Запропоновано аналітичний метод обмеження складності нейро-нечітких моделей, що забезпечують гарантовану точність їх реалізації при апроксимації функцій, які мають дві і більше похідних. Метод дозволяє визначати необхідне мінімальне число параметрів для систем із застосуванням нечіткої логіки та нейронних моделей.

Проведена оцінка необхідного числа нейронів (термів) моделі, що забезпечують необхідну точність наближення площі модельної кривої до системної на ділянках апроксимації функції. Оцінку похибки апроксимації отримано по залишковим членам розкладання в формі Лагранжа площ апроксимованої системної функції в ряд Маклорена. Отримані результати дозволяють визначати необхідне число ділянок апроксимації та кількість нейронів (термів) для забезпечення заданої відносної і абсолютної похибки апроксимації.

Проведена оцінка необхідного числа нейронів (термів), що забезпечують необхідну точність реалізації моделі по максимальному відхиленню між системною і модельною кривими на ділянці апроксимації. Це дозволяє обирати, в залежності від заданої необхідної точності, число термів нечітких змінних, вхідних і вихідних змінних, лінгвістичних правил, координат модальних значень на осях вхідних і вихідних змінних.

Для перевірки правильності запропонованих рішень проведено моделювання системних кривих в середовищі Matlab/Simulink, яке підтвердило гарантовану точність їх реалізації у відповідності до приведених раніше аналітичних розрахунків.

Отримані результати можуть бути застосовані в сучасних інтелектуальних технічних системах керування, контролю, діагностики та прийняття рішення. Використання запропонованих методів по вибору і використанню мінімальної кількості термів (нейронів) сприятиме зменшенню затребуваної обчислювальної потужності в нелінійних системах

Ключові слова: апроксимація, гарантована точність, нечітка логіка, нейронні мережі, імітаційне моделювання

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CONSTRUCTION OF AN ANALYTICAL METHOD FOR LIMITING THE COMPLEXITY OF NEURAL-FUZZY MODELS WITH GUARANTEED ACCURACY

B. Sytnik
PhD, Associate Professor*
E-mail: sitnik@kart.edu.ua

V. Bryksin
PhD, Associate Professor*
E-mail: vladimir.bryksin@gmail.com

S. Yatsko
PhD, Associate Professor**
E-mail: ua.yatsko@gmail.com

Y. Vashchenko
PhD, Senior Lecturer**
E-mail: yaroslav.vashchenko@gmail.com

*Department of Information Technologies***
Department of Electrical Energetics, Electrical Engineering and Electromechanics*
***Ukrainian State University of Railway Transport
Feierbakh sq., 7, Kharkiv, Ukraine, 61050

1. Introduction

It is known that movable technical objects with nonlinear and variable parameters (for example, vehicles, robots, drones) functioning under the influence of disturbances, depending on weight, wind, trajectory, and profile, the obstacles of varying intensity [1]. To execute control over them, it is necessary to nonlinearly correct the parameters in the settings of control system, to form the assigned motion trajectories, to limit the required motion speed in a speed control setpoint, etc. [2].

The use of conventional digital PID-controllers in this case turns out to be not effective enough, given a significant number of random, chaotic, non-linear and fuzzy variable parameters in the characteristic equations of a closed system [3].

To set the desired nonlinear motion trajectories of movable objects and to nonlinearly correct the parameters for setting the controllers in control systems, a guaranteed accuracy in the implementation of their models is needed [4].

In cases when the application of conventional methods to control the system proves to be not efficient enough due to a lack of precise knowledge about the object of control, specialized methods of calculation and decision making are used. These algorithms include adaptive technologies, fuzzy logic, artificial neural networks, genetic algorithms.

2. Literature review and problem statement

In [5], authors derived a simplified analytical assessment of the minimally required number of approximation sections in the curve of motion trajectory of second order by the trapezoidal method with guaranteed accuracy. Such an estimate is missing for curves of higher orders [6]. In order for the law of control to account for all variables that affect the process of control, the adaptive [7], situational [8], fuzzy [9], neural and neural-fuzzy model and regulators [10], as well as genetic algorithms [11] are used. The specified algorithms can improve the properties of control and decision-making systems; however, the efficiency of their functioning is defined by the optimal structure capable of solving the predefined class of tasks. The effectiveness of methods for neural-fuzzy modelling and control improves if they are used in combination with methods based on intelligent machines [12].

An analysis of papers in this field [13–15] has revealed a lack of analytical dependences for determining the complexity of models depending on the assigned accuracy of their implementation.

Paper [13], for example, is based on the research from psychology on that a person with average capabilities can simultaneously keep in memory from 5 to 9 information granules (terms). Thus, when implementing a fuzzy production model, the number of terms for the input and output variables should be chosen from 5 to 7. The number r in production rules is proposed to be derived from formula:

$$r = z^{\omega},$$

where z is the number of fuzzy sets, ω is the number of inputs of x_i model.

The number of parameters p for assigning the membership functions is recommended to choose from formula:

$$p = r + \omega \cdot z = z^{\omega} + \omega \cdot z,$$

and when implementing a model using ANN, the initial number of neurons m in the intermediate layer of ANN is recommended to choose from formula:

$$m = \sqrt{pl},$$

where p and l are the numbers of input and output neurons, respectively.

However, a given recommendation does not make it possible to minimize the number of terms for fuzzy variables or the number of neurons in the intermediate layer of ANN depending on the required accuracy for model implementation.

In [14], the number of neurons m in the intermediate layer of ANN is recommended to be adjusted until achieving the desired accuracy of approximation. For genetic algorithms [15], in the process of ANN learning, achieving the required accuracy of simulation in search procedures also requires the knowledge of the required number of chromosomes, mutations, distances between them, etc.

Thus, as noted, existing methods for determining the minimally required number of terms or neurons fail to provide for a guaranteed accuracy of model implementation.

The solution to a given problem can be obtained based on estimating the area of a model curve approaching the system one or from the maximum deviation between the system and model curves at the section of function approximation.

3. The aim and objectives of the study

The aim of this study is to develop an analytical method for limiting the complexity of neural-fuzzy models that provide for a guaranteed accuracy of their implementation during approximation of functions. This must ensure a decrease in redundancy, as well as the rational selection of the number of terms for fuzzy variables or neurons depending on the assigned required accuracy.

To accomplish the aim, the following tasks have been set:

– to devise a procedure for estimating the required number of neurons (terms) in a model that provide for the necessary precision in approximating the area of a model curve to the system one at the sections of function approximation;

– to devise a procedure for estimating the required number of neurons (terms), which provide for the necessary accuracy of model implementation based on the maximum deviation between the system and model curves at the section of approximation;

– to consider practical implementation of constructing a fuzzy model and a model that uses ANN for a system with the assigned dependence.

4. Model of the system and the development of a method for limiting the complexity of neural-fuzzy models with guaranteed accuracy

Consider a model of the system that is described by a function of high order in the form:

$$y_c = f_c(x) = a_l x^{2l} + a_{l-1} x^{2l-1} + \ldots + a_1 x^1 + a_0, \tag{1}$$

that has $2l$ derivatives, for instance, for $2l \geq 2$:

$$y_c = \mu\sqrt{x_c}, \quad y_c = \mu x_c^{l+1}, \quad x^2 + y^2 = r^2,$$

$$y_c = A\sin(\mu x) \text{ etc.} \tag{2}$$

An example of the chart $y_c = f_c(x)$ for system curve $y_c = \sqrt{x_c}$ and the model curves implemented by the method of rectangles (1) and the trapezoidal method (2) is shown in Fig. 1.

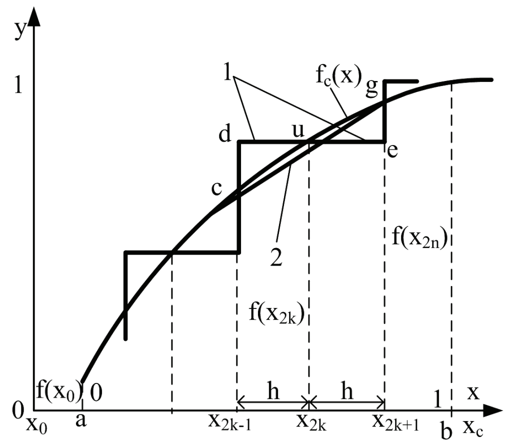

Fig. 1. Dependence charts of approximated curve and approximating curves

4. 1. Estimation of the number of neurons (terms) such that a model curve approaches the system curve

To assess the required number of neurons (terms) in a model that provide for the necessary precision of approximating the area of a model curve to the system curve at the sections of function approximation, we estimated an error of approximation. An estimate of the approximation error can be obtained based on the residual terms of decomposition, in the Lagrange form, of the areas of system function $f_c(x)$ into a Maclaurin series. The abscissa coordinate of the modeled system function x_c and the dimensionless relative abscissa coordinate of model function x are related via interrelation:

$$\frac{(x-0)}{(1-0)}=\frac{(x_c-a)}{(b-a)}, \quad (3)$$

hence:

$$n\Delta_x=nh=\frac{n(x_c-a)}{(b-a)}=\frac{n\Delta_{x_c}}{(b-a)}=1. \quad (4)$$

To calculate the area under curve $f_c(x)$ at the assigned accuracy at segment $(-\Delta_{x_c}, \Delta_{x_c})$, that is the determined integral – the primary function $F_c(x)$, using the interpolation polynomial by Taylor or Maclaurin.

Given the fact that the primary function $F_c(x)$ has three derivatives at segment $(-\Delta_{x_c}, \Delta_{x_c})$, we decompose this function into a Maclaurin series with a residual term in the Lagrange form:

$$F_c(x)=\int_{-\Delta_{x_c}}^{\Delta_{x_c}} f_c(x)dx=F_c(\Delta_{x_c})-F_c(-\Delta_{x_c})=$$
$$=\left[F_c(0)+F_c^{(1)}(0)+F_c^{(2)}(0)\frac{x_c^2}{2}+F_c^{(3)}(x_{2k})\frac{x_c^3}{6}\right]\Bigg|_{-\Delta_{x_c}}^{\Delta_{x_c}}. \quad (5)$$

Considering (5):

$$F_c^{(1)}(0)=f_c(0),\quad F_c^{(2)}(0)=f_c^{(1)}(0),\quad F_c^{(3)}(x_{2k})=f_c^{(2)}(x_{2k}),$$

$$\int_{x_{2k}-h}^{x_{2k}+h} f_c(x)dx\approx 2f_c(x_{2k})\Delta_{x_c}+\frac{\Delta^2 f_c(x_{2k})}{\Delta_{x_c}^2}\frac{(\Delta_{x_c})^3}{3}=$$
$$=2f_c(x_{2k})\Delta_{x_c}+f_c^{(2)}(x_{2k})\frac{(b-a)^3}{3n^3}=2f_c(x_{2k})\Delta_{x_c}+\Delta_c. \quad (6)$$

Thus, the absolute error of approximation Δ_c is:

$$\Delta_c=\frac{\Delta^2 f_c(x_{2k})}{\Delta_{x_c}^2}\frac{(b-a)^3}{3n^3}=\Delta^2 f_c(x_{2k})\frac{(b-a)}{3n}. \quad (7)$$

We obtain from (7) the number n of approximation sections (8) at curve $f_c(x)$ of a neural or fuzzy model at section (a, b) with the assigned error Δ_c approximated based on the formula of rectangles (9) or a trapezoidal formula (10):

$$n=\Delta^2 f_c(x_{2k})\frac{b-a}{\Delta_c}, \quad (8)$$

$$F_c(x)=\int_a^b f(x)\mathrm{d}x=$$
$$=\frac{b-a}{n}\left[f(x_0)+f(x_2)+\ldots+f(x_{2n-2})\right]+\Delta_c, \quad (9)$$

$$F_c(x)=\int_a^b f(x)\mathrm{d}x=\frac{b-a}{n}\left[\begin{array}{l}\frac{f(x_0)+f(x_{2n})}{2}+f(x_2)+\\+f(x_4)+\ldots+f(x_{2n-2})\end{array}\right]+\Delta_c. \quad (10)$$

Similarly, given the absolute error Δ along segment $(-h, h)$, at $a=0$ and $b=1$, we compute area $F(x)$ under curve $f(x)$ by using a decomposition of the primary function $F(x)$ into a Maclaurin series with a residual term in the Lagrange form:

$$F(x)=\int_{-h}^{h} f(x)\mathrm{d}x=F(h)-F(-h), \quad (11)$$

where

$$-h=x_{2k}-x_{2k-1},\quad h=x_{2k+1}-x_{2k}.$$

$$\int_{-h}^{h} f(x)\mathrm{d}x=F(h)-F(-h)=$$
$$=\left[F(0)+f(0)h+f^{(1)}(0)\frac{h^2}{2}+f^{(2)}(x_{2k})\frac{h^3}{6}\right]-$$
$$-\left[F(0)+f(0)h+f^{(1)}(0)\frac{h^2}{2}-f^{(2)}(x_{2k})\frac{h^3}{6}\right]=$$
$$=2f(0)h+f^2(x_{2k})\frac{h^3}{3}=f(0)2h+\frac{d^2 f(x_{2k})}{d(x_{2k})^2}\frac{(2h)^3}{24}, \quad (12)$$

where x_{2k} is some point along segment$(-h, h)$.

Since:

$$\frac{d^2 f(x_{2k})}{d(x_{2k})^2}\approx\frac{\Delta^2 f(x_{2k})}{\Delta x_{2k}^2}\frac{(h)^3}{3}, \quad (13)$$

$$\Delta x_{2k}^2=h=\frac{1}{n},$$

$$\Delta^2 f(x_{2k})=f(x_{2k})-2f(x_{2k-1})+f(x_{2k-2}), \quad (14)$$

then

$$\int_{-h}^{h} f(x)\mathrm{d}x\approx 2f(x_{2k})h+\frac{\Delta^2 f(x_{2k})}{h^2}\frac{(h)^3}{3}=2f(x_{2k})h+\Delta, \quad (15)$$

where the absolute approximation error Δ is equal to:

$$\Delta=\frac{\Delta^2 f(x_{2k})h}{3}=\Delta^2 f(x_{2k})\frac{1}{3n}. \quad (16)$$

We obtain from (16) the number n of approximation sections (17) at curve $f(x)$ of a neural or fuzzy model [7] at section (0, 1) with the assigned absolute approximation error Δ, using the formula of rectangles (18) or a trapezoidal formula (19):

$$n=\frac{\Delta^2 f(x_{2k})}{3\Delta}=\frac{1}{h}=\frac{1}{x_{2k}-x_{2k-1}}, \quad (17)$$

$$F(x)=\int_0^1 f(x)\mathrm{d}x=\frac{1}{n}\left[f(x_0)+f(x_2)+\ldots+f(x_{2n-2})\right]+\Delta, \quad (18)$$

$$F(x)=\int_a^b f(x)\mathrm{d}x=$$
$$=\frac{1}{n}\left[\frac{f(x_0)+f(x_{2n})}{2}+f(x_2)+f(x_4)+\ldots+f(x_{2n-2})\right]+\Delta. \quad (19)$$

Denote the relative error of approximation:

$$\delta = \frac{3\Delta}{\Delta^2 f(x_{2k})}, \qquad (20)$$

hence:

$$\delta = \frac{\delta\Delta^2 f(x_{2k})}{3}.$$

For example, for function $y=\mu x^2$, $\Delta^2 f(x_{2k})=0.02\mu$ for $n=10$, $h=0.1$, and for any two adjacent sections:

$$\frac{d^2 f(x_{2k})}{d(x_{2k})^2} = \frac{\Delta^2 f(x_{2k})}{h^2} = 2\mu,$$

$$\Delta = \frac{\delta\Delta^2 f(x_{2k})}{3} = \delta\frac{0.02\mu}{3} = 0.00067\delta\mu.$$

We obtain from (17):

$$n = \frac{1}{h} = \frac{1}{x_{2k} - x_{2k-1}} = \frac{1}{\delta}. \qquad (21)$$

We obtain from expression (21) coordinates for the abscissa of modal values for the next point x_{2k+1} in a model curve based on the coordinate for the preceding point x_{2k-1} and the assigned value for error δ:

$$x_{2k} = x_{2k-1} \pm \delta. \qquad (22)$$

Table 1 gives, for $x_{max}=1$, $x_{min}=0$, the dependence of number n of the required sections of approximation and the number of neurons or terms m for fuzzy variables x, y depending on the preset values for approximation errors δ or Δ.

Table 1

Dependences of approximation error on the number of sections, neurons, or terms

δ	0.5	0.2	0.1	0.05	0.02	0.01
Δ	0.00335μ	0.00134μ	0.00067μ	0.000335μ	0.000134μ	0.000067μ
n	2	5	10	20	50	100
m	3	6	11	21	51	101

4. 2. Estimation of the number of neurons (terms) to ensure the accuracy of model implementation

At the next stage of research, we estimated the required number of neurons (terms), which provide for the necessary accuracy of model implementation based on the maximum deviation between the system and model curves at the section of approximation. The model of this system y will be searched for in the form of sections along straight lines that pass through points with coordinates x_1, y_1 and x_2, y_2, located along the approximated curve with the assigned maximum error of deviation Δ_{max}. Write down the equation of the straight line passing through two points of the curve with coordinates x_1, y_1 and x_2, y_2:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}. \qquad (23)$$

Considering (1), substitute in (12):

$$\frac{y - f(x_1)}{f(x_2) - f(x_1)} = \frac{x - x_1}{x_2 - x_1}. \qquad (24)$$

We obtain as a result:

$$y = \frac{x - x_1}{x_2 - x_1}\left[f(x_2) - f(x_1)\right] + f(x_1). \qquad (25)$$

Approximation error Δ will be determined as the maximum deviation between the system $f_c(x)$ and model $f(x)$ curves along the approximation section from correlation (26):

$$\Delta = y - y_c = \\ = \frac{x - x_1}{x_2 - x_1}\left[f(x_2) - f(x_1)\right] + f(x_1) - f_c(x). \qquad (26)$$

The maximum of error Δ_{max} is determined from condition:

$$\frac{\partial\Delta}{\partial x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{\partial f_c(x)}{\partial x} = 0, \qquad (27)$$

we obtain the dependence of coordinate of the next point x_2 based on the coordinate for the preceding point of curve x_1 and the assigned value for maximum error Δ_{max}.

An example of the implementation of the modeled system of guaranteed accuracy, described relative to the coordinate origin 0 (0, 0) by the equation of a non-central curve of second order in the form:

$$a_2 x^2 + 2a_1 x + a_0 + 2b_0 y = 0, \qquad (28)$$

was considered by authors in [5]. Transform of equation (28) is a curve (parabola), shifted relative to the coordinate origin:

$$y = f(x) = -\frac{a_2 x^2}{2b_0} - \frac{2a_1 x}{2b_0} - \frac{a_0}{2a_1}.$$

The transformation of a parabola equation (28) and the transfer of coordinate origin to point O' with coordinates:

$$x_c = x + \frac{a_1}{a_2};\quad y_c = y + \frac{a_0}{2a_1} - \frac{a_1^2}{2b_0 a_2};$$

$$O'\left(-\frac{a_1}{a_2}; \frac{a_1^2}{2b_0 a_2} - \frac{a_0}{2b_0 a_2}\right)$$

produces the modeled system, described by the equation of parabola (29).

$$y_c = \mu x^2, \qquad (29)$$

where $\mu = 1/2p = -a_2/2b_0$, p is the distance from the focus to the directrices of parabola.

The model of this system in [5] was found in the form of sections along straight lines that pass through points with coordinates x_1, y_1 and x_2, y_2 of parabola (29) with the assigned maximum error Δ_{max} of deviation between curve (29) and straight line (25). Write down the equation of the straight line passing through two points with coordinates x_1, y_1 and x_2, y_2. Considering (29), substitute in (25):

$$y_1 = \mu x_1^2, \text{ and } y_2 = \mu x_2^2.$$

The obtained result is:

$$\frac{y-\mu x_1^2}{\mu(x_2^2-x_1^2)}=\frac{x-x_1}{x_2-x_1}\cdot\frac{x_2+x_1}{x_2+x_1}=\frac{(x-x_1)(x_2+x_1)}{x_2^2-x_1^2},$$

hence:

$$y-\mu x_1^2=\mu(x-x_1)(x_2+x_1)=\mu(xx_2+xx_1-x_1x_2-x_1^2)$$

or

$$y=\mu\left[(x_1+x_2)x-x_1x_2\right]. \quad (30)$$

Error of approximation Δ is determined as a difference:

$$\Delta=y-y_c=\mu\left[(x_1+x_2)x-x_1x_2-x^2\right]. \quad (31)$$

The maximum of error is determined from condition:

$$\frac{\partial\Delta}{\partial x}=\mu\left[x_1+x_2-2x\right]=0,$$

hence:

$$x=\frac{x_1+x_2}{2}. \quad (32)$$

Substitute (32) into (31) and obtain the dependence of coordinates of the next point x_2 along the curve on the coordinate for the preceding point x_1 and the assigned value for maximum error Δ_{max}:

$$\Delta_{max}=\mu\left[\frac{(x_1+x_2)(x_1+x_2)}{2}-x_1x_2-\frac{(x_1+x_2)^2}{4}\right],$$

$$4\Delta_{max}=\mu\left[(x_1+x_2)^2-4x_1x_2\right],$$

$$4\Delta_{max}=\mu\left[(x_1^2+2x_1x_2+x_2^2-4x_1x_2\right]=$$
$$=\mu\left[(x_1^2-2x_1x_2+x_2^2\right]=\mu\left[(x_1-x_2)^2\right]=\mu h^2,$$

$$h=x_1-x_2=2\sqrt{\frac{\Delta_{max}}{\mu}},$$

$$x_2=x_1\pm 2\sqrt{\frac{\Delta_{max}}{\mu}}.$$

In a general case:

$$x_{i+1}=x_i\pm 2\sqrt{\frac{\Delta_{max}}{\mu}}. \quad (33)$$

Considering the notation:

$$x_{max}-x_{min}=b-a,\ \ x_{i+1}-x_i=h,\ \ h=\frac{b-a}{n},$$

the required number of approximation sections n and the number of neurons or terms m for fuzzy variables x, along the section of the curve from $x_{min}=a=0$ to $x_{max}=b=1$ was derived from expressions:

$$n\geq\frac{b-a}{2\sqrt{\frac{\Delta_{max}}{\mu}}}=\frac{1}{h}=\frac{1}{2\sqrt{\frac{\Delta_{max}}{\mu}}}, \quad (34)$$

$$m=n+1, \quad (35)$$

$$n^2\geq\frac{1}{\frac{4\Delta_{max}}{\mu}}=\frac{1}{\delta}, \quad (36)$$

$$\delta\geq\frac{1}{n^2}, \quad (37)$$

$$\Delta_{max}\geq\frac{\mu}{4n^2}. \quad (38)$$

Table 2 gives, depending on the number of approximation sections n and the number of terms m for fuzzy variables x and y, the values derived for a relative error of approximation δ and the absolute error of approximation Δ_{max}. By choosing the required values for δ or Δ_{max}, we obtain values for n or m, which warrant the required accuracy of model implementation.

Table 2

Dependences of errors on the number of approximation sections, terms, and fuzzy variables

n	1	2	3	4	5	10	20
m	2	3	4	5	6	11	21
δ	1	0.25	0.11	0.0625	0.04	0.01	0.0025
Δ_{max}	0.25μ	0.0625μ	0.0275μ	0.016μ	0.01μ	0.0025μ	0.000625μ

5. Practical implementation of construction of a fuzzy model and a model using ANN

Consider an example of building a fuzzy model for the system with the assigned dependence $y_c=\mu x^2$. The membership functions of values for the input and output parameters of the model take the form, shown in Fig. 2, with the assigned system of rules:

R_i: IF $x_i=T_{xi}$ THEN $y_i=T_{yi}$.

Fig. 2, *a* shows a dependence chart $y_c=\mu x^2$ (curve 1). It is obvious that at uniform location along the *XY* axes of coordinates for modal values of terms for the input variable x and the output variable y, the model (approximating) curve 2 will be a straight line. In this case, the nodal points of the model curve will be at the points of intersection of coordinates for their modal values. That would lead to a large error in fuzzy model 2.

We suggest that the coordinates of modal values for the terms of the output variable in the approximating curve of fuzzy model $y_i=f(x_i)$ (T_{y0}, T_{y1}, T_{y2}, T_{y3}) should be chosen from condition (Fig. 2, *b*):

$$y_i=\mu x_i^2. \quad (39)$$

In this case, the nodal points of model curve 2 would be located along curve 1 unevenly, which would provide for the assigned accuracy of modeling.

Fig. 3 shows the implementation of the model using ANN.

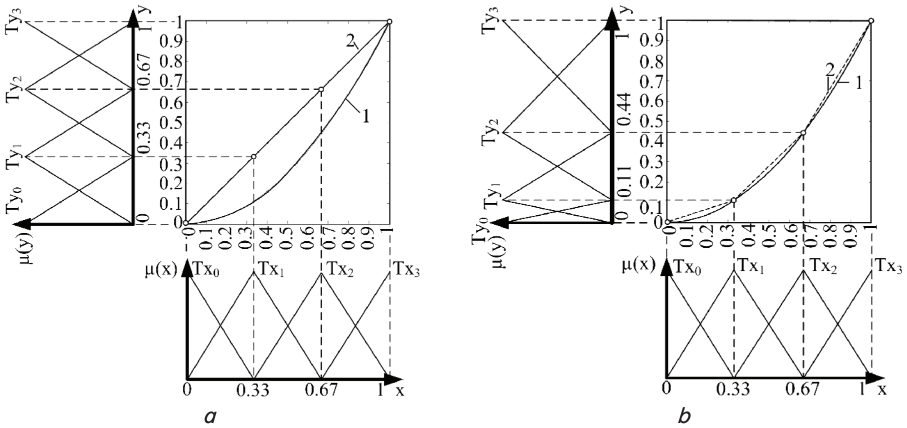

Fig. 2. Values for triangular terms of input variable x (T_{x0}, T_{x1}, T_{x2}, T_{x3}) and the output variable y (T_{y0}, T_{y1}, T_{y2}, T_{y3}), whose modal values are located along the XY coordinate axes: a – evenly; b – unevenly

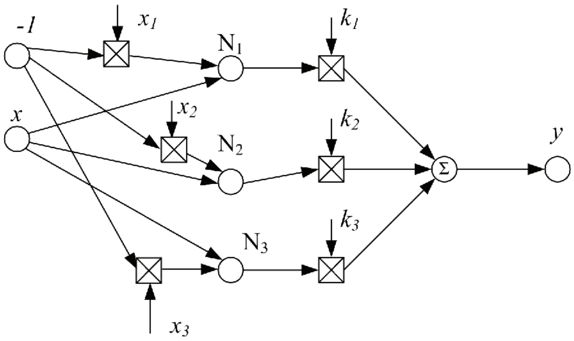

Fig. 3. A neural model of guaranteed accuracy

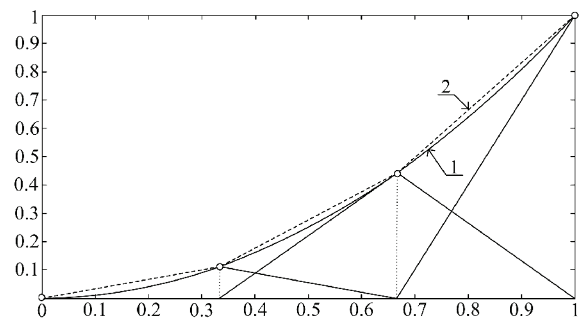

Fig. 4. Dependence charts for the modelled curve $y_c=\mu x^2$ (curve 1) and model curve 2 of the neural model

Fig. 4 shows the dependence charts of the modelled curve (curve 1) and model curve 2 of the neural model, representing a sum of triangular functions of activating all neurons, shifted by the magnitude x_i.

Choose coordinates of the modal values for functions of the activation of neurons from condition (39), determining k_i from coefficients that are selected from $k_i = y_{ci}/x_i^2$, and y_{ci} – from modal values for the modeled curve at approximation nodes x_i (Fig. 4).

In a given case, the nodal points of model curve 2 will be located along curve 1, which would provide for the assigned accuracy of modeling.

To test validity of the proposed solutions, we simulated different approximation algorithms in the environment Matlab/Simulink.

The results of simulation at approximation by fuzzy logic are shown in Fig. 5, by artificial neural networks – in Fig. 6.

The imitation simulation results indicate that the approximation error corresponds to values derived from analytical calculations.

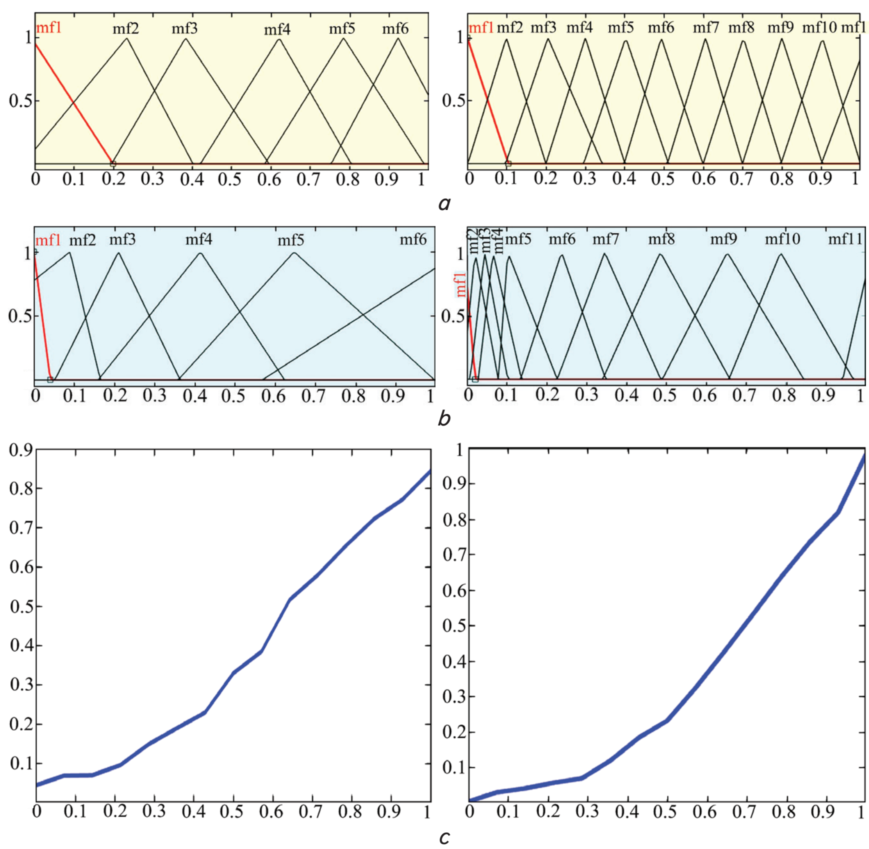

Fig. 5. Results of simulating the approximation by fuzzy logic: a – terms of input variable x; b – terms of output variable y; c – shape of the approximated curve

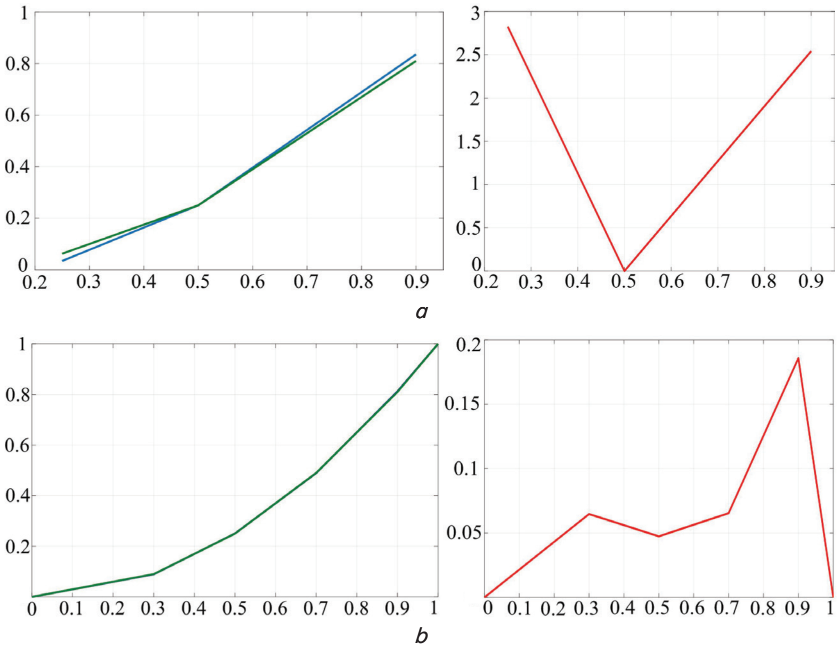

Fig. 6. Results of simulating the approximation by artificial neural networks (left) and an approximation error (right): *a* — for the curve assigned by 3 points; *b* — for the curve assigned by 10 points

6. Discussion of results of studying the analytical method for limiting the complexity of neural-fuzzy models of guaranteed accuracy

Therefore, we have proposed a solution on the choice of coordinates for the modal values of terms in fuzzy models, the number n of approximation sections and the number m of membership functions T_i of terms in a fuzzy model, the number of activation functions of neurons N_i. Computation of the specified parameters is possible in line with formulae (21) to (22) and (33) to (39), depending on the assigned accuracy of model implementation.

In contrast to methods, examined in chapter 2, which imply the selection of values (settings) for parameters of neural-fuzzy models, the developed method solves this task analytically while assigning guaranteed accuracy. The advantage of the proposed method is the universality of its implementation in order to limit the complexity of fuzzy and neural models of systems, described by functions that have *2l* derivatives.

The factors that remain unaddressed in this study include the uncertainty of the impact of noise and disturbances on the accuracy of a system curve approximation and their comparison to existing methods.

The further advancement of research could be accounting for the dynamics of work of neural-fuzzy models' elements over time.

8. Conclusions

1. We have devised a procedure for estimating the required number of neurons (terms) in a model that would provide for the necessary accuracy such that the area of a model curve approaches the system one. Evaluation of approximation error is performed based on the residual terms from decomposition of areas of the system function into a Maclaurin series.

2. We have devised a procedure for estimating the required number of neurons (terms), which provide for the required accuracy of model implementation based on the maximum deviation between the system and model curves at the approximation section. The derived dependences of error on the number of approximation sections and the number of terms for fuzzy variables make it possible to assign a predefined level of guaranteed accuracy for the implementation of models.

3. We have considered the solution to the applied task on constructing a fuzzy model and a model using ANN for a system with the assigned dependence $y_c = \mu x^2$. The reported results from imitation simulation indicate exact match between the results and the estimated analytical values.

That testifies to the validity of the devised approach and the possibilities for its use in applied problems.

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INCREASING THE SHARE OF CORRECT CLUSTERING OF CHARACTERISTIC SIGNAL WITH RANDOM LOSSES IN SELF-ORGANIZING MAPS

S. Shapovalova
PhD, Associate Professor*
E-mail: lanashape@gmail.com
Yu. Moskalenko
Postgraduate student
E-mail: yuramuv@gmail.com
*Department of Automation of Designing of Energy Processes and Systems
National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»
Peremohy ave., 37, Kyiv, Ukraine, 03056

Проведено аналіз способів оптимізації алгоритмів функціонування нейронних мереж Кохонена – карт самоорганізації (Self-organizing map – SOM) за швидкістю навчання та часткою коректної кластеризації. Визначено ефективну оптимізацію карт самоорганізації за другим критерієм – Enhanced Self-Organizing Incremental Neural Network (ESOINN). Визначено, що у випадку неповного вхідного сигналу, тобто сигналу з втратами в невідомі моменти часу, частка коректної кластеризації неприпустимо низька на будь-яких алгоритмах SOM, як базових, так і оптимізованих.

Неповний сигнал представлено як вхідний вектор нейронної мережі, значення якого подані єдиним масивом тобто без урахування відповідності моментів втрат поточним значенням і без можливості визначення цих моментів. Запропоновано та програмно реалізовано спосіб визначення відповідності неповного вхідного вектора до вхідного шару нейронів для підвищення частки коректного розпізнавання. Спосіб засновано на пошуку мінімальної відстані між поточним вхідним вектором та вектором-ваг кожного з нейронів. Для зменшення часу роботи алгоритму запропоновано оперувати не окремими значеннями вхідного сигналу, а їх неподільними частинами та відповідними групами вхідних нейронів. Запропонований спосіб реалізовано для SOM та ESOINN. Для доведення ефективності реалізації базового алгоритму SOM проведено його верифікацію з існуючими аналогами інших розробників.

Розроблено математичну модель для формування прикладів повних сигналів навчальної вибірки на основі еталонних кривих другого порядку та сформовано навчальну вибірку. За цією навчальною вибіркою було проведено навчання всіх нейронних мереж, реалізованих з використанням запропонованого способу та без нього. Розроблено схему імітації втрат та згенеровано тестові вибірки для обчислювальних експериментів на неповних сигналах.

На основі експериментів доведено ефективність запропонованого способу для класифікації за неповним вхідним сигналом на основі карт самоорганізації як для реалізацій базового алгоритму SOM, так і для ESOINN

Ключові слова: карта самоорганізації, SOM, ESOINN, нейронні мережі Кохонена (Kohonen), сигнал з втратами, втрати в часовому ряді, класифікація за характеристичним сигналом

1. Introduction

One of the basic problems in systems of object control in terms of input signal of their characteristics is classification problem. If the signal is represented as a time series, it is expedient to use neural networks to ensure high level of recognition accuracy. When a part of a signal or some of its values are lost at unknown time points, it is impossible