 6 are the standard indicator diagrams deployed based on the crankshaft rotation angle adjusted to the upper dead point  $\varphi=0$ .

The base of any calculation of processes is the balance of energy within a given process in the region bounded by the walls of combustion space (Fig. 2, 3).

### 4. 3. Thermodynamic model of engine: fundamental ratios

Underlying the calculation is a system of three equations: equation of the law of conservation of matter, equation of the law of conservation of energy, and equation of the thermodynamic state of a working body. The first equation reflects the balance of masses. The second is the record of the first law of thermodynamics. The third is the equation of state of a perfect gas.

The mass of the working body in the cylinder is the sum

$$m = m_{nh} + m_{zh}, \tag{1}$$

$m_{nh}$  of the mixture that did not burn, and mass  $m_{zh}$  of the mixture that was burnt. Upon differentiation, equation (1) transforms to ratio

$$\frac{dm}{d\varphi} = \frac{dm_{nh}}{d\varphi} + \frac{dm_{zh}}{d\varphi}. \tag{2}$$

And because in the process of mixture combustion (in line with the law of conservation of amount of substance, the law of indestructibility of matter)  $dm/d\varphi=0$ , then

$$\frac{dm_{zh}}{d\varphi} = -\frac{dm_{nh}}{d\varphi}. \quad (3)$$

Fig. 7 shows charts for the course of the mean (throughout the entire workspace) value for the coefficient  $\lambda$  of excess air depending on the engine's crankshaft angle of rotation. Here 1, 2, ..., 6 are, respectively,  $p_e=0$  MPa ( $\tan \alpha=0.088$  1/degree), 0.10 (0.167), 0.18 (0.258), 0.30 (0.417), 0.45 (0.555), 0.62 (1.177)). The magnitude  $\lambda$  was determined from formula

$$\lambda = \frac{m - (m_{pl} + m'_{pl0})}{l_0 (m_{pl} + m'_{pl0})}, \quad (4)$$

where  $m$  is the mass of the working body located within the cylinder;  $m_{pl}$  is the mass of the fuel fed over a stroke;  $m'_{pl0}$  is the part of fuel, burned over a preceding stroke, contained in the residual gases;  $l_0$  is the theoretically required amount of air for the stoichiometric combustion of unit mass of fuel.

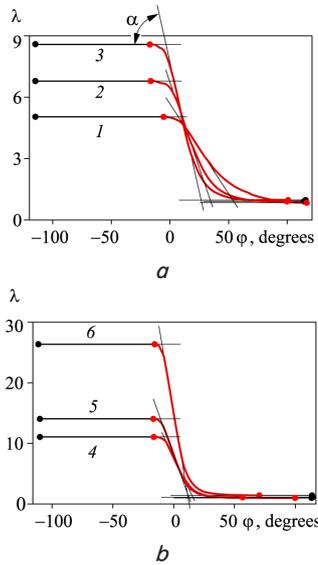


Fig. 7. Charts of change in the excess air coefficient: *a* – under small loads; *b* – under large loads

Ratios (1) to (4) do reflect the conservation of matter.

A change in the mass of a working body in the combustion zone is calculated from formula

$$\frac{dm_{zh}}{d\varphi} = -\frac{m}{m - m'_{pl0}(1 + \lambda l_0)} (1 + \lambda l_0) \frac{dm_{pl}}{d\varphi}, \quad (5)$$

where  $m_{pl}$  is the mass of burnt fuel (that participated in heat generation).

For the zone in which there was no combustion, a working body state's thermodynamic equation takes the form

$$pV_{nh} = m_{nh}R_{nh}T_{nh}, \quad (6)$$

where  $R_{nh}=\text{const}$  is the gas constant in the mixture, which is determined from

$$R_{nh} = \frac{m_{pv}}{m_{nh}}R_{pv} + \frac{m_{ppl}}{m_{nh}}R_{p?} + \frac{m_{pl0}}{m_{nh}}R_{pl0}, \quad (7)$$

where  $m_{pv}$  and  $R_{pv}$  are the mass and the gas constant of air;  $m_{ppl}$  and  $R_{ppl}$  are the mass and the fuel vapor gas constant;  $m_{pl0}$  and  $R_{pl0}=R_{pl0}(T_{nh}, \lambda_{zh}, p)$  are the mass and the gas constant of residual gases;  $V_{nh}$  is the volume of the non-burnt mixture;  $T_{nh}$  is the temperature inside the specified "passive" zone. By differentiating expression (6) and taking into consideration (7), we obtain ratios

$$p \frac{dV_{nh}}{d\varphi} + V_{nh} \frac{dp}{d\varphi} = R_{nh}T_{nh} \frac{dm_{nh}}{d\varphi} + m_{nh}R_{nh} \frac{dT_{nh}}{d\varphi} + m_{nh}T_{nh} \frac{dR_{nh}}{d\varphi}. \quad (8)$$

The law of conservation of energy for the specified zone takes the form

$$h_{nh} \frac{dm_{nh}}{d\varphi} - \frac{dQ_{znh}}{d\varphi} = \frac{d(m_{nh}u_{nh})}{d\varphi} + p \frac{dV_{nh}}{d\varphi}, \quad (9)$$

where  $h_{nh}=u_{nh}+R_{nh}T_{nh}$  is the specific enthalpy of a working gas;  $dQ_{znh}/d\varphi$  is the heat transmitted from a zone in which there is no combustion to the walls of the combustion chamber and to a combustion zone;  $p \cdot dV_{nh}/d\varphi$  is the work required to change the volume of the zone in which there is no combustion, caused by the piston movement;  $u_{nh}$  is internal energy.

The internal energy of the zone in which there is no combustion depends on the coefficient of excess air and temperature of the working gas, and corresponds to equation

$$\frac{d(m_{nh}u_{nh})}{d\varphi} = m_{nh} \frac{dV_{nh}}{d\varphi} + u_{nh} \frac{dm_{nh}}{d\varphi}, \quad (10)$$

Given that the zone under consideration is composed of air, residual gases, and gasoline vapor, we shall obtain:

$$m_{nh}u_{nh} = m_{pv}u_{pv} + m_{ppl}u_{ppl} + m_{pl0}u_{pl0}, \quad (11)$$

where  $u_{pv}$  is the specific internal energy of air;  $u_{pl0}$  is the specific internal energy of residual gases;  $u_{ppl}$  is the specific internal energy of fuel vapors.

By substituting  $m_{pv}=m_{nh}-m_{ppl}-m_{pl0}$  in equation (11), we obtain:

$$u_{nh} = \left(1 - \frac{m_{ppl}}{m_{nh}} - \frac{m_{pl0}}{m_{nh}}\right) u_{pv} + \frac{m_{ppl}}{m_{nh}} u_{ppl} + \frac{m_{pl0}}{m_{nh}} u_{pl0}. \quad (12)$$

Equation of state for the zone of combustion takes the form

$$pV_{zh} = m_{zh}R_{zh}T_{zh}. \quad (13)$$

The index «zh» indicates, as agreed, the fact that one or another magnitude relates to the zone of combustion.

Upon differentiation of expression (13), we derive ratio

$$p \frac{dV_{zh}}{d\varphi} + V_{zh} \frac{dp}{d\varphi} = R_{zh}T_{zh} \frac{dm_{zh}}{d\varphi} + m_{zh}R_{zh} \frac{dT_{zh}}{d\varphi} + m_{zh}T_{zh} \frac{dR_{zh}}{d\varphi}. \quad (14)$$

A gas constant, corresponding to the combustion zone, depends on temperature, an excess air coefficient  $\lambda$  and pressure  $p$ :  $R_{zh}=R_{zh}(T_{zh}, \lambda_{zh}, p)$ . The law of energy conservation in this zone is expressed by ratio

$$\frac{dQ_{pl}}{d\phi} - h_u \frac{dm_{nh}}{d\phi} - \frac{dQ_{znh}}{d\phi} = \frac{d(m_{zh}u_{zh})}{d\phi} + p \frac{dV_{zh}}{d\phi}, \quad (15)$$

where  $dQ_{pl}/d\phi = h_u dm_{pl}/d\phi$  is the energy of fuel, released as a result of fuel combustion;  $dQ_{znh}/d\phi$  is the heat that is transmitted from the combustion zone to the walls of combustion chamber and to the zone in which there is no combustion;  $p \cdot dV_{zh}/d\phi$  is the work required to change the volume of the combustion zone, caused by the piston movement.

The internal energy of combustion depends on temperature, an excess air coefficient, and pressure  $u_{zh} = u_{zh}(T_{zh}, \lambda_{zh}, p)$ , and is in line with equation

$$\frac{d(m_{zh}u_{zh})}{d\phi} = m_{zh} \frac{du_{zh}}{d\phi} + u_{zh} \frac{dm_{zh}}{d\phi}. \quad (16)$$

The ratios (5) to (16), taken together, comprehensively enough reveal the actual current energy balance in the engine's working space, as well as peculiarities of its description.

#### 4. 4. Analytical description of heat formation

In gasoline engines with forced ignition, heat formation is typically a process that one can quite properly describe analytically using an exponential ratio [2, 14]

$$z = 1 - \exp(a\tau^{m+1}), \quad (17)$$

where  $z = Q_t/Q_{tc}$  is the relative (specific) heat formation;  $Q_t$  is the current heat formation;  $Q_{tc}$  is the total potentially possible heat formation over a working stroke;  $m > 0$  is the characteristic indicator for the quality of fuel combustion;  $a$  is the constant that characterizes the completeness of fuel combustion;

$$\tau = \frac{t - t_s}{t_e - t_s} = \frac{\phi - \phi_s}{\phi_e - \phi_s} \quad (18)$$

– relative (abstract, dimensionless) time.

Relative time is determined through current time  $t$ , the moment of start  $t_s$  and the moment  $t_e$  of completion of the fuel combustion process within a workspace. Here,  $\phi_s$  and  $\phi_e$  are the rotation angles of the crankshaft, corresponding to the moments  $t_s$  and  $t_e$  of the start and completion of the fuel combustion process.

According to (17), (18), a change in the intensity of heat formation over an abstract time is described by formula

$$\begin{aligned} \frac{dz}{d\tau} &= \tau_0 \frac{dz}{dt} = \\ &= -a(m+1)\tau^m \exp(a\tau^{m+1}) = a(m+1)\tau^m(z-1), \end{aligned} \quad (19)$$

where  $\tau_0 = t_e - t_s$  is the duration of the process of heat formation within the engine working space. It is obvious that

$$\frac{dz}{d\tau} = \frac{1}{Q_{tc}} \frac{dQ_t}{d\tau} = \frac{\phi_e - \phi_s}{Q_{tc}} \frac{dQ_t}{d\phi}. \quad (20)$$

As an example, Fig. 8 shows dependence (19) charts at different values for parameters  $a$  and  $m$  (a minimum of the maximum value for magnitude  $dz/dt$  is achieved if  $\tau = 1/e$ ). The actual heat release in the process of fuel combustion under six modes of engine operation is represented (with a certain advance) by charts in Fig. 10 ( $Q_{pl}$  – heat from the combustion of fuel;  $n_e$  – rotation frequency of the engine shaft;  $p_e$  – average effective pressure).

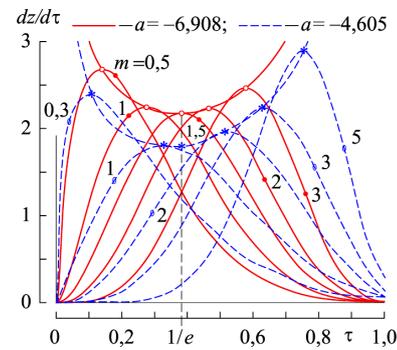


Fig. 8. Charts, identified analytically, representing heat formation in the engine cylinder

Typically, one unconditionally accepts [14] that over a working process one and the same proportion of fuel is always burnt  $z_k = 0.999$ , and thus, definitely,  $a = \ln(1 - z_k) = \ln(1 - 0.999) = -6.908$  (Fig. 8). However, why not compare the dependences, obtained by model-experimental means, which reflect the actual course of the intensity of heat formation in the engine cylinder (Fig. 9), to the ones, analytically identified by description ((19), (20)) provided  $a = -6.908$ . It is obvious that we now have good reasons to consider the magnitude  $a$  as one more modal, rather than formal purely analytical, parameter (or even a regime characteristic) for the process of fuel combustion in the engine cylinder. Specifically, if  $z_k = 0.99$  (Fig. 8), an essentially different equality  $a = -4.605$  would hold. Instead, parameter  $\tau_0$  is to be appropriately deprived of the status of being (as if) a physically real duration of the combustion process, in order to interpret it as a measure of asymptotic behavior of the process of heat production.

#### 4. 5. Heat transfer–heat release

Since the temperature of a working body in the zones is different, it is appropriate to determine the heat transfer–heat release for both zones separately. The basic object of empirical and theoretical research is typically a coefficient of heat release.

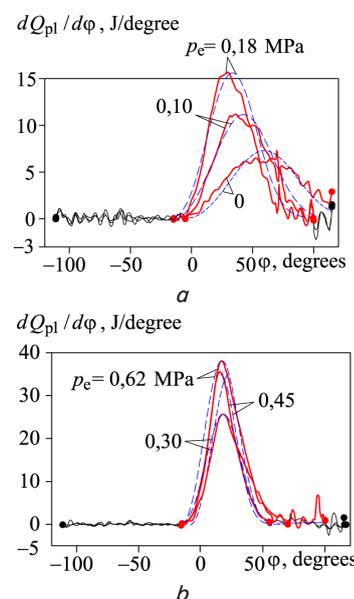


Fig. 9. Experimentally identified characteristics of heat formation in the engine (dashed lines – possible approximations (19)): a – under small loads; b – under large loads

The flow of heat (the amount of heat per unit time)  $Q$  from a fluid medium (gas) with the higher temperature  $T$  to a wall with a lower temperature penetrates the boundary layer of this same medium (gas) by thermal conductivity. According to the law of thermal conductivity by Fourier

$$dQ = -\lambda_3 \left( \frac{\partial T}{\partial n} \right)_{n \rightarrow 0} dA, \tag{21}$$

where  $A$  is the area of the heat transfer surface;  $n$  is the length of the normal to the wall's surface. The Newton's "law" also holds

$$dQ = \alpha(T_\infty - T_w)dA, \tag{22}$$

where  $\alpha = \lambda_g / \delta'_T$  is the coefficient of heat transfer;  $T_\infty$  and  $T_w$  are the temperatures of gas space (away from the wall) and the wall itself;  $\lambda_g$  is the coefficient of gas thermal conductivity;  $\delta'_T$  is the width of the temperature layer. In the case of a forced convection, along with a temperature boundary layer, there is a boundary layer of the flow, in which velocity  $w$  of gas in the direction of the wall descends from the value  $w_\infty$  away from the wall to null directly at the wall. Based on (21) and (22), one can obtain the so-called differential equation of heat transfer

$$\alpha = -\frac{\lambda_g}{T_\infty - T_w} \left( \frac{\partial T}{\partial n} \right)_{n \rightarrow 0}, \tag{23}$$

that opens a possibility, at a known temperature field in a fluid medium, to determine the coefficient of heat transfer (the method to directly measure this coefficient is not known). Pilot computer calculations testified to a greater flexibility-adequacy of the Woschni model. A change in the coefficient (23) of heat transfer due to a change in the angle of rotation of the engine crankshaft, which corresponds to the information given in Fig. 4–7, is demonstrated in Fig. 10.

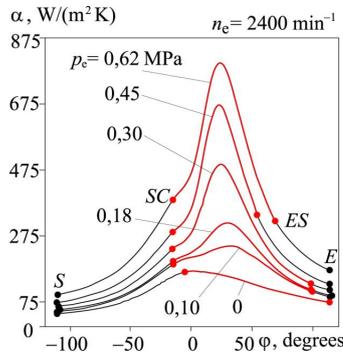


Fig. 10. Charts of change in a heat transfer coefficient

It is obvious that the character of change in a heat transfer coefficient under different modes of engine operation is in some sense identical. However, this character is so special that it defies representation by simple analytical tools (by approximation).

#### 4. 6. Certain types of external heat transfer

Based on information about the value for a heat transfer coefficient, one can estimate a differential heat transfer to the cylinder's walls  $dQ_c$  (22) whose course is shown in Fig. 11:

$$dQ_c = dQ_{cp} + dQ_{ck} + dQ_{cg}, \tag{24}$$

where  $dQ_{cp}$  is the heat transfer to the piston;  $dQ_{ck}$  is the heat transfer to the head (lid) of the cylinder;  $dQ_{cg}$  is the heat transfer to the sleeve of the cylinder. Individual components of heat transfer (24) to the walls of the cylinder are determined from formulae

$$\begin{aligned} dQ_{cp} &= \alpha A_{cp}(T - T_{cp})dt; & dQ_{ck} &= \alpha A_{ck}(T - T_{ck})dt; \\ dQ_{cg} &= \alpha A_{cg}(T - T_{cg})dt, \end{aligned} \tag{25}$$

where  $A_{cp}, A_{ck}, A_{cg}$  are the areas of, respectively, the piston, the head (lid) of the cylinder, sleeve. Patterns in these three quantities are shown in Fig. 12–14. It is appropriate to emphasize that determining the areas of surfaces of heat transfer  $A_{cp}, A_{ck}$ , appearing in expressions (25), is quite a challenge.

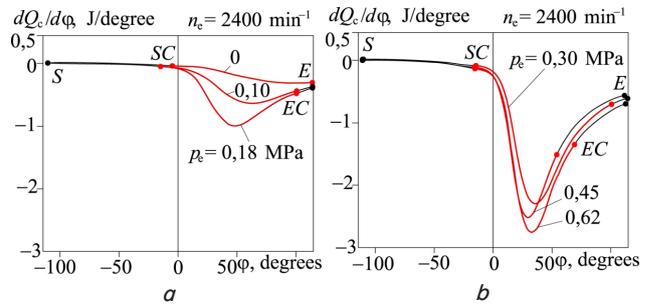


Fig. 11. Differential heat transfer, total to the surface of the cylinder: *a* – under small loads; *b* – under large loads

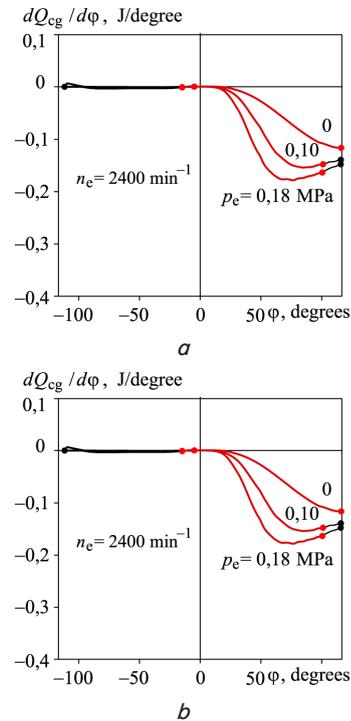


Fig. 12. Differential heat transfer to the surface of the cylinder's sleeve: *a* – under small loads; *b* – under large loads

A fundamentally important role in the engine working process also belongs to heat transfer from a zone of combustion to the walls of combustion space and to the zone in which there is no combustion

$$\frac{dQ_{czh}}{d\varphi} = \alpha_{zh} A_{czh} (T_{zh} - T_c) + k A_{ff} (T_{zh} - T_{nh}) + \epsilon_A (A_{czh} - A_{ff}) \left[ \left( \frac{T_{zh}}{100} \right)^4 - \left( \frac{T_c}{100} \right)^4 \right] \quad (26)$$

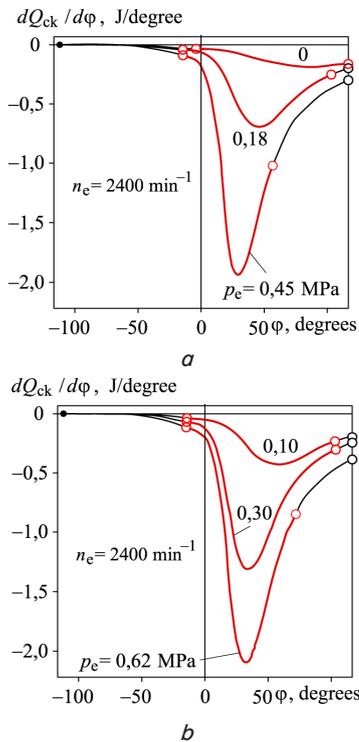


Fig. 13. Differential heat transfer to the surface of the cylinder's lid: *a* – under small loads; *b* – under large loads

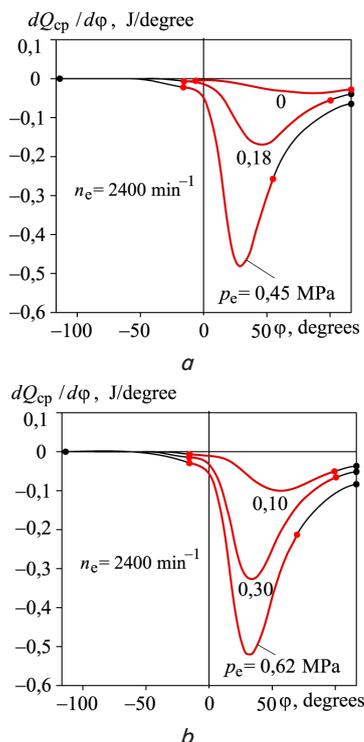


Fig. 14. Differential heat transfer to the surface of the piston's head: *a* – under small loads; *b* – under large loads

Not less important is the heat transfer from a zone in which there is no combustion to the walls of the combustion chamber and to the combustion zone:

$$\frac{dQ_{cni}}{d\varphi} = \alpha_{nh} A_{cni} (T_{nh} - T_c) + k A_{ff} (T_{nh} - T_{zh}) \quad (27)$$

In formulae (26), (27), the following designations are used:  $T_{zh}$ ,  $T_{nh}$  is the temperature of the respective zone;  $A_{czh}$ ,  $A_{cni}$  is the area of the border of the respective zone;  $A_{ff}$  is the flame front area;  $\alpha_{zh}$ ,  $\alpha_{nh}$  is the coefficient of heat transfer of the respective zone between gas and a wall;  $\epsilon_c$  is the coefficient of thermal emission of soot (particulates) in a working gas;  $k$  is the coefficient of the flame front's thermal penetration. Parameter  $k$  determined from formula  $k=1/(1/\alpha_{zh}+1/\alpha_{nh})$ .

#### 4. 7. Internal heat exchange

A determining characteristic for a two-zone model is the characteristic of internal heat exchange  $Q_{vn}=Q_{vn}(\varphi)$  between the earlier specified zones, Fig. 15. It follows from Fig. 15 that in terms of intensity the internal and external heat exchanges (Fig. 11) are qualitatively and quantitatively mutually similar. This indicates that the use of a two-zone model to determine heat transfer is completely justified and appropriate.

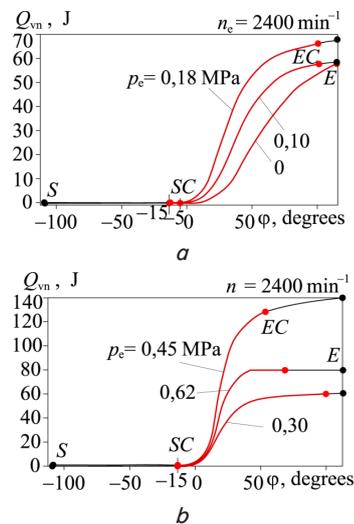


Fig. 15. Characteristics of internal (intra-zone) heat exchange: *a* – under small loads; *b* – under large loads

It is quite difficult to determine the magnitudes such as  $A_{czh}$ ,  $A_{cni}$  and  $A_{ff}$  that vary so significantly over time ((26), (27)). An idea about a flame propagation and a change in the volumes and shapes of both zones could be obtained only by using high-speed video recording. Typically, for the examined engines, there is no possibility to obtain this kind of series of photographs. Thus, we have to rely on the information about flame propagation in test engines-analogs, which have a similar combustion chamber. The areas of zones are formally determined by analogy, applying ratios

$$A_{czh} = \frac{V_{zh}}{V} A, \quad A_{cni} = \frac{V_{ni}}{V} A, \quad (28)$$

where  $A$  is the current area of the entire surface of the combustion space. In the end, expressions (28) could be specified in different ways.

It is possible to define the course of the process of heat formation by following the first law of thermodynamics and taking into consideration the expressions for determining the internal energy of gases in both zones:

$$\begin{aligned} & \frac{dQ_{pl}}{d\varphi} - (u_{nh} + R_{nh}T_{nh}) \frac{dm_{nh}}{d\varphi} - \frac{dQ_{zh}}{d\varphi} = \\ & = m_{zh} \left( \frac{du_{zh}}{dT_{zh}} \frac{dT_{zh}}{d\varphi} + \frac{du_{zh}}{dp} \frac{dp}{d\varphi} \right) + u_{zh} \frac{dm_{zh}}{d\varphi} + p \frac{dV_{zh}}{d\varphi}, \end{aligned} \quad (29)$$

$$\begin{aligned} & (u_{nh} + R_{nh}T_{nh}) \frac{dm_{nh}}{d\varphi} - \frac{dQ_{zh}}{d\varphi} = \\ & = m_{nh}c_{epv} \frac{dT_{nh}}{d\varphi} + u_{nh} \frac{dm_{nh}}{d\varphi} + p \frac{dV_{nh}}{d\varphi}. \end{aligned} \quad (30)$$

In order to derive  $dQ_{pl}/d\varphi$  from these two equations, one must assign in advance such magnitudes as  $dT_{zh}/d\varphi$ ,  $dT_{nh}/d\varphi$ ,  $dm_{zh}/d\varphi$ ,  $dm_{nh}/d\varphi$ ,  $dV_{zh}/d\varphi$ ,  $dV_{nh}/d\varphi$ .

A change in volume can be determined from ratio  $dV/d\varphi = dV_{nh}/d\varphi + dV_{zh}/d\varphi$ . A change in mass is calculated using (5). Assuming  $m / (m - m'_{p0}(1 + \lambda_0)) = A_1$ , we shall obtain:

$$\frac{dm_{zh}}{d\varphi} = A_1(1 - \lambda_0) \frac{dm_{pl}}{d\varphi}. \quad (31)$$

Ratios (29) to (31) together constitute the analytical supplement to the experimental information flow that should be sent to a computer. The character of change in heat transfer in the process of fuel combustion has been represented above by charts in Fig. 10.

### 5. Discussion of results of studying the processes of heat formation and heat transfer

Among the set of analytical-experimental studies, one can select such in which basic regime parameters  $n_e$  and  $p_e$  are almost (up to the second sign after the decimal point) equal. Next, one can average additional regime parameters corresponding to them. Among them, it makes sense to select the following:  $p_{sr}$ ,  $T_{sr}$  and  $\alpha_{sr}$  are the values for pressure, temperature, and a coefficient of heat transfer, averaged over a zone of high pressure;  $p_{max}$ ,  $T_{max}$  and  $\alpha_{max}$  are the maximum values for pressure, temperature, and a heat transfer coefficient. These parameters for the six modes of engine operation are summarized in Table 1.

Among the parameters listed in the Table are the dimensionless quantities  $p_{max}/p_{sr}$ ,  $T_{max}/T_{sr}$ ,  $\alpha_{max}/\alpha_{sr}$ ,  $K = \alpha_{sr}T_{sr}/(p_{sr}c_m)$ ,  $K_m = \alpha_{max}T_{max}/(p_{max}c_m)$ . Here,  $c_m = Sn_e/30$  [m/s] is the average speed of a piston,  $S$  is the piston motion. Here,  $S = 82.6$  mm,  $n_e = 2400 \text{ min}^{-1}$  and the average piston speed accepts a value of  $c_m = 6.608$  m/s. These parameters make it possible to formally compare the different modes of engine operation and to recognize the peculiarities of processes of heat formation/heat consumption under different loads. The magnitude of  $T_{max}/T_{sr}$ , for example, decreases with increasing  $p_e$  (the weight of the local, so to speak, is reduced), while the magnitudes of  $p_{max}/p_{sr}$  and  $\alpha_{max}/\alpha_{sr}$ , by contrast, increase (a local extremum becomes more pronounced against a general, let us put it this way, background). Parameters  $K$  and  $K_m$  define engine operation modes as rather similar. And we can still argue on that the mode under load  $p_e = 0.30$  MPa is the most heat-intensive in a sense that it is matched by the largest value for magnitude  $K$ . Measure  $KK_m$  defines a special mode under load  $p_e = 0.62$  MPa.

Table 1

Regime parameters

$p_e$ , MPa	0	0.10	0.18	0.30	0.45	0.62
$p_0$ , MPa	0.0971	0.0971	0.0971	0.0971	0.0971	0.0971
$p_{max}$ , MPa	0.3423	0.4965	0.7599	1.2601	1.8162	2.3212
$p_{sr}$ , MPa	0.2033	0.2847	0.3706	0.5144	0.6900	0.9363
$p_{max}/p_{sr}$	1.68	1.74	2.05	2.45	2.63	2.48
$T_0$ , K	303.4	306.3	306.1	306.3	312.1	308.4
$T_{max}$ , K	1,820.92	1,967.01	2,069.46	2,277.44	2,341.46	2,223.53
$T_{sr}$ , K	863.49	1,054.62	1,123.72	1,224.18	1,287.72	1,226.62
$T_{max}/T_{sr}$	2.11	1.87	1.84	1.86	1.82	1.81
$\alpha_{max}$ , W/(m <sup>2</sup> K)	170	220	315	495	675	800
$\alpha_{sr}$ , W/(m <sup>2</sup> K)	107.0	133.6	164.0	211.0	265.5	339.5
$\alpha_{max}/\alpha_{sr}$	1.59	1.65	1.92	2.35	2.54	2.36
$K$	0.0688	0.0749	0.0753	0.0760	0.0750	0.0673
$K_m$	0.137	0.132	0.130	0.135	0.132	0.116
$KK_m$	0.0094	0.0099	0.0098	0.0102	0.0099	0.0078

Applying mainly the diagrams of change in temperature and pressure makes it possible to achieve a high praxeological level of modeling working processes in the rapid internal combustion engine.

In the future there are no obstacles to improve the proposed methodology for the model-experimental representation of processes of heat formation and heat consumption. Of course, it would be possible to apply a model that implies the division into three, or more, zones, but in that case one would need to manage a much larger number of unknown parameters. One could also resort to describing heat formation using a set of two, and more, Wiebe functions.

In general, the issue related to rational (optimal) correlation and combination in a single model of both the virtual and natural is not yet resolved to the end. One should seek to ensure that the full-scale model workspace of the internal combustion engine is standardized based on specific criteria. That would make it possible to align the parallel cooperation between different research centers and to organize a continuous exchange of the acquired scientific information. An example that could be considered is the technology of using standardized test running cycles for determining a car excellence – its operational efficiency, fuel consumption, environmental friendliness.

Based on the general theoretical description of the phenomenon of heat transfer, one typically relies on the Newton's law and the theory of similarity of the processes of forced convective heat exchange with the obligatory use of the notion of a heat transfer coefficient. In this case, the main driver (the difference of potentials) is a temperature gradient (difference). But other potentials could also serve the main drivers. Moreover, a coefficient of convective heat transfer in a natural environment is only an abstraction, without which, however, it is not possible to navigate the virtual medium. Estimating such a theoretically meaningful magnitude employs indirect measurements. Thus, it would be reasonable to eliminate this notion in the theory of heat transfer. And that, of

course, will require a certain adjustment of the scientific paradigm that would involve efforts by many scientists. The interpretation of model views, proposed in this work, might be a convenient basis for the implementation of this grand plan.

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## 6. Conclusions

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1. Application of a two-zone model to study processes inside the engine is appropriate, effective, useful. In the case of a homogeneous interpretation of pressure and temperature, it becomes possible to refuse the analytical control over the so-called chemical equilibrium of combustion products. In fact, in this case, there is no reason that would predetermine a materials exchange between the two zones, and thus a heat transfer to the walls of the workspace could be determined using an example of a single-zone model.

2. A mathematical model of thermodynamic processes in the engine working space could be adjusted to rather simple narratives that have praxeological attributes. Specifically, the analytical component of the model itself should be appropriately built based on the classical analytical ratios, reflecting the law of conservation of matter, the law of conservation of energy, the law of heat transfer, the equation of the thermodynamic state of a working body. The desired flexibility in the model is provided through the simulation in the programming environment of interaction between two zones, as well as between them and the environment, into which the engine workspace is divided. The model acquires specificity as it collects information from a real information space in real time based on

the theory of similarity. The essentially different means of simulation function together as a single unit, rather than serving each individually to solve a particular separate problem.

3. The most rational description of the process of heat formation is the analytical Wiebe representation, which is characterized by substantial visibility, structural simplicity, qualitative/quantitative adequacy and which allows further improvement.

4. An analysis of patterns in the course of processes of heat transfer/heat release, applying the proposed model-experimental means, testifies to a notable manifestation of internal heat exchange under almost all possible modes of engine operation. Moreover, the internal flows of heat are not related transparently enough to a load on the engine, and the prediction of their manifestation cannot be elucidated by any general theory. In addition, the character of change in a heat transfer coefficient under different modes of engine operation is so special in general that it defies modern capabilities of analytical representation. The same applies when one argues about heat transfer to the outside. Interestingly, the differential heat transfer to the surface of a cylinder sleeve, in the qualitative sense, is better connected to the pattern of temperature in the workspace, while the differential heat transfer to the surface of a cylinder lid a piston head – to the pattern of pressure in the working body. However, classic theory disregards this at all. All the foregoing allows us to recognize the exceptional praxeologicity in the technology of modelling working processes in the rapid internal combustion engine given that this model incorporates an actual workspace and that there does not exist a more efficient alternative.

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