1. Introduction

Information redundancy is the main condition for ensuring the reliability of the data obtained from fire extinguishing system sensors. As a rule, each sensor has insufficiently high characteristics that determine data reliability. These characteristics include the probability of correct detection, non-detection of fire and false alarm.

To improve these characteristics, parallel and serial information redundancy is used. However, both parallel and serial information redundancy implies a number of problems associated primarily with the cost of the system of parallel-connected sensors, time of information aging and operational reliability of the sensors whose information is read consistently in time.

Therefore, it is proposed to introduce parallel information redundancy by means of nested modules. The module of controlling fire alarm sensors is a set of n parallel-used sensors, in which the decision on the presence of fire is made according to the «n-out-of-m» criterion. The number n is chosen small n=3, 4, 5. To increase the total number of parallel-used sensors, the method of nested modules is used, where not primary sensors are combined into the module, but the same modules. Moreover, the resulting module can be used as a primary source for the subsequent modular combination. So, the modular connection of 2, 3, etc. hierarchy levels of sensors connection can be created. This method is called the method of nested modules. In this way, the number of connected primary sensors can be increased. At the same time, there is also an alternative way of connecting the same large number of sensors directly to the information consumer without first combining them into modules.

The method of sensors combination by the method of nested modules provides high probability characteristics of...
information and controlling system

an information system with significantly lower economic expenses compared with the alternative method of sensors combination. In this case, the problem of operational reliability of the structures connecting a large number of primary sensors is solved much easier.

In accordance with airworthiness requirements, aircraft fire refers to dangerous flight situations, the probability of which does not exceed 10⁻¹⁰. Failures of the fire alarm system also refer to dangerous flight situations. These failures are mainly divided into non-detection and false alarm, reaching about 75%. A false alarm is caused by insufficient reliability of the actuators and sensors. Therefore, search for a solution to increase the reliability of fire alarm systems is an urgent scientific and technical problem. Thus, the relevance of this study consists in the need to improve the reliability of information that comes from fire alarm sensors, in order to increase fire reliability and safety.

2. Literature review and problem statement

The analysis of structures of information control systems (ICS) of aircrafts (AC) shows that their efficiency depends on the reliability of information on the basis of which the relevant decisions are made. Therefore, it is necessary to take appropriate measures to increase the reliability of information in ICS.

Solutions to the problem of data validation during transmission and processing in process control systems are proposed in [1]. In it, the principles and methods of using statistical redundancy data for solving problems related to information reliability control based on the root-mean-square minimum error criterion under various distribution laws are considered. In this case, the issues of obtaining reliable information by the operator in the decision-making process are not considered in this work. Therefore, it is advisable to study the issue of increasing the reliability of information before it arrives to the operator to make a decision.

In [2], the reliability monitoring scheme for active fault-tolerant control systems using the method of stochastic modeling is proposed. In this work, previous fault detection and localization data are used to update the transition characteristics in the reliability model. It also addresses issues of parallel information processing, but without simultaneous accounting of probability characteristics of the control object. Basically, the issue of monitoring the reliability and fault tolerance of control systems using the method of stochastic modeling is solved. Thus, issues of reliability of technical systems are considered in terms of monitoring, but not decision support.

The study of quantitative error estimates of the lambda method in predicting the mean time to first failure and the failure rate of technical systems depending on the element base reliability, system complexity and operation life is carried out in [3]. At the same time, the work does not consider the effect of reliability of technical systems on the accuracy of the information they provide.

In [4], the method is proposed for determining the distribution of the limiting condition of time by the excess of the diagnostic parameter, which determines the accuracy of maintaining a zero state. The change in the magnitude of deviation occurs as a result of destructive processes during operation. To estimate the deviation of the increase rate in a probabilistic sense, the Fokker-Planck equation was used to determine the residual life of onboard devices. Such methods are advisable to use in the study of mechanical systems, but evaluation of modern electronic systems requires a different approach.

Determination of failure components that may lead to the degradation of the entire production system is investigated in [5], where the probabilistic model is based on the Erlang distribution, and the failure rate of elements is subject to an exponential law. These are failures, which in turn can lead to degradation of the entire production system. Thus, the probability model is investigated, which is constructed for the case when random variables are distributed according to the Erlang law, and the failure rate of elements – exponentially. In this case, reliability assessment of the information received is not performed.

Estimation of safe time of fire detection required to start fire extinguishing procedures, based on the method for estimating the time of fire detection by temperature detectors, rate of temperature rise, light scattering detectors such as smoke, is given in [6]. In this case, the paper does not provide solutions for the construction and selection of optimal fire alarm structures.

In [7], the analysis of alarm systems to identify the causes of false alarms is carried out and several solutions to reduce their rate are discussed. In this case, the paper does not consider the reliability of the incoming information.

In [8], the method of fault diagnosis, based on the fault detection and diagnosing procedure is proposed. This method relies on the use of adaptive filters developed using non-linear geometric approach. This allows increasing the noise immunity of the system. The above method enables fault detection, but it does not address the issue of the influence of faults on the reliability of information received from the system.

In [9], the issues of increasing the efficiency of information systems by means of serial information redundancy are considered. It is shown that a priori probability of fire detection depends on the quality of sensors for the given a posteriori probability values and number of repeated requests. The disadvantage of the proposed solutions is the high coefficient of non-detection of the monitored parameter.

The paper [10] is aimed at improving the reliability of data by parallel information redundancy. Mathematical models of parallel information redundancy systems are obtained. The studies show that parallel information redundancy allows creating reliable information systems with low information capabilities of individual information sources. The disadvantage of the proposed solutions is a high false alarm rate.

The most progressive method of increasing the information reliability in ICS is serial and parallel information redundancy. In this case, serial redundancy reduces the false alarm probability, and parallel – non-detection probability. Modular information redundancy is the development and expansion of parallel information redundancy and allows you to simultaneously reduce false alarm and non-detection probabilities.

Despite the extensive list of works on information reliability, it can be noted that there are no works on non-detection and false alarm probability reduction in terms of providing the crew with reliable information. Providing reliable information to the crew is especially necessary when making decisions in complex and dangerous flight situations. This is especially important when localizing and processing aircraft engine fires, when the pilot needs to make a decision about
shutting down the engine, making an emergency landing. However, fire non-detection leads to an emergency situation and may lead to the loss of an aircraft with crew members and passengers. False alarm of the alarm system leads to large economic losses associated with shutting down the running engine, making an emergency landing, etc., and can lead to an aircraft accident. Therefore, there is a need to study new ways of forming alarm system structures by the principle of «nested modules». This method is more efficient than the basic method of parallel information redundancy.

3. The aim and objectives of the study

The aim of the work is to develop reliable structures of the aircraft fire alarm system (FAS) by means of nested modules to support crew decision making.

To achieve the aim, the following objectives were set:
- to develop mathematical models for the construction of the modules $M_{2,3}, M_{2,4}$ and their hierarchies;
- to conduct a comparative analysis of the reliability characteristics of the proposed nested modular circuits;
- to assess the cost efficiency of nested modular circuits in comparison with the corresponding parallel information redundancy circuits.

4. Mathematical model for constructing nested modules of the fire alarm system

ICS efficiency depends significantly on the reliability of the information that is the basis for crew decision making. Available information alarm systems, primary information sensors have a fuzzy operation threshold. As a result, information from a real sensor always comes with a certain degree of reliability, which can be characterized by three probability states [11]:

- $a$  — correct event detection probability;
- $b$  — false alarm probability;
- $d$  — event non-detection probability.

Such a system of probability states is rather fully described by trinomial distribution, which is an extension of the binomial distribution.

The probability $p_{k(k, m, n, k, i)}$ that $k$ out of $n$ sensors will not detect the controlled phenomenon at all, $m-k$ sensors issue a false alarm and $n-m$ will give correct information about the controlled phenomenon is described by the following expression [12]:

$$p_{k(k, m, n, k, i)} = C_n^a a^n C_n^b b^m C_n^d d^{n-m}$$

(1)

and

$$a + b + d = 1.$$  

(2)

The probability characteristics $a_{kd}, b_{kd}$ and $d_{kd}$ for $n$ parallel redundant information sensors can be determined from trinomial distribution. If the majority rule is introduced, according to which information is received only when at least $Q$ sources assert about the presence of a controlled characteristic, then dependencies (3) can be derived from the trinomial distribution expression. These dependencies determine the probability $a(n, Q)$ of correct detection of monitored information, as well as the probabilities $b(n, Q)$ and $d(n, Q)$ of false alarm and non-detection, respectively [11]:

$$a_{kd} = 1 - (b + d)^n = \frac{Q^n}{n!} \sum_{w=0}^{Q} C_w^n \sum_{v=0}^{Q-n} C_v^n b^w d^{n-w},$$

$$b_{kd} = \sum_{w=Q}^{n} C_w^n b^w d^{n-w},$$

$$d_{kd} = \sum_{w=0}^{Q} C_w^n b^w d^{n-w}.$$  

(3)

where $v$ is the number of sources giving correct information about the controlled characteristic; $w$ is the number of sources giving false information about the controlled characteristic.

For $Q=1$ and $b+d=1-a$, expressions (3) are greatly simplified:

$$a_{s1} = 1 - (1 - a)^n;$$

$$b_{s1} = (1 - a)^n - d^n;$$

$$d_{s1} = d^n.$$  

(4)

The most effective principle of parallel redundancy is the majority rule, according to which information coming from $n$ sensors will be the most reliable when at least $m$ out of $n$ sensors confirm its reliability and it is necessary that the ratio between $m$ and $n$ hold:

$$m = n/2.$$  

(5)

If condition (5) holds, the probabilities $p_{kd}$ of non-detection of the controlled event and the probabilities $p_{kd}$ of false alarm will be approximately equal:

$$p_{kd} = p_{kd}$$  

(6)

The overall information reliability, correct detection probability $p_{kd}$ increase with an increase in the number of sensors $n$, and the probabilities $p_{kd}$ and $p_{kd}$ under this condition decrease significantly.

Of course, information reliability $p_{kd}$ significantly depends on the quality of information sensors, assessed by the probability $a$ of correct detection of a single sensor, as well as on the operational reliability of this sensor, estimated probability of its failure-free operation.

As a rule, there is a trend in engineering according to which sensors with high functional characteristics have low operational reliability. This pattern follows from the fact that high functionality requires significant complication of the information sensor equipment. This entails an increase in the cost of manufacture, reduction in operational reliability and, consequently, an increase in the cost of operation.

The way out of these contradictory technical conditions can obviously be found by applying parallel redundancy of a large number of sensors $n$ with low functional characteristics and high operational reliability. At the same time, low functional characteristics of the sensor are estimated by the low probability $a$ of correct detection and high probability $b$ of false alarm and $d$ non-detection of the controlled event. Such sensors are generally characterized by exceptional design simplicity and, as a result, high operational reliability.

An increase in the number $n$ of parallel-used, inefficient, but operationally reliable sensors will compensate their low probability characteristics $a$ of correct detection. At the same time, this will lead to the creation of informative, highly efficient, operationally reliable and inexpensive information
systems, according to the principle «create reliable structures from unreliable information sources».

The studies show that such highly efficient information systems can be created using the principle of modular redundancy, the essence of which will be considered on specific examples.

5. Development of structures of aircraft fire alarm systems

Fig. 1 shows the primary module \( M_{2,3} \) of the information system consisting of three fire alarm sensors implementing the majority rule «2-out-of-3» for deciding on the correct-ness of the message.

![Diagram](image)

The probability \( p_{2,3a} \) of correct detection, the probability \( p_{2,3na} \) of non-detection and the probability \( p_{2,3fa} \) of false alarm for the information module (Fig. 1) is estimated by the following relations:

\[
\begin{align*}
p_{2,3a} &= a^3 + 3a^2b + 3ab^2 + b^3 + 6abd; \\
p_{2,3na} &= d^3 + 3bd^2 + 3ad^2; \\
p_{2,3fa} &= b^3 + 3b^2d + 3ab^2;
\end{align*}
\]  

(7)

where \( a \), \( b \) and \( d \) are the probabilities of correct detection, false alarm and non-detection of a sensor, respectively. All sensors on the circuit (Fig. 1) are denoted by \( x_i \).

On the basis of the module (Fig. 1), it is possible to build a more efficient information system \( N_{6,9} \), shown in Fig. 2, combining 9 information sources \( x_i \) and implementing the majority rule «6-out-of-9». The circuit (Fig. 2) is performed on the modules \( M_{2,3} \) (Fig. 1) and its probability characteristics \( p_{6,9a}, p_{6,9na}, p_{6,9fa} \), respectively, determining the probabilities of correct detection, non-detection and false alarm, are evaluated by the following recursive dependencies:

\[
\begin{align*}
p_{6,9a} &= x_{123}^3 + 3x_{123}^2p_{2,3a} + 3p_{2,3a}^2p_{2,3a} + 6p_{2,3a}^3p_{2,3a}x_{123} + 6p_{2,3a}p_{2,3a^2}x_{123}^2; \\
p_{6,9na} &= y_{123}^4 + 3p_{2,3na}x_{123} + 3p_{2,3na}^2x_{123} + 3p_{2,3na}^3x_{123} + 6p_{2,3na}^4x_{123} + 6p_{2,3na}^2x_{123}^2 + 6p_{2,3na}p_{2,3na^2}x_{123}^3; \\
p_{6,9fa} &= x_{123}^3 + 3x_{123}^2p_{2,3fa} + 3p_{2,3fa}^2p_{2,3fa} + 6p_{2,3fa}^3p_{2,3fa}x_{123} + 6p_{2,3fa}p_{2,3fa^2}x_{123}^2; \\
p_{6,9na} &= x_{123}^3 + 3x_{123}^2p_{2,3na} + 3p_{2,3na}^2x_{123} + 3p_{2,3na}^3x_{123} + 6p_{2,3na}^4x_{123} + 6p_{2,3na}^2x_{123}^2 + 6p_{2,3na}p_{2,3na^2}x_{123}^3.
\end{align*}
\]  

(8)

Thus, the information module \( N_{6,9} \) is based on the 21st AND circuit, combines 9 sources \( x_i \) and implements the majority rule «6-out-of-9» for deciding on the receipt of correct message.

An alternative option of systems of parallel redundancy of sources \( x_i \) with the majority rule «6-out-of-9» for deciding on message reliability is shown in Fig. 3.

![Diagram](image)

The probabilities \( p_{6,9a} \) of correct detection, \( p_{6,9na} \) non-detection and \( p_{6,9fa} \) false alarm, as shown in Fig. 3, are evaluated by the general formulas:
The number of AND circuits in the circuit (Fig. 3) is determined by the number of combinations of 6-out-of-9 and is equal to $C_6^9 = 84$, i.e., 4 times the number of AND circuits in the system (Fig. 2). Thus, the circuit (Fig. 3) having similar probability characteristics $p_1, p_2, p_3$, will be about 4 times more expensive than the circuit (Fig. 2) and the operational reliability in such a complex circuit is expected to be significantly worse.

Consequently, the circuit (Fig. 3) compared to the circuit (Fig. 2) will be more expensive both in manufacture and in operation.

In accordance with the modular principle of parallel redundancy circuits, Fig. 4 shows the circuit $L_{18,27}$, combining 27 sources $x_i$ and implementing the majority rule «18-out-of-27», and information sources for this circuit will be modules $N_{69}$ (Fig. 2).

![Fig. 4. Information module $L_{18,27}$ of the majority rule «18-out-of-27» for deciding on the receipt of correct message](image)

Naturally, the probability characteristics of the module $L_{18,27}$ (Fig. 4) namely, the probabilities $p_{18,27,ed}$ of correct detection, $p_{18,27,nd}$ of non-detection and $p_{18,27,fa}$ of false alarm can be estimated according to the general expressions:

$$p_{18,27,ed} = 1 - \sum_{i=1}^{27} C_{27}^i a^i (1-a)^{27-i};$$
$$p_{18,27,nd} = \sum_{i=0}^{27} C_{27}^i a^i (1-a)^{27-i};$$
$$p_{18,27,fa} = \sum_{i=0}^{27} C_{27}^i b^i (1-b)^{27-i}.$$  \hspace{1cm} (13)

Given that the majority rule «18-out-of-27» is the same for both circuits (Fig. 4, 5), information sources $x_i$ have identical probability characteristics $a, b$ and $d$, it is easy to show the validity of the following identities: $p_{18,27,ed} = p_{18,27,nd}$; $p_{18,27,ed} = p_{18,27,fa}$.

Thus, it can be argued that the circuits (Fig. 4, 5) are identical in their functionality, but the number of AND circuits for implementation of the circuit (Fig. 4) is 129, and for implementation of the circuit (Fig. 5) it is determined by the number of combinations $C_{27}^6$ and will be equal to $C_{27}^6 = 246675$, i.e., the circuit (Fig. 5) is practically unsuitable for implementation.

![Fig. 5. Alternative option of parallel redundancy of sources $x_i$ with the majority rule «18-out-of-27»](image)
Similarly, it is possible to show significant advantages of the modular principle of parallel redundancy for the module $M_{2,4}$ given in the circuit (Fig. 6), implemented on 6 AND circuits and combining 4 information sources $x_i$.

$$p_{12,24,i} = p_{2,3,i}^4 + 4p_{2,3,i}^2 p_{2,4,i}^2 + 6p_{2,3,i}^2 p_{2,5,i}^2 +
+ 4p_{2,3,i} p_{2,4,i} p_{2,5,i}^2 + 4p_{2,3,i}^2 p_{2,4,i} p_{2,5,i}^2 +
+ 12p_{2,3,i}^2 p_{2,4,i}^2 p_{2,5,i}^2 + 6p_{2,3,i}^2 p_{2,4,i}^2 p_{2,5,i}^2 +
+ 12p_{2,3,i}^2 p_{2,4,i}^2 p_{2,5,i}^2 ;$$

$$p_{12,24,i} = p_{2,3,i}^4 + 4p_{2,3,i}^2 p_{2,4,i}^2 + 6p_{2,3,i}^2 p_{2,4,i}^2 +
+ 12p_{2,3,i}^2 p_{2,4,i}^2 ;$$

$$p_{12,24,i} = p_{2,3,i}^4 + 4p_{2,3,i}^2 p_{2,4,i}^2 + 6p_{2,3,i}^2 p_{2,4,i}^2 .$$

By analogy with the modules $N_{12,3}$ (Fig. 2, 3), it is possible to construct a modular circuit $L_{12,27}$ (Fig. 8) on the modules $N_{12,24}$, which is performed on 174 AND circuits, combines 96 information sources $x_i$ and implements the majority rule «48-out-of-96».

The probability characteristics $p_{12,24,ab}, p_{12,24,ad}, p_{12,24,fa}$ of the circuit (Fig. 7) are estimated by the following expressions:

$$p_{0,96,ad} = p_{12,24,ad}^4 + 4p_{12,24,ad}^2 p_{12,24,fa} + 6p_{12,24,ad}^2 p_{12,24,fa}^2 +
+ 4p_{12,24,ad} p_{12,24,fa}^2 + 4p_{12,24,ad} p_{12,24,fa}^2 +
+ 12p_{12,24,ad} p_{12,24,fa}^2 + 12p_{12,24,ad} p_{12,24,fa}^2 +
+ 6p_{12,24,ad} p_{12,24,fa}^2 + 12p_{12,24,ad} p_{12,24,fa}^2 ;$$

$$p_{0,96,fa} = p_{12,24,fa}^4 + 4p_{12,24,fa}^2 p_{12,24,fa} + 4p_{12,24,fa}^2 p_{12,24,fa}^2 ;$$

$$p_{0,96,fa} = p_{12,24,fa}^4 + 4p_{12,24,fa}^2 p_{12,24,fa} + 4p_{12,24,fa}^2 p_{12,24,fa}^2 .$$

In accordance with trinomial probability distribution, it is easy to show that the probability characteristics, namely the probability $p_{2,4,ad}$ of correct detection, the probability $p_{2,4,fa}$ of non-detection and the probability $p_{2,4,fa}$ of false alarm, are estimated using the following formulas:

$$p_{2,4,ad} = a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + 4a^3 d +
+ 12a^2 bd + 12ab d^2 + 6a^2 d^2 + 12abd d^2 ;$$

$$p_{2,4,ad} = d^4 + 4bd^3 + 4a^3 d ;$$

$$p_{2,4,fa} = b^4 + 4b^3 d + 6b^2 d^2 .$$

By analogy with the modules $M_{2,4}, N_{12,9}$ (Fig. 1, 2), it is possible to construct a modular circuit $N_{12,24}$ (Fig. 7) on the modules $M_{2,4}$, which is performed on 42 AND circuits, combines 24 information sources $x_i$ and implements the majority rule «12-out-of-24».

The probability characteristics $p_{12,24,ab}, p_{12,24,ab}, p_{12,24,fa}$ of the circuit (Fig. 7) are estimated by the following expressions:

$$M_1 \& M_2 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_1 \& M_4 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_3 \& M_5 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_7 \& M_8 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_7 \& M_4 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_9 \& M_4 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_9 \& M_4 \rightarrow 1 \rightarrow N_{12,24}$$

$$M_9 \& M_4 \rightarrow 1 \rightarrow N_{12,24}$$

Fig. 6. Information module $M_{2,4}$ of the majority rule «2-out-of-4» for deciding on the receipt of correct message

Fig. 7. Information module $N_{12,24}$ of the majority rule «12-out-of-24» for deciding on message reliability
The circuits (Fig. 7, 9) have identical probability characteristics \( p_{12,24}, p_{12,24,48}, p_{12,24,48,6} \), but implementation of the circuit \( N_{12,24} \) (Fig. 7) requires 42 AND circuits, and implementation of the circuit \( F_{12,24} \) (Fig. 9) – \( 2704156 \) circuits, which is almost impossible.

6. Results of evaluation and study of reliability characteristics of fire alarm systems

Fig. 10, \( a, b \), respectively, show graphical dependencies \( p_{2,3,4}(a), p_{6,9,12,24}(a), p_{18,27,6,9,12,24}(a), p_{48,96,6,9,12,24}(a) \) of system modular probabilities of correct detection on the probability \( a \) of correct detection of a separate fire alarm sensor. They are the main defining functional characteristics of the modular circuits \( M_{2,3}, N_{6,9}, L_{18,27}, M_{2,4}, N_{12,24}, L_{48,96} \). The analysis of the graphs shows that with the use of modules with large \( n \), the functional quality of these modules is significantly improved even with poor quality (low probability \( a \)) of individual sensors.

For example, when \( a=0.5 \), the probability of correct detection for the module \( M_{2,3} \) is 0.6875; for \( M_{2,4} \) – 0.9063; for \( N_{6,9} \) – 0.8687; for \( N_{12,24} \) – 0.9999; for \( L_{18,27} \) – 0.9753; for \( L_{48,96} \) – 0.99999. And with high-quality sensors, \( a=0.9 \),
the probabilities of correct detection for the module \( M_{2,3} \) is 0.9855; for \( M_{2,4} \) is 0.9994; for \( N_{9,9} \) is 0.9997; for \( N_{12,24} \) is 0.999999; for \( L_{18,27} \) is 0.999999; for \( L_{48,96} \) is 0.9999999.

This suggests that the idea of building highly efficient fire alarm systems with parallel nested redundancy is quite real and is waiting for technical implementation.

For a more accurate estimation, efficiency increase factors can be introduced:

\[
\begin{align*}
    k_{1a}(a) &= \frac{p_{a,3d}(a)}{p_{a,0d}(a)}; \\
    k_{2a}(a) &= \frac{p_{a,2b}(a)}{p_{a,0d}(a)}; \\
    k_{1b}(a) &= \frac{p_{1,2b}(a)}{p_{1,3d}(a)}; \\
    k_{2b}(a) &= \frac{p_{1,2d}(a)}{p_{1,3d}(a)};
\end{align*}
\]

Accordingly, for the modular circuits \( N_{9,9}, L_{18,27}, N_{12,24}, L_{48,96} \). Graphic dependencies \( k_{1a}(a), k_{2a}(a), k_{1b}(a), k_{2b}(a) \) are shown in Fig. 11. The analysis of the given graphs shows that the efficiency of modular circuits depends on the number of hierarchy levels of nested modules connection. Conducting such a study, the optimal number \( k \) of hierarchical connections can be found.

![Graph](image)

**Fig. 11.** Graphic dependencies of efficiency increase factors \( k_{1a}(a), k_{2a}(a), k_{1b}(a), k_{2b}(a) \) of modular circuits

Increasing the number of hierarchies \( k \) significantly increases the probability of correct detection even at very small values of \( a \). Also, an increase in the number \( k \) is associated with a significant complication of parallel redundancy circuits. According to the comparison of modular and alternative circuits implementing the same majority rule «m-out-of-n», modular circuits are greatly superior in design simplicity and technical implementation capabilities.

In this work, two types of basic modules \( M_{2,3} \) and \( M_{2,4} \), are studied, which determined the following sequences of nested modular circuits: \( M_{2,3}, N_{9,9}, L_{18,27}, M_{2,4}, N_{12,24}, L_{48,96} \). By the method of mathematical induction, general dependencies can be derived: \( M_{m,n}, N_{9,9}, L_{18,27}, F_{2,3} \). The indices \( m, n \) in which are determined by the expressions:

\[
\begin{align*}
    m_1 &= m C_n^m; m_2 = m C_n^m; \ldots \; m_k = m C_n^m; \\
    n_1 &= n C_n^m; n_2 = n C_n^m; \ldots \; n_k = n C_n^m.
\end{align*}
\]  

(17)

where \( n_1, n_2, \ldots, n_k \) are the total numbers of connecting the primary information sources with probabilities \( a \) of correct detection, probabilities \( b \) of false alarm and probabilities \( d \) of non-detection, respectively, for the \( k \)-th level of the hierarchy of nested modules; \( m_1, m_2, \ldots, m_k \) are the values of the majority rule «m-out-of-n», respectively, for the \( k \)-th level of the hierarchy of nested modules; \( k \) is the number of hierarchy levels of nested modules.

By the method of mathematical induction, it is easy to derive general formulas for determining the number \( V_k \) of AND circuits in modular circuits and the number \( V_k \) in alternative circuits.

For example, for the first level of the hierarchy \((k=1)\), i.e., for the circuits \( N_{9,9}, L_{18,27} \), the total number \( V_1 \) of AND circuits is determined by the dependency:

\[
V_1 = m \cdot C_n^n + m.
\]  

(18)

For the second level \((k=2)\), i.e., for the circuits \( F_{2,3}, F_{2,4} \), the total number \( V_2 \) of AND circuits is determined by the relationship:

\[
V_2 = V_1 \cdot C_n^n + m = C_n^n \cdot (C_n^n + m + m) + m.
\]  

(19)

Accordingly, for alternative circuits, the total numbers \( V_a1, V_a2 \) of AND elements for the first \((k=1)\) and second \((k=2)\) circuits \( F_{2,3} = F_{2,4}, F_{2,3} = F_{2,4} \) are determined by the dependencies:

\[
\begin{align*}
    V_a1 &= C_a^n + n \cdot C_a^n, \\
    V_a2 &= C_a^n + n \cdot C_a^n.
\end{align*}
\]  

(20)

where:

\[
\begin{align*}
    m_a &= m \cdot C_n^n; \; n_a = n \cdot C_n^n, \\
    m_a &= m(C_n^n)^2; \; n_a = n(C_n^n)^2.
\end{align*}
\]

Now it is easy to get the formula dependencies of the total economic gain \( W \), which consists in reducing the number of AND circuits for the implementation of the majority rule «k-out-of-m» using nested modules for the first level of hierarchy:

\[
W_1 = \frac{V_a1}{V_1} = \frac{C_a^n}{C_n^n + m + m}.
\]  

(21)

For the second level of hierarchy:

\[
W_2 = \frac{V_a2}{V_2} = \frac{C_a^n}{C_n^n \cdot (C_n^n + m + m)}.
\]  

(22)

The results of the comparison of the modular circuits \( N_{9,9}, L_{18,27}, L_{48,96} \) and their alternative options \( F_{2,3}, F_{2,4} \) are summarized in Table 1.

<table>
<thead>
<tr>
<th>Basic module</th>
<th>Hierarchy level</th>
<th>Nested modules</th>
<th>Number of AND circuits</th>
<th>Type of alternative circuits</th>
<th>Number of AND circuits</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{2,3} )</td>
<td>1</td>
<td>( N_{9,9} )</td>
<td>21</td>
<td>( F_{2,3} )</td>
<td>84</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( L_{18,27} )</td>
<td>129</td>
<td>( F_{2,3} )</td>
<td>246675</td>
<td>1912</td>
</tr>
<tr>
<td>( M_{2,4} )</td>
<td>1</td>
<td>( N_{12,24} )</td>
<td>42</td>
<td>( F_{2,4} )</td>
<td>2704156</td>
<td>64380</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( L_{48,96} )</td>
<td>174</td>
<td>( F_{2,4} )</td>
<td>643510^22</td>
<td>3.698 \times 10^23</td>
</tr>
</tbody>
</table>
Comparison data of modular circuits with their alternative circuits given in Table 1 indicate that the technical implementation of the principle of majority parallel information redundancy should be carried out only on the basis of modular circuits.

7. Discussion of the results of the study of reliability characteristics of information systems

Synthesis of systems of parallel redundancy of fire alarm sensors is cost-effectively provided by the modular principle of their construction. With the help of the modular principle of information systems, it is possible to create highly efficient information systems from simple, operationally reliable and inexpensive sensors with limited functionality, defined by relatively low accuracy characteristics.

The modules $M_{2,4}$ are more efficient than the modules $M_{2,3}$:

- to obtain almost identical values of probabilities $p_1$ of correct detection for the modules $M_{2,3}$, two levels of the hierarchical structure of nested modules are required, and only one level for the modules $M_{2,4}$, as shown in the graphs (Fig. 10);
- implementation of the 2-level hierarchy of nested modules $M_{2,3}$ requires 129 AND circuits, and implementation of the 1-level of the hierarchy of nested modules $M_{2,4}$ – 42 AND circuits, i.e., three times less to obtain identical results, as can be seen from Table 1.

At the lowest quality of sensors, when $a = 0.5$, the probability of correct detection for the module $M_{2,3}$ is $p_1 = 0.69$, and for $M_{2,4}$, the probability is $p_1 = 0.91$. This suggests that there is a fundamental possibility of creating highly efficient information modules of the type $M_{2,3}$, from low-informative, simple fire alarm sensors with high operational reliability.

In the study of the economic gain factor $W$, the following results were obtained. For the module $X_{10}$ compared to $F_{10}$, the gain increased 4 times, for the module $L_{18,27}$ compared to $F_{18,27}$ – almost 2,000 times, and for the module $N_{12,24}$ compared to $F_{12,24}$ – almost 6,500 times. This suggests that the basic module $M_{2,4}$ is superior to the basic module $M_{2,3}$ in operational reliability and economic efficiency.

The main advantage of the proposed method is a significant increase in the probability of correct detection and a decrease in the probability of non-detection and false alarm. The limitations of the proposed method are as follows. At high $M_{2,3}$, there is no need to introduce several hierarchies in the system, and one can be limited to the first hierarchy, since this provides high probability characteristics with minimum values of sensor quality.

Aircraft fire alarm system sensors operate in a complex environment, especially engine fire alarm sensors: high temperatures, high vibration levels and other factors that reduce performance. The proposed method of nested modules ensures a sufficient level of information reliability even with a significant deterioration in the quality of sensors.

The use of the proposed method for constructing the fire alarm system implies structural changes in the available one, which, in its turn, incurs additional costs for re-equipment of available systems and change of production technology for new systems.

The prospects for research in this direction is the creation of a generalized mathematical model with the help of which one can build any structural circuit of modular redundancy. Thus, it is possible to expand the use of modular redundancy for other information management systems, such as nuclear power plants, «smart» house, and in other areas of smart technologies.

8. Conclusions

1. The structure of the aircraft fire alarm system using nested modules is developed. Such a system makes it possible to ensure high operational reliability of connecting a large number of fire alarm system sensors into a single highly efficient information system. The mathematical model of parallel information redundancy by the principle of nested modules and their implementation on logic gates are developed. This model shows theoretical and practical importance, both for the production and operation of fire alarm systems. Mathematical dependencies of the system’s probability characteristics on the probability of correct detection of a separate sensor for the basic modules $M_{2,3}$ and $M_{2,4}$ and their hierarchies are obtained. The corresponding graphs are presented. Based on the graphs, it is concluded that the module $M_{2,4}$ and its hierarchies are superior to the module $M_{2,3}$, since it provides the best probability characteristics of the fire alarm system.

2. The comparative analysis of probability characteristics, namely, for modules $M_{2,3}$ and $M_{2,4}$ showed that the use of modules with a large number of sensors $n$ significantly improves the functionality of these modules even with low quality ($a < 0.5$) of individual sensors. Mathematical formulas for evaluating the efficiency of the hierarchy are obtained. The analysis of the mathematical dependencies shows that the efficiency of modular circuits increases with an increase in the number of hierarchy levels of nested modules. According to the comparison of modular circuits and alternative circuits implementing the same majority rule «m-out-of-n», modular circuits greatly benefit in terms of design simplicity and technical implementation capabilities.

3. The dependencies to assess the economic gain of the methods used are proposed. The obtained dependencies can be used as a technique for selecting the structures of fire alarm systems built on the basis of nested modules. The obtained results are of practical importance and can be used in the design of reliable and safe structures of fire alarm systems.

References

1. Introduction

New mega-trends in information and communication technologies demand greater flexibility, greater throughput capacity, and a faster performance from networks. However, further increases in the infrastructure of networks are only a temporary solution and they lead to complications in network management as well. That is why conventional