1. Introduction

Automatic control of nonlinear objects is a well-known scientific problem in the theory of automatic control. Synthesis of nonlinear automatic control systems (ACSs) typically requires taking into account the features of each particular object. A known method for synthesizing nonlinear ACSs is the method of inverse dynamics [1]. It makes it possible to synthesize ACSs of high dynamic accuracy provided there is a precise model of the dynamics of a nonlinear object of control. The synthesis problem is even more complicated in cases when ACS of a nonlinear object operates under conditions of uncertainty: structural, parametrical, or disturbing influences. The inverse controller of ACS requires the application of appropriate compensation contours. Otherwise, uncertainties would lead to a significant deterioration in the quality of ACS.

A typical example of a nonlinear object, which operates under conditions of uncertainty, is a tethered underwater remotely operated vehicle (ROV). Tethered ROVs operate as part of underwater complexes with flexible tethers [2]. The conditions of uncertainty are formed by the following factors. First, these are parametric uncertainties. The mathematical models of ROVs are developed sufficiently enough to synthesize ACSs, for example, by the method of inverse dynamics. However, the procedure for determining their parameters is complicated, it requires conducting pool and marine full-scaled tests, and its implementation is not always possible in practice [3]. Second, these are the uncertainties
created by the environment. An underwater vehicle moves in water; only an approximate assessment of its effect on ROV is possible. Third, ROV is mechanically connected via a flexible tether – a tether-cable (TC) – to a surface support vessel (SV). Any displacements of SV affect the motion of ROV. It is possible to assess these effects using an appropriate mathematical model of flexible tether [4]. However, the accuracy of such an assessment depends strongly on measuring the movements of SV, a water flow diagram, parameters of TC, etc. The cumulative impact of uncertainties in the control of non-linear objects, specifically ROVs, could lead to a significant deterioration in the quality of automatic control: the emergence of overregulation, a static control error. Overregulation poses a threat of ROV hitting the ground or underwater objects. The existence of a static error compromises quality or completely prevents underwater operation completion. Improving the method of inverse dynamics for operations under conditions of uncertainty would make it possible to synthesize high-precision ACSs of nonlinear objects, including remotely operated underwater vehicles.

2. Literature review and problem statement

Paper [5] proposed ACS of spatial movement of ROV based on a PID controller. The authors suggested an algorithm for positioning the vehicle relative to the underwater object by determining the distance based on the image from a video camera. The quality of control processes depends on the adjustment of parameters for the controller.

Study [6] proposed solving a task on the motion control of ROV by using ACS based on a multidimensional adaptive PID controller. Parameters for the controller are set by a neural network. The process of adjusting the proposed system is complicated by a possibility for the value of a loss function to enter the vicinity of a local minimum.

Authors of [7] proposed ACS of spatial movement of RUV with a high dynamic accuracy. Motion control of the vehicle is carried out by a multidimensional PID controller whose robust properties are enabled by entering the mode of “sliding”. The process of control of the ROV motion when using such a system is accompanied by the effect of a high-frequency stepwise change in the signals of control of propulsion devices, which leads to a decrease in the duration of their service life.

A multidimensional motion speed controller for unmanned vehicle under the mode of sliding is proposed in [8]. Compensation for the effect of the attached masses of the environment on velocity motion parameters is executed by decomposing the matrix of inertial characteristics of the object of control. The authors note that this controller is feasible only at low motion speeds of the object and in the absence of external disturbances.

Paper [9] suggested ACS of spatial movement of ROV based on a fuzzy controller. The quality of control processes, which are enabled by such a system, is significantly reduced under conditions of action of external disturbances.

Study [10] proposed ACS of spatial movement of ROV based on a neural-fuzzy controller that could compensate for the impact of nonlinearity of propulsion devices of the type “a zone of insensitivity”. However, high accuracy of control process is achieved by reducing the dynamic range of propulsion devices.


Authors of [12] proposed a motion control system for the system “manipulator – underwater vehicle” based on a linear robust controller. The impact of uncertainties and external disturbances is eliminated by the circuit that includes a mathematical model of the control object. The accuracy of model parameters significantly affects the quality of transients.

An analysis of the scientific literature reveals the following. Control of non-linear moving objects under conditions of uncertainty employs methods for synthesizing robust controllers. However, they do not provide for a high dynamic accuracy of control, or operate under the mode of a high-frequency switching of controlling influence. To ensure precise control, one would need information on the structure and parameters of a mathematical model, as well as on the numerical values for disturbing influences. The necessity to synthesize ACS, capable to ensure high accuracy of control under conditions of uncertainty, predetermines the relevance of our study.

3. The aim and objectives of the study

The aim of the study is to improve the method of inverse dynamics by collecting uncertain influences into a separate variable and to find its numerical value using a gradient method. This would make it possible to synthesize automatic control systems of non-linear objects with high dynamic accuracy under conditions of uncertainty.

To accomplish the aim, the following tasks have been set:
- to improve the classic method of inverse dynamics in order to synthesize ACSs of nonlinear objects under conditions of uncertainty;
- to synthesize the law of compensation for uncertainty;
- to synthesize a precise ACS of a one-dimensional motion of ROV;
- to examine the operation of the synthesized ROV ACS under conditions of uncertainty.

4. Materials and methods for the improvement of the method of inverse dynamics for automatic control of a nonlinear object under conditions of uncertainty

4.1. Classic method of inverse dynamics

Let the dynamics of a one-dimensional object of control be described by a nonlinear differential equation of the N-th order [1]:

\[ f \left( y^{(n)}, \lambda^{(s)}, u \right) = 0; \]

\[ n = 0, 1, 2, \ldots, N; \quad s = 0, 1, 2, \ldots, S; \quad q_s = 0, 1, 2, \ldots, Q_s, \]

where \( y \) is the controlled magnitude, \( \lambda_s \) is the disturbing influence, \( S \) is the total number of disturbing influences, \( Q_s \) is the order of the \( s \)-th disturbing influence, \( u \) is the controlling influence, \( N \) is the order of control object. Upper indexes \( (n) \) and \( (q_s) \) denote the orders of derivatives for time.

The magnitude \( g^{(n)} \) is the senior derivative from the controlled magnitude of the object. It represents the velocity of change in phase coordinate \( y^{(N-1)} \) and explicitly depends on controlling influence \( u \).
For the movement of an object along the desired trajectory, the following condition should be satisfied at any time point:

\[ y_d^{(N)}(t) = y^{(N)}(t), \]

where \( y^{(N)} \) is the desired value for the senior derivative from the controlled magnitude.

Parameter \( y^{(N)} \) is determined from the reference model that relates the controlled magnitude \( y \) to control task \( y_d(t) \). A reference model is assigned by a differential equation with the desired dynamic characteristics:

\[ y_d^i = \Phi \left( y^{(i)}, y_d^{(i)} \right); \quad i = 0, 1, 2, \ldots, N - 1; \quad j = 0, 1, 2, \ldots, J, \quad (1) \]

where \( f \) is the senior derivative from control task \( y_d \).

To ensure the high dynamic accuracy of control, the reference model contains derivatives from control task \( y_d \) in this case, the condition \( f \geq N \) must be met [13].

The mathematical model of the control object yields its inverse model:

\[ u = f \left( y^{(N)}, y^{(i)}, \lambda^{(s)} \right), \]

\[ i = 0, 1, 2, \ldots, N - 1; \quad s = 0, 1, 2, \ldots, S; \quad q = 0, 1, 2, \ldots, Q. \]

The desired value for a controlling influence is found by substituting \( y^{(N)} \) instead of \( y^{(N)} \) in the inverse model of the object:

\[ u = f \left( y^{(N)}, y^{(i)}, \lambda^{(s)} \right). \]

The input of the inverse model must receive information on disturbing influences. If the inverse model is not reliable or there is no information about the disturbance, ACS will not be able to ensure a high dynamic accuracy of control.

### 4.2. Synthesis of the generalized structure of control law to operate under conditions of uncertainty

Let the dynamics of a control object be exactly described by the following nonlinear differential equation:

\[ y^{(N)} = f_{\text{ref}} \left( u, y^{(i)}, \lambda^{(s)} \right). \]

Then the ideal controlling influence, which could ensure the high dynamic accuracy of control, can be determined based on a reference model (1), and the absolutely precise inverse model of a control object:

\[ u_{\text{ideal}} = f_{\text{inv. ref}} \left( y^{(N)}, y^{(i)}, \lambda^{(s)} \right). \]

When constructing mathematical models for control objects, certain assumptions are made. As a result, the dynamics of a control object is modeled imprecisely:

\[ y_{\text{est}}^{(N)} = f_{\text{est}} \left( u, y^{(i)}, \lambda^{(s)} \right); \]

\[ y_{\text{est}}^{(N)} \neq y^{(N)}. \]

According to the method of inverse dynamics, controlling influence is determined based on the reference model (1) and the imprecise model of a control object \( f_{\text{est}}(\cdot) \):

\[ u_{\text{est}} = f_{\text{inv. est}} \left( y_{\text{est}}^{(N)}, y^{(i)}, \lambda^{(s)} \right). \]

\[ p = 0, 1, 2, \ldots, P; \quad q = 0, 1, 2, \ldots, Q, \]

where \( \lambda_p \) is the disturbing influence whose magnitude is determined.

In a general case, \( u_{\text{est}} \neq u_{\text{ideal}} \). Quality of control would depend on the extent to which \( f_{\text{inv. est}}(\cdot) \) matches \( f_{\text{inv. ref}}(\cdot) \) and how insignificant the unidentified disturbing influences are.

We shall introduce an uncertain component of controlling influence so that it is possible to equate \( u_{\text{est}} \) to \( u_{\text{ideal}} \):

\[ u_n - u_s = u_{\text{ideal}}, \]

where \( u_s \) is the uncertain component of the controlling influence.

Then, the ideal signal of control can be obtained based on the imprecise inverse model of control object under uncertain disturbing influences:

\[ u_{\text{ideal}} = f_{\text{inv. est}} \left( y^{(N)}, y^{(i)}, \lambda^{(s)} \right) - u_s. \]

Parameter \( u_s \) changes dynamically in the process of control and characterizes the manifestation of all the uncertainties of control object in totality.

The reference model (1) and the inverse model (3) form the generalized law of control of a nonlinear one-dimensional object under conditions of uncertainty.

### 4.3. Synthesis of the law for the compensation of uncertainties

Because parameter \( u_s \) is uncertain, then it is impossible to obtain at the output from ACS controller a signal of control equal to \( u_{\text{ideal}} \). Given this, \( u_s \) is proposed to be determined from the condition of approaching: \( u \to u_{\text{ideal}} \):

\[ u = f_{\text{inv. est}} \left( y_{\text{est}}^{(N)}, y^{(i)}, \lambda^{(s)} \right) - u_s; \quad u_D \to u_s, \]

where \( u_D \) is the signal to compensate for uncertainties.

Parameter \( u_D \) defines the extent to which the inverse model \( f_{\text{inv. est}}(\cdot) \) differs from the ideal inverse model \( f_{\text{inv. ref}}(\cdot) \) of control object. At any point in time, it has to compensate for the structural and/or parametrical inaccuracies in the mathematical model of control object, inaccurately measured and/or uncertain disturbing influences.

In order to ensure that \( u_D \to u_s \), we suggest ensuring \( y_{\text{est}}(t) \to y(t) \). In this case, \( y_{\text{est}}(t) \) must be determined based on the model of control object taking into account the parameter \( u_D \):

\[ y_{\text{est}}^{(N)} = f_{\text{est}} \left( u + u_s, y_{\text{est}}^{(i)}, \lambda^{(s)} \right). \]

Given this, we propose introducing to the structure of ACS a model of control object \( f_{\text{est}}(\cdot) \). The structure of such an ACS is shown in Fig. 1.

The structure of ACS’s controller includes the reference model, the control object inverse model, the control object dynamics model, and the uncertainties compensation unit.

The task of the latter is to compute magnitude \( u_s \) in order to ensure \( y_{\text{est}}(t) \to y(t) \).

The law of compensation for uncertainties, which would ensure \( y_{\text{est}}(t) \to y(t) \) and on whose basis the uncertainties compensation unit in ACS would work, will be synthesized by the method of minimizing local functionals [14]. To be
minimized is functional \( G(u_D) \), which is a normalized energy of the senior derivative from the output of model for the control object’s dynamics:

\[
G(u_D) = \frac{1}{2} \left[ y_m^{(N)} - y_a^{(N)} \right]^2, \tag{5}
\]

where \( y_m^{(N)} \) is the desired value for the senior derivative from the output of control object dynamics model.

The magnitude \( y_m^{(N)} \) is derived from a reference model of the circuit that compensates for uncertainties:

\[
y_m^{(N)} = \Phi \left( y_a^{(i)}, y_D^{(j)} \right); \quad i = 0, 1, 2, \ldots, N-1; \quad j = 0, 1, 2, \ldots, J. \tag{6}
\]

If one draws an analogy with the reference model (1), then the control task in (6) is the magnitude \( y \), and a controlled parameter is the magnitude \( y_m \).

An absolute minimum of functional \( \min G(u_D) = 0 \) is reached under condition \( u_D = u_a \). To this end, parameter \( u_a \) should be determined, which contradicts the set task to synthesize ACS under conditions of uncertainty. Therefore, at each point in time \( t \geq 0 \), the value for functional \( G(u_D) \) must be within a small vicinity of the extremum-minimum.

A signal of the compensation for uncertainties \( u_D \) will be computed via a gradient search for an extremum \( \min G(u_D) \) to 0:

\[
\frac{du_a(t)}{dt} = \lambda \left( \frac{\partial G(u_a)}{\partial u_D} \right), \quad \lambda = \text{const};
\]

\[
\frac{\partial G(u_D)}{\partial u_D} = -\left( y_m^{(N)} - y_a^{(N)} \right) \frac{\partial y_a^{(N)}}{\partial u_D},
\]

where \( \lambda \) is the parameter employed to assign the rate of a gradient search.

The differential law of the compensation for uncertainties takes the following form:

\[
\frac{du_a(t)}{dt} = \sigma k \left( y_m^{(N)} - y_a^{(N)} \right); \quad \sigma = \text{sign} \left( \frac{\partial y_a^{(N)}}{\partial u_D} \right); \quad k > 0, \tag{7}
\]

where \( k \) is the coefficient that defines the speed of performance of the circuit to compensate for uncertainties; \( \sigma \) is the parameter that implements the rule of signs, which ensures stability of the circuit to compensate for uncertainties [14].

Performance speed of circuit (7) must be much higher than the performance speed of reference model (6). However, a significant increase in the coefficient \( k \) can cause the loss of stability in the process of solving numerically the differential equations of the circuit to compensate for uncertainties. Thus, parameter \( k \) is chosen so that the performance speed of circuit (7) is much slower than the time quantization period in the computer implementation of ACS controller. Next, we set the performance speed for the reference model of a compensation circuit (6). And the last step is to assign the performance speed of a reference model of ACS (1); it should be much lower than the performance speed for the reference model of a compensation circuit (6).

The differential law of compensation (7) allows a decrease in order. If it is integrated over time under zero initial conditions, then we obtain

\[
\begin{align*}
    u_a(t) &= \alpha k \left( y_m^{(N)} - y_a^{(N)} \right); \quad k > 0; \\
    y_m^{(N)} &= \Phi \left( \int y_a(t) dt; y_a^{(j)} \right), \quad j = 0, 1, 2, \ldots, J.
\end{align*}
\tag{8}
\]

The reference model (6) was also integrated and introduced to the structure of the law (8). The law of compensation (8) allows for ACS to work under conditions of uncertainty and, at the proper choice of reference models, provides for a high dynamic accuracy of control of a nonlinear object of the \( N \)-th order. Its application makes it possible to improve ACSs that were synthesized by the method of inverse dynamics. Thus, the classic method of inverse dynamics has been further elaborated in order to synthesize ACSs that operate under conditions of uncertainty.

### 4. 4. Synthesis of ACS of a tethered remotely operated underwater vehicle under conditions of uncertainty

#### 4. 4. 1. Mathematical model of ROV

The studied control object represents ROV of the project “Freight self-propelled underwater carrier” [15]. Its one-dimensional vertical motion is described by differential equations of a propulsion device and of translational motion of the hull of ROV, which build system [13]:

\[
\begin{align*}
    J \ddot{\omega} &= M_{em}(\omega) - k_r \omega - M_p(\omega, \dot{\omega}) \\
    M_{em} &= c_1 (u - c_2 \omega) \\
    (m + \lambda) \ddot{y} &= nF_p(\omega, \dot{\omega}) - F_b(\dot{y}) - F_e(x, y, z) \\
    F_b &= \text{sign}(\dot{y}) 0.5 p S_k \beta g^2.
\end{align*}
\tag{9}
\]

where \( J \) is the moment of inertia for the elements of a propulsion device reduced to propeller; \( \omega \) is the rotation speed of the propeller; \( M_{em} \) is the electromagnetic torque of electric motor reduced to the propeller; \( u \) is the control signal; \( k_r \) is the coefficient of resistance to the rotation of the electric motor’s rotor in a liquid dielectric; \( M_p \) is the hydrodynamic braking torque of the propeller; \( y \) is the coordinate of ROV (controlled magnitude); \( \lambda \) is the mass of ROV; \( \lambda \) is the attached mass of water; \( n \) is the...
number of vertical propulsion devices, at one-dimensional motion, we assume that each propulsion device receives the same controlling influence \( u \); \( F_p \) is the propeller’s resistance (controlling force); \( F_h \) is the hydrodynamic resistance force of ROV’s hull; \( F_u \) is the disturbing force of TC; \( \rho \) is water density; \( S_p \) is the characteristic area of ROV; \( \text{sign}(\cdot) \) is the function of taking a sign. Points above variables indicate time derivatives.

Disturbing influence \( F_u \) is modeled using a mathematical model of TC dynamics [4]. Coefficients \( k_p, k_u \) are derived from the mathematical model of a direct current electric motor [16]. Dependences \( M_u(\omega, \dot{y}) \) and \( F_p(\omega, \dot{y}) \) represent a mathematical model of the propeller and are essentially non-linear. They include a variable \( \omega \dot{y} \) and the coefficients that are dependent nonlinearly on \( \omega \) and \( \dot{y} \) and are determined from the curves of propeller action [3].

Parameters for the ROV mathematical model are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of ROV’s hull (three-axial ellipsoid):</td>
<td></td>
</tr>
<tr>
<td>length ( l )</td>
<td>2.7 m</td>
</tr>
<tr>
<td>width ( w )</td>
<td>1.3 m</td>
</tr>
<tr>
<td>height ( h )</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Characteristic area of hull ( S_p )</td>
<td>2.76 m²</td>
</tr>
<tr>
<td>Coefficient of hydrodynamic resistance ( k_d )</td>
<td>1.45</td>
</tr>
<tr>
<td>ROV mass ( m )</td>
<td>1.129 kg</td>
</tr>
<tr>
<td>Attached mass of water at vertical motion ( \lambda )</td>
<td>1.891 kg</td>
</tr>
<tr>
<td>Buoyancy of ROV</td>
<td>Null</td>
</tr>
<tr>
<td>Propulsion device</td>
<td></td>
</tr>
<tr>
<td>Moment of inertia of propulsion device ( J )</td>
<td>0.086 kg·m²</td>
</tr>
<tr>
<td>Gain coefficient of controlling influence ( k_u )</td>
<td>310 V</td>
</tr>
<tr>
<td>Range of permissible controlling influences of propulsion device ( u )</td>
<td>([-1, 1])</td>
</tr>
<tr>
<td>Coefficient of resistance to the rotation of electric motor’s rotor in a liquid dielectric ( k_m )</td>
<td>0.017 N·m·s⁻¹</td>
</tr>
<tr>
<td>Coefficient ( c_1 )</td>
<td>0.217 N·m·V⁻¹</td>
</tr>
<tr>
<td>Coefficient ( c_2 )</td>
<td>1.891 V·s⁻¹</td>
</tr>
<tr>
<td>Number of vertical propulsion devices ( n )</td>
<td>4</td>
</tr>
<tr>
<td>Range of thrusts of propulsion device (in mooring mode)</td>
<td>([-126, 205]) N</td>
</tr>
<tr>
<td>Tether-cable</td>
<td></td>
</tr>
<tr>
<td>Length of tether-cable</td>
<td>100 m</td>
</tr>
<tr>
<td>Diameter of cross-section of tether-cable</td>
<td>20 mm</td>
</tr>
<tr>
<td>Coefficient of normal component of TC hydrodynamic resistance</td>
<td>1</td>
</tr>
<tr>
<td>Coefficient of tangential component of TC hydrodynamic resistance</td>
<td>0.1</td>
</tr>
<tr>
<td>Position of ROV in the horizontal plane:</td>
<td></td>
</tr>
<tr>
<td>coordinate ( x )</td>
<td>0 m</td>
</tr>
<tr>
<td>coordinate ( y )</td>
<td>0 m</td>
</tr>
<tr>
<td>Horizontal motion speed of TC in water flow</td>
<td>1.5 m/s</td>
</tr>
<tr>
<td>Disturbing influence of TC ( F_u )</td>
<td>to 220 N</td>
</tr>
</tbody>
</table>

The parameters for the electric motor and propeller are chosen according to a mathematical model of the propulsion device with a capacity of 500 W [16]. A large value for the attached mass of water is due to the flat shape of ROV’s hull [3]. The mathematical model of the propulsion device in the general case is of second order. However, electrical processes in the propulsion device proceed much faster than mechanical processes. Therefore, it is possible to use a model of the first order, which predetermines the form of a differential equation of the propulsion device [17].

### 4. 4. 2. Synthesis of the system of automatic control of ROV under conditions of uncertainty

In order to synthesize ACS by the method of inverse dynamics, it is necessary to have an inverse model of the control object. We shall assume that based on the results of identification we have obtained the following model of the control object’s dynamics:

\[
K_u \dot{y} = K_m u - \text{sign}(y) K_y \dot{y}^2; \quad K_m = 3020 \text{ kg};
\]

\[
K_y = 700 \text{ N}; \quad K_y = 2049 H \cdot (\text{m/s})^{-2}; \quad K_y = 700 \text{ N};
\]

(10)

where \( K_{m,u,y} \) are the coefficients of the model.

The model presented is the object of the second order. Its structure and parameters are deliberately chosen such that they are significantly different from the structure and parameters of the control object (9). First, they differ by one order, which introduces structural uncertainty. Second, that part of the model that is responsible for forming the controlling force has been greatly simplified, and the chosen coefficient \( K_u \) is a generalized estimate of the thrust of vertical propulsion devices. This introduces the structural and parameter uncertainties. It is worth noting that parameters \( K_m \) and \( K_y \) match the object (9) and were computed based on data from Table 1. Third, model (10) does not account for the disturbing influences exerted by the tether-cable \( F_u \). That creates conditions for the uncertainty of disturbing influences.

We shall choose a reference model of ACS in the form of a linear differential equation of second order with a unity gain factor and a unity dampening factor. Take the senior derivative from it:

\[
\ddot{y}_d = \frac{2}{T_i^2} (\dot{y}_d - \dot{y}) + \frac{1}{T_i} (y_d - y).
\]

(11)

where \( T_i \) is the time constant of ACS reference model.

Based on (4), we write down the equation of controlling function that compensates for uncertainties:

\[
u = \frac{1}{K_m} (K_u \dot{y} + \text{sign}(y) K_y \dot{y}^2) - u_p.
\]

(12)

To compute \( u_p \), we choose a reference model for the compensation circuit in the form of a second-order differential equation with a unity gain factor and a unity factor of dampening. We shall take the senior derivative from the reference model – the desired acceleration \( \ddot{y}_d \) – and integrate it:

\[
\dot{y}_{ds} = \ddot{y}_d + \frac{2}{T_m} (y - y_s) + \frac{1}{T_m} \int (y - y_s) dt,
\]

(13)

where \( T_m \) is the time constant of the reference model for the compensation circuit.

Based on (8) and (10), we write down the law of uncertainties compensation:
\[ u_d = \sigma k (y_m - y_u); \quad \sigma = 1; \quad k = \frac{cK_s}{K_{y_m}}; \quad c > 1; \]
\[ \dot{y}_u = \frac{1}{K_k} [K_s (u + u_d) - \text{sign}(\dot{y}_u) K_s \dot{y}_u^2], \]

(14)

where \( c \) is the parameter based on which one selects \( k \) according to recommendations [14].

The law of control of a nonlinear second-order object under conditions of uncertainty consists of:

- reference model of ACS (11);
- controlling function (inverse model of the object) (12);
- the reference model of the compensation circuit (13);
- the law of uncertainties compensation (14).

The time constant \( T_{rm} \) is derived based on the following. Because the object of control (9) is of third order, \( T_{rm} \) should be several times larger than the time constant of propulsion device \( T_p \). Otherwise, the process that compensates for uncertainties would be unstable. The estimation of the dynamics of transient processes in a propulsion device has made it possible to establish that \( T_p \approx 0.25 \text{ s} \). Given this, we choose \( T_{rm} = 0.75 \text{ s} \). Time constant \( T_p \) is selected to be several times greater than \( T_{rm} \); \( T_p = 4.5 \text{ s} \). Assuming \( c = 10 \text{ s}^{-1} \), we obtain \( k = 69 \text{ m}^{-1} \). A set of the derived parameters forms the settings for ACS controller.

Control law (14) contains an integrator. Its work shall be organized according to the principle “integration under condition”. If \( u \) exceeds the permissible limits, the integrator will retain the last value that it received at its output prior to entering the zone of saturation. That will make it possible to adjust the operation of ACS in the presence of nonlinearity of the type of constraint [18].

5. Results of studying the ACS with high dynamic accuracy under conditions of uncertainty

By using computer simulation, we examined the dynamics of ACS transients.

The operation of ACS under conditions of uncertainty at a stepwise change in the control task is illustrated in Fig. 2, a.

The output of model \( y_m \) is not given because it almost completely coincides with the output of object \( y \). This means that the circuit that compensates for uncertainties fulfills the set task and ensures that the dynamics of model (10) approaches the dynamics of control object (9). Upon exiting the saturation mode (Fig. 2, b) and eliminating the error, the controlled magnitude \( y \) matches task \( y_g \) (Fig. 2, a).

The operating zone of ROV is limited by the length of TC: \( y \in [0,100] \text{ m} \). Thus, hereafter a relative value for the error is given relative to this range. The duration of a transition process after controlling influence \( u \) exits the zone of saturation and until the error of control is within a 1% corridor, does not exceed 4 s. There is no overregulation.

The operation of ACS under conditions of uncertainty when a control task is changed in line with a harmonious law is shown in Fig. 3, a.

In a given case, the output of model \( y_m \) is also not given because it almost completely coincides with the output of object \( y \).

The duration of a transition process after controlling influence \( u \) exits the zone of saturation and until the error of control is within a 1% corridor, does not exceed 5 s. Following this, \( g(t) \) is asymptotically approaching \( y_g(t) \) with the error not exceeding 0.01%. That testifies to the high dynamic accuracy of the synthesized ACS at a dynamic change in \( y_g(t) \) under conditions of uncertainty.

6. Discussion of results of synthesizing the system of automatic control of a non-linear object of high dynamic accuracy

The synthesized ACS makes it possible to control essentially non-linear objects under conditions of uncer-
tainty. In this case, the character of a signal to compensate for uncertainty allows the estimation of adequacy of the inverse model of a control object: the more precisely the model of a control object is identified, the smaller the value for a signal of compensation $w_d$. However, this statement holds in the absence of the uncertain external disturbances.

The synthesized control law implies the measurement or computation of derivatives from the control object. In some cases, this is a disadvantage, however, it is not critical for ROVs since they are equipped with sensors of coordinates and speed.

This work has shown that the improved method of inverse dynamics ensures a high dynamic accuracy of control, including under conditions of structural uncertainty when the order of the object is higher than the order of control law. However, this is possible only under condition for selecting the time constants for the reference models of control law that are several times larger than the time constants for the higher orders of control object. That imposes some limitations on the performance speed of ACS.

It is possible to improve the performance speed of ACS by synthesizing a control law using the method of inverse dynamics with the compensation for uncertainties of the respective $N$-th order. However, in this case, it is necessary to measure, among others, a derivative from the controlled magnitude of order ($N$−1). In terms of control of ROV, that means that one needs to measure (or compute) its acceleration. Since such a possibility is technically feasible, then the improved method of inverse dynamics appears promising in order to synthesize high-precision ACSs of marine moving objects.

Simulation results have demonstrated high dynamic accuracy of ACS at a stepwise and sinusoidal change in the controlling influence under conditions of structural and parametrical uncertainties in a control object model and under the influence of uncertain disturbances from a tether-cable. The resulting ACS is planned to be implemented into the control system of ROV of the project “Freight self-moving underwater carrier”. Advancing the obtained results implies the synthesis of a multi-dimensional ACS to control the spatial motion of ROV under conditions of uncertainty.

7. Conclusions

1. We have improved the method of inverse dynamics by introducing to the composition of the inverse control law a signal of compensation in order to synthesize high-precision nonlinear systems of automatic control under conditions of uncertainty.

2. We have synthesized the law of uncertainty compensation based on the application of an imprecise model of control object and the method of minimizing local functionals. Its application makes it possible to maintain high dynamic accuracy of the inverse control law at operation under conditions of uncertainty.

3. We have synthesized the system of automatic control of vertical motion of the remotely operated underwater vehicle of the project “Freight self-moving underwater carrier”, based on the improved method of inverse dynamics. It ensures that the controlled parameter asymptotically approaches the task of control, including at a dynamic change in the latter under conditions of structural and parametrical uncertainties in a control object’s model and under the influence of the uncertain disturbances from the tether-cable.

4. By using computer simulation, we examined the work of the synthesized system of automatic control of one-dimensional motion of the remotely operated underwater vehicle. The simulation results have demonstrated that the duration of transition processes beyond the zone of saturation did not exceed 5 s. At a dynamic change in the control task, a deviation of the controlled parameter did not exceed 0.01 %. That testifies to the high dynamic accuracy of the synthesized ACS under conditions of uncertainty.

References


