

Вирішальним фактором успіху під час ведення сучасних бойових дій є боротьба за перевагу у повітрі. Основним завданням сторони, що обороняється, є недопущення завоювання противником переваги у повітрі. Для цього застосовуються різноманітні засоби ураження. За критерієм недопущення завоювання противником переваги у повітрі прийнято задане співвідношення сил авіації сторін, яке визначається за їх бойовими потенціалами. Для обґрунтування потрібного бойового складу сил для дій у повітряному просторі у статті розроблені відповідні методичні положення. При цьому сукупність різноманітних засобів, які діють в інтересах боротьби за перевагу у повітрі, розглядається як система ураження військ і об'єктів противника.

В рамках дослідження було вирішено чотири задачі.

При вирішенні першої задачі здійснена декомпозиція системи на компоненти, науковим результатом якої є отримання морфологічного зрізу системи. Це дозволило визначити взаємовплив дій компонентів системи на завдання втрат бойових потенціалом протидіючих сторін.

Друга задача дослідження присвячена розробленню методичного підходу до обґрунтування бойового складу сил для боротьби у повітрі, який ґрунтується на розрахунках збережених бойових потенціалів компонентів протидіючих сторін. Отримані структурні схеми методики обґрунтування потрібного бойового складу сил для недопущення завоювання або підсилення переваги противника у повітрі. Потрібний склад сил визначається за критерієм співвідношення сил авіації сторін на кінець бойових дій з використанням методу ітерацій.

Результатом вирішення третьої задачі є отримання математичних виразів розрахунку збережених бойових потенціалів компонентів сторін на кінець бойових дій, які є основою розробленої методики.

При вирішенні четвертої задачі розглянуто порядок застосування розробленої методики на прикладі визначення складу ударної авіації для недопущення завоювання противником переваги у повітрі.

Розроблені методичні положення доцільно використати при створенні відповідного спеціального математичного програмного забезпечення для використання органами військового управління.

Ключові слова: бойовий склад, співвідношення сил, системний аналіз, перевага у повітрі

UDC: 358.4:355.354

DOI: 10.15587/1729-4061.2019.163082

DEVELOPMENT OF METHODOLOGICAL PROVISIONS REGARDING THE SUBSTANTIATION OF THE COMBAT STRUCTURE OF FORCES FOR ACTIVITIES IN THE AIRSPACE

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1. Introduction

In modern military conflicts, fighting in the air space becomes a decisive factor for achieving success during warfare. The past experience shows that the party that initiated an armed conflict typically achieved supremacy in the air and tried as soon as possible to strengthen such an advantage, thereby creating favorable conditions for the ground forces. Under such conditions, the main task for the party attacked is to prevent the strengthening of enemy's superiority in the air.

This objective can be accomplished by: aviation units, anti-aircraft missile troops (AMT), units of air defense troops (ADT), rocket troops and artillery (RTA), army aviation (AA) of ground forces (GF), ADT units of naval forces (NF), units of electronic warfare (EW).

During warfare the opposing sides would exchange missile and air strikes (MAS) against troops and facilities, which could involve strike aircraft, cruise missiles, ballistic missiles for operational-tactical and tactical purposes. Reflection MAS are repelled by AMT, fighter aviation (FA), ADT forces, NF ADT.

Accomplishing a task for preventing the enemy's superiority in airspace is estimated based on the ratio of the parties' aviation forces to the assigned duration of warfare. The calculation of the ratio of forces by opposed parties typically takes into consideration the combat potentials of armament, whose magnitudes are proportional to the average damage that these samples could cause to opposing troops (forces) over the assigned duration of fighting (operations, battle) [1]. The calculation of these magnitudes implies a comparison between the samples of weapons in terms of their contribution to the result of hostilities, which makes it possible to derive the weights that are considered to be the combat potentials of samples of weapons [2]. The obtained weights are typically normalized to match a reference sample of weapons, whose combat potential is accepted to equal unity. When calculating the ratio of the parties' forces, one reference sample is used for own weapons and for the enemy's weapons.

To determine the ratio of forces by opposed parties based on the results of warfare, the losses in their combat potentials are applied.

A criterion for preventing the enemy's superiority in the air is the predefined ratio of aviation forces by parties, which implies the involvement of all the forces that take part in a fight against an opponent in the air.

To ensure superiority in the air, the opposing sides would strive for the coordinated use of all forces to strike the means both in the air and on the ground. In this case, more effective means of destruction and ways of fighting would be used. Consideration of these factors is a necessary condition for the substantiation of a combat structure of forces for preventing the enemy's superiority or its strengthening in the air.

The results of the armed fight for superiority in the air in many respects determine the success of an operation (combat) in general.

A diversity in forces that act in the interests of fighting in airspace predetermines the need for methodical provisions that could substantiate the required combat composition of forces to prevent enemy's superiority, or its strengthening, in the air, which would consider their integrated application.

The required structure of forces for activities in the airspace is determined by the bodies of military control when preparing an operation (combat).

The relevance of our research relates to the requirements put forward by bodies of military control to obtain substantiated quantitative estimates of the structure of forces to fight in the air under current situation and given the limitations in armament.

2. Literature review and problem statement

Paper [3] reports a procedure for predicting the losses of troops and facilities due to enemy's air strikes, in which the effectiveness of activities by ADT forces is accounted for based on the assigned probability of hitting the target. No issues regarding the evaluation of losses by enemy's aircraft due to activities by AMT, FA AC, ADT GF were discussed, which is explained by the development of a simplified procedure. Paper [4] shows that the projected ratio of forces and means from opposing sides in the course of air operations can be determined using the exponential dependence on its initial value. In this case, the overall losses by a group of forces at the end of hostilities are determined by the ratio of the initial general combat capabilities of the parties. The use of

such an approach is impossible in principle when determining the losses of certain types of forces that take part in the fight for superiority in the air. This is predetermined by the absence of a possibility to obtain the appropriate coefficients that are applied in the exponential dependences.

Work [5] reports a research into the allocation of forces and determining the sequence of MAS against troops and enemy's facilities; it calculates the ratio of aviation forces from opposed parties. It is shown that reducing this ratio can be achieved by a rational sequence of MAS. However, the work fails to determine the necessary structure of forces to prevent enemy's aircraft superiority during combat, there are no dependences for determining the losses of forces by opposed parties.

Methodological aspects of substantiating the effectiveness of armed forces were considered in paper [6]. The required effectiveness of troops at the beginning of combat is determined by the necessary magnitude of averted damage (preserving the combat potential) over a certain period of the operation. No issues related to determining the structure of forces for activities in airspace were considered.

Study [7] suggested determining the quantitative and qualitative ratio of aviation units when fighting for air supremacy by taking into consideration their combat capabilities and their readiness factors. However, in this case the activities by ground forces of ADT were not considered. The procedure for substantiation the required structure of forces by operational and tactical aviation at a dangerous strategic direction that solves the task of the initial period of war [8] also fails to consider the activities of ADT ground forces by an enemy.

Paper [9] solves the task on determining the combat structure of tactical aviation and AMT in the form of the inverse problem of qualimetry. The paper defines such a combat structure of tactical aviation and AMT at which combat objectives would be accomplished with the results that are not less than those assigned at minimum cost of resources. However, the task on preventing the enemy's air supremacy was not considered.

Substantiation of the quantitative and qualitative structure of the anti-aircraft missile weaponry of AMT unit, reported in [10], disregards, when repelling enemy's MAS, the activities of FA and ADT GF troops, which makes it impossible to obtain a proper estimation.

The issues on substantiation of combat and qualitative structure of an aviation group for carrying out effective air operations have been addressed in many studies by Ukrainian and foreign authors. Paper [11] outlines an evolution of concepts on the use of aircraft to defeat the enemy's objects. It has been proven that one of the doctrines to use aviation is a "mechanistic view on war", which implies a thorough mathematical substantiation of forces and means, but the procedure itself has not been provided. Article [12] considered in detail the issue on the development of intelligence as an element of combat support for aviation activities, but neither its implementation nor substantiation of the required structure have been provided. Paper [13] argues that the destruction or suppression of enemy air defense has long been a central element of any operation (fight) because it ensures air supremacy. It is shown that in modern operations from 15 % to 30 % of the total flights perform tasks on suppression of ADT, but there are no any mathematical dependences on the choice of priority objects and on calculations of the required forces and means for activities in airspace.

Work [14] considers an air operation and its objectives as the interaction between dynamic systems. It is proven that

the initial allocation of resources does not always lead to accomplishing the goal. It is proposed to redeploy the means in the course of an operation by using methods from the theory of games. At the same time, no tasks concerning air supremacy were considered.

Article [15] emphasized the relevance of research into the theory of planning military operations from the standpoint of the theory of complex systems. It is proven that the methodology in this field of science can evaluate the ultimate result based on the dynamics of changes in the process of conducting a military operation. It is proposed to use multiagent models for revealing the dynamics in a battlefield. However, no military action aimed at fighting in the air was examined.

Article [16] is interesting from the perspective of the theme of our research, in which it is proposed to derive estimates for the potentials of armed struggle based on the evaluation of uncertainty and planning the capabilities, which also testifies to the relevance of research into substantiation of combat structure of forces for activities aimed at achieving superiority in the air.

Given the current state of aviation in the armed forces of Ukraine, of interest is the study by Polish scientists [17] who substantiate the composition and structure of the information and technological support system for managing military aviation by using statistical methods and techniques of expert polls. The study failed to address the issue on employing other forces in the struggle for air supremacy.

It follows from an analysis of the above studies that there remain the unresolved issues related to determining the losses by opposing sides in an armed fight for air supremacy, taking into consideration the complex nature of application of different forces. The cited papers did not consider the issue on determining a combat force structure in order to prevent the enemy's superiority, or its strengthening, in the air. This allows us to argue that it is expedient to undertake a study aimed at developing methodological provisions for estimating the losses of forces that are engaged in a fight for air supremacy, as well as for substantiating their combat structure.

3. The aim and objectives of the study

The aim of this study is to devise a procedure for substantiating the required combat composition of forces that are engaged in the fight with an enemy in the air in order to prevent the enemy's superiority, or its strengthening, in the air.

To accomplish the aim, the following tasks have been set:

- to decompose the system of destruction troops and objects based on the morphological pattern, taking into consideration the destruction of disparate forces in determining the ratio of aircraft forces of opposing sides;

- to devise a methodological approach to the substantiation of the required combat structure of forces to fight in the air;

- to derive mathematical expressions for calculating the saved combat potentials in the components of opposing sides at the stages of combat activities to determine the ratio of aviation forces;

- to consider the procedure for determining the required combat structure of forces for activities in airspace using an example of determining the structure of strike aviation.

4. Methods and procedure to substantiate the structure of forces for activities in airspace

4.1. Decomposition of the system of destruction of troops and objects based on the morphological pattern for taking into consideration the activities of different forces

Application of different forces that act to prevent the enemy's superiority, or its strengthening, in the air is interconnected, they are coordinated based on a unified principle. The totality of such forces matches the known properties that characterize systems, namely: integrity and division, connectedness, organization, existence of a system property, inherent to the totality of forces in general [18]. That makes it possible to consider the totality of acting forces as a complex organizational and technical system for military purposes, studying which should employ the principles of systems approach.

According to a systemic approach, solving a task under consideration implies its description, setting the criteria, decomposition of the problem, composition of components, finding a solution [19].

The degree of decomposition of a problem into its component parts is defined by the purpose of research, by the necessity to consider those factors that affect the functioning of the system, as well as by the possibility to synthesize a system when resolving the problem. Application of systems analysis [20] also implies dismemberment (decomposition) of the system into component elements (subsystems) in order to study its structure and properties.

Decomposition of a system into components is carried out based on "strata". To substantiate the combat structure of forces that act to prevent the enemy's superiority, or its strengthening, in the air, the decomposition of the system is advisable to carry out based on a morphological pattern that matches the dismemberment of the system based on a functional attribute, that is, according to the tasks that are performed by subsystems [19]. The tasks that should be executed by subsystems in order to prevent the enemy's superiority, or its strengthening, in the air, are to defeat its troops and objects.

A morphological pattern for the system aimed at defeating enemy's troops and objects is shown in Fig. 1.

The system under consideration has M components (types of mass destruction), $m = \overline{1, M}$ subsystem for defeating enemy's means at deployment sites and on positions and L components (types of mass destruction), $l = \overline{1, L}$ subsystem for defeating enemy's means in the air, which are shown in Fig. 1. It is assumed that the opposing system for defeating our troops and objects can be represented by the same structure and contains $r = \overline{1, R}$ and $s = \overline{1, S}$ respective components.

Application of systems approach makes it possible, by evaluating the impact of each component on the ratio of aviation forces, to determine the required structure of combat forces, which are engaged in fighting an air enemy, in order to meet requirements for a given criterion.

According to the character of the armed struggle, in the substantiation of combat structure of forces for activities in airspace it is advisable to consider a temporal pattern of the system for defeating enemy's troops and objects, which is a predicted sequence of exchanging MAS by opposing sides.

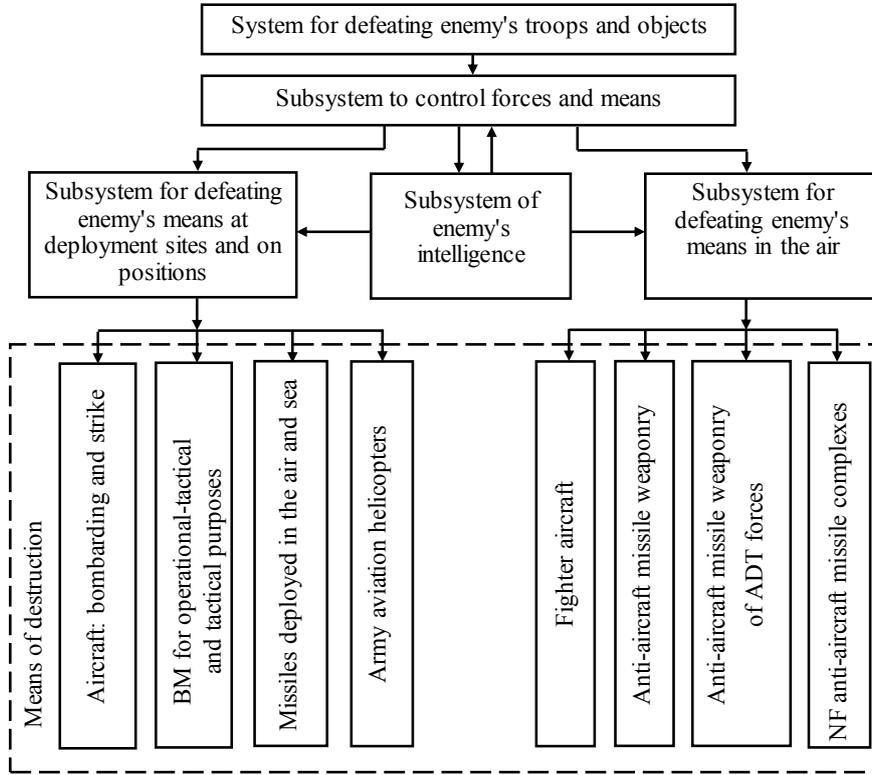


Fig. 1. Morphological pattern of system for defeating enemy's troops and objects

4. 2. Methodological approach to the substantiation of the required combat structure of forces to fight in the air

During substantiation of the combat structure of forces, military activities are split into n stages ($i = \overline{1, n}$) so that at each stage MAS are executed only by one of the opposing sides [4]. Forecasting the stages during a military action (a sequence of exchanging MAS by opposing sides) is performed by experts (specialists) based on the results from estimating the situation. A sequence of MAS exchange by opposing sides is mapped in the diagram shown in Fig. 2.

Stage number, i	1	2	3	...	$n-2$	$n-1$	n
Enemy's MAS				...			
Our forces' MAS				...			
MAS duration	t_1	t_2	t_2	...	t_{n-2}	t_{n-1}	t_n
Duration of military action	T						

Fig. 2. Representation of sequence of MAS exchange by opposing sides

When conducting a study, one evaluates a change in the ratio of aviation forces and the general ratio of sides' forces according to the stages in a military action. For this purpose, one determines the losses of combat potentials in the components of opposing forces in the course of the task to repel MAS. When planning MAS, a considerable part of forces (fighter aviation, military aircraft, ballistic and cruise missiles) by opposing sides (especially starting at early MAS) is allocated to destruct (suppress) the means of air defense, and aircraft on air fields, helicopters, BM on starting positions, which directly affects the combat potential of the forces taking part in a military action. The combat potential of fighter aviation, destroying aviation, helicopters, reduces due to the use of ADT means to repel strikes, and BM and CM due to

using them for their purposes (expenditure).

To perform MAS, such objects are selected that have the greatest combat potential. Allocation of forces for destruction of objects is carried out using the iterative procedures [3, 5].

Results of the calculations are used to determine the saved combat potentials in the components of enemy forces and own troops at each stage, which makes it possible to estimate the ratio of aviation forces C_n^{av} and the general correlation of forces C_n of opposing sides at the end of the military action at duration T . If C_n^{av} is less than or is equal to the assigned value for ratio C_{adj}^{av} which corresponds to preventing the enemy's superiority, or its strengthening, in the air, it is considered that the objective of military action is achieved by our forces. Otherwise, additional forces would be required to fight the air enemy, implying the repeated calculations, that is, we use the method of successive approximations (iterations), which makes it possible to

determine the required combat structure of forces for preventing the enemy's superiority, or its strengthening, in the air.

Accomplishing the goal to prevent the enemy's superiority, or its strengthening, in the air is facilitated by the application of a rational sequence of MAS against enemy's troops and facilities [4]. To study this factor, it is necessary to change the sequence of MAS against enemy's troops and objects, and to estimate, over duration of military action (T), the ratio of the saved combat potentials of aviation forces and the saved combat potentials of all forces from opposing sides.

According to the proposed methodological approach, we have devised a procedure for the substantiation of the required combat structure of forces to prevent the enemy's superiority, or its strengthening, in the air, whose structural diagram is shown in Fig. 3.

Underlying the procedure are the mathematical expressions for calculating the saved combat potentials in the components of systems for defeating troops and objects from opposing sides.

4. 3. Mathematical expressions for calculating the saved combat potentials in the components of opposing sides

The initial combat potentials (CP) of the r -th and s -th components of enemy's forces and the m -th and l -th component of our forces are determined from formulae:

$$\begin{aligned}
 CP_{r_0} &= n_{r_0} \cdot C_r, \quad r = \overline{1, R}; \\
 CP_{s_0} &= n_{s_0} \cdot C_s, \quad s = \overline{1, S}; \\
 CP_{m_0} &= n_{m_0} \cdot C_m, \quad m = \overline{1, M}; \\
 CP_{l_0} &= n_{l_0} \cdot C_l, \quad l = \overline{1, L},
 \end{aligned}
 \tag{1}$$

where $n_{r0}, n_{s0}, n_{m0}, n_{l0}$ is the initial number of means of destruction of the r -th and s -th components of enemy's forces, and the m -th and l -th components of our forces, respectively; C_r, C_s, C_m, C_l are the combat potentials for types of means of destruction in the respective components.

When a certain component is missing in a force unit, its combat potential equals zero.

The general initial combat potentials of enemy forces CP_0^{ef} and our forces CP_0^{of} are determined as follows:

$$CP_0^{ef} = \sum_r CP_{r0} + \sum_s CP_{s0}; \quad r = \overline{1, R}; \quad s = \overline{1, S},$$

$$CP_0^{of} = \sum_m CP_{m0} + \sum_l CP_{l0};$$

$$m = \overline{1, M}; \quad l = \overline{1, L}. \quad (2)$$

The original ratio between combat potentials of opposing sides is calculated from formula

$$CP_0 = \frac{CP_0^{ef}}{CP_0^{of}}. \quad (3)$$

The original ratio between combat potentials of aviation forces from opposing sides is calculated from formula

$$C_0^{av} = \frac{\sum_r CP_{r0} + CP_{s0}}{\sum_m CP_{m0} + CP_{l0}}; \quad r = \overline{m=1; 4}; \quad s = \overline{l=1}. \quad (4)$$

The combat potentials of components of enemy forces that can be engaged in MAS, as well as our forces that may be involved to repel it, at the i -th stage of military action are determined from formulae:

$$CP_{ri} = CP_{r(i-1)}^{sv} (1 - K_{pri}) K_{Rri}, \quad r = \overline{1, R};$$

$$CP_{si} = \xi_{si} CP_{s(i-1)}^{sv} (1 - K_{psi}) K_{Rsi}, \quad s = 1;$$

$$CP_{li} = CP_{l(i-1)}^{sv} (1 - K_{pli}) K_{Rli}, \quad l = \overline{1, L}. \quad (5)$$

Similarly, at a MAS by our forces:

$$CP_{mi} = CP_{m(i-1)}^{sv} (1 - K_{pmi}) K_{Rmi}, \quad m = \overline{1, M};$$

$$CP_{li} = \xi_{li} CP_{l(i-1)}^{sv} (1 - K_{pli}) K_{Rli}, \quad l = 1;$$

$$CP_{si} = CP_{s(i-1)}^{sv} (1 - K_{psi}) K_{Rsi}, \quad s = \overline{1, S}, \quad (6)$$

where $CP_{r(i-1)}^{sv}, CP_{s(i-1)}^{sv}, CP_{l(i-1)}^{sv}, CP_{m(i-1)}^{sv}$ are the saved combat potentials of respective components at the $(i-1)$ -th stage of military action; ξ_{si}, ξ_{li} is the share of fighter aviation by an enemy and our forces, intended to support strike aviation; $K_{pri}, K_{psi}, K_{pmi}, K_{pli}$ are the reserve ratios; $K_{Rri}, K_{Rsi}, K_{Rmi}, K_{Rli}$ are the combat readiness coefficients.

For example, for aircraft, one can accept a reserve ratio of 0–0.1; readiness factor, 0.90–0.95 [3].

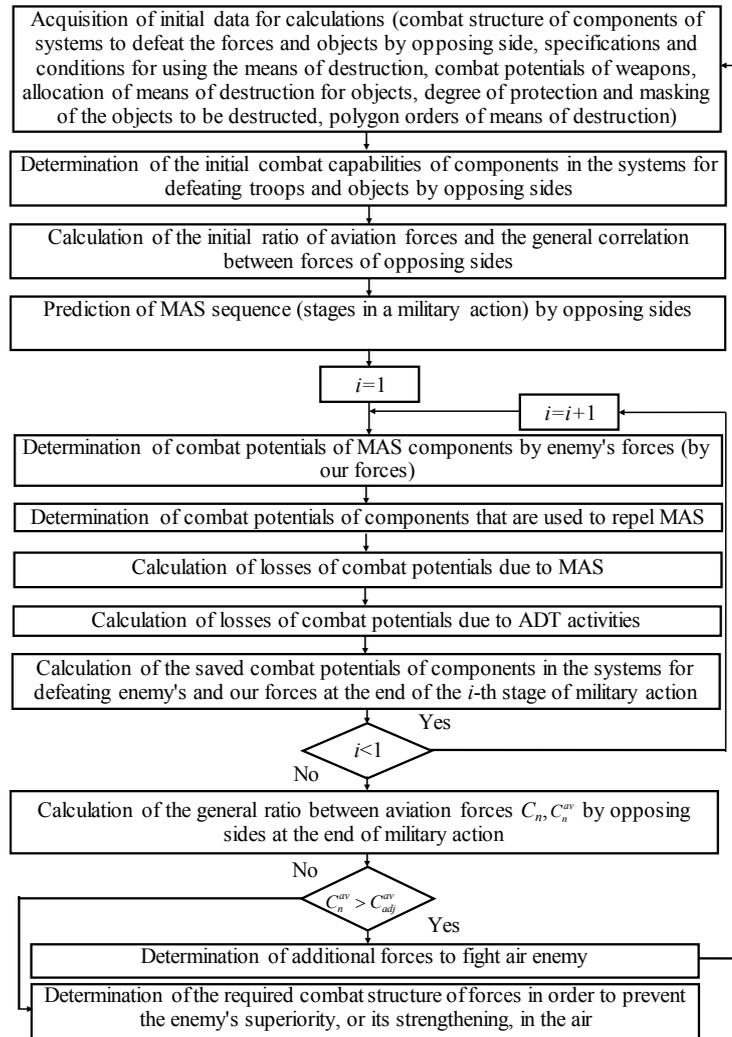


Fig. 3. Structural diagram of the procedure for substantiating the required combat structure of forces in order to prevent the enemy's superiority, or its strengthening, in the air

For BM and CM, reserve ratios are used to determine part of BM and CM, which is supposed to be utilized during subsequent stages of military action.

For the first stage of military action, combat potentials $CP_{r(i-1)}^{sv}, CP_{s(i-1)}^{sv}, CP_{l(i-1)}^{sv}, CP_{m(i-1)}^{sv}$ correspond to their original values.

An important role in the procedure whose structure is shown in Fig. 3 belongs to determining the losses of combat capabilities in the components of systems to defeat troops and objects from opposing sides at the stages of military action.

Mathematical expectation of losses in combat potentials of the r -th and s -th components from the structure of enemy's MAS due to the effect of the l -th component of our forces is determined from formulae:

$$\Delta CP_{ri} = P_{1rli} \cdot CP_{ri}, \quad r = \overline{1, R};$$

$$\Delta CP_{si} = P_{1sli} \cdot CP_{si}, \quad s = 1, \quad (7)$$

where P_{1rli}, P_{1sli} is the mean probability of destroying a separate target of the r -th and s -th component from the structure of enemy's MAS by the l -th component of our forces when repelling the enemy's MAS at the i -th stage of military action.

In the general case, for land-based and marine ADT forces (components 2–4), the mean probability of destroying a separate target of the r -th component from the structure of MAS is determined from known expression [21]

$$P_{1r} = P_d \cdot P_h [1 - (1 - R_{1r})^d], \quad (8)$$

where P_d is the probability of detecting a target prior to the boundary when the complex is capable of hitting it; P_h is the probability of hitting a target; R_{1r} is the probability of hitting a target of the r -th component by a single missile taking into consideration the process of AMC functioning; d is the number of missiles that are intended for hitting a target.

The probability P_d is determined by processing statistical data on the detection of air targets under different conditions (in the absence or presence of obstacles).

The probability of hitting an air target by AMC with a single missile is a characteristic of the complex, which is determined based on the results of ground-based tests and military practice involving actual targets.

The application of EW means for the suppression of AMC is accounted for through coefficient $K_x = 0.5 - 0.8$ [6], which is specified in the source data for the procedure. In this case, the probability $R_{1r} = K_x R'_{1r}$ (R'_{1r} is the probability of hitting a target of the r -th component with a single missile). The number of missiles that are intended to attack a target is determined by the regulations.

Determining the probability of attacking a target P_h by an AMC unit is performed using methods from the theory of mass service.

For n AMC with small areas of attack (components 3 and 4 in the subsystem for defeating enemy's means in the air, and when repelling a BM attack (component 2), the probability P_{hl} is determined from formula by Erlang [21, 22]

$$P_{hl} = 1 - \frac{\alpha^{n_l}}{\sum_{k=0}^{n_l} \frac{\alpha^k}{k!}}; \quad l=2-4, \quad (9)$$

where $\alpha = \lambda t_{hl}$; λ is the density of targets in the raid; t_{hl} is the average time of attacking the targets by a complex of the l -th component.

Taking into consideration that BM and CM act during MAS separately, in determining the probability P_{hl} for these targets, only their density in the raid is considered.

Expressions for determining the magnitudes for ΔCP_{ni} , ΔCP_{si} can be written in the following form:

$$\begin{aligned} \Delta CP_{ni} &= CP_{ri} \cdot P_d \cdot K_{pl} \cdot P_{hrl} [1 - (1 - R_{1r})^{d_i}] b_{li}, \\ \Delta CP_{si} &= CP_{si} \cdot P_d \cdot K_{pl} \cdot P_{hsl} [1 - (1 - R_{1s})^{d_i}] b_{li}, \end{aligned} \quad (10)$$

where K_{pl} is the factor of participation of complexes from the l -th component in repelling MAS (determined based on the results of military exercises and simulation of military activities).

When calculating, it is typically accepted that: for AMC AMT $K_{pl2} = 0.5$; for FA $K_{pl1} = 0.3$; for the anti-aircraft ADT complexes of GF and NF $K_{pl3} = K_{pl4} = 0.2$.

In the process of repelling an attack, there is a decrease in the capabilities of ADT forces to repel it. Therefore, we introduced a coefficient b_{li} to dependence (10), which char-

acterizes the capability of ADT forces to repel a strike by means of air attack.

In determining the probability of attacking planes and helicopters P_{hrl} , P_{hsl} by complexes with large zones of attack ($l=2$), it is necessary to take into consideration the average time that a target spends in the area of attack (waiting time), that is, it is necessary to consider the system of mass service with an expectation.

The probability of destroying a target from the r -th enemy's component by a single group of fighters is determined from formula [23]

$$P_{1r} = P_d \cdot P_s \cdot P_{ed} [1 - (1 - R_{1r})^z], \quad (11)$$

where P_s is the probability that until the moment a target enters the border of fighters' flight there is at least one free guidance channel and at least one fighter; P_{ed} is the probability of guiding a fighter (group) on the target; R_{1r} is the probability of hitting a target from the r -th component of enemy forces with a single fighter (based on the results of field tests); z is the number of fighters in a group.

The probability of servicing P_s , as well as the probability of attacking P_h , is determined from a formula by Erlang [21] depending on the number of combat channels. In this case, $\alpha = \lambda T_{gc}$, where T_{gc} is the duration of a guidance cycle.

The probability of guiding a fighter on target

$$P_{ed} = \Phi \left(\frac{\Delta Q_p}{\sigma_Q} \right), \quad (12)$$

where Φ is the Laplace distribution function; ΔQ_p is the permissible guidance error on the course; σ_Q is the mean quadratic error of guiding to the course.

One can accept that fighters will be distributed for attacking the targets from the r -th and s -th components of enemy forces in accordance with the ratio of their combat capabilities at MAS. In this case, mathematical expectations for the magnitudes of losses in combat potentials from the r -th and s -th components of enemy due to the activities of our fighter aviation forces ($l=1$) can be determined from formulae:

$$\begin{aligned} \Delta CP_{ni} &= C_r \frac{CP_{ri}}{CP_{ri} + CP_{si}} \cdot P_{1rl} \frac{n_{li}}{z}, \\ \Delta CP_{si} &= C_s \frac{CP_{si}}{CP_{ri} + CP_{si}} \cdot P_{1sl} \frac{n_{li}}{z}; \quad r=1;4; \quad s=1; \quad l=1, \end{aligned} \quad (13)$$

where n_{li} is the number of fighter jets, which are used at the i -th stage of military action.

Similar dependences (10), (13) are used to determine the loss of combat capabilities by the components of our forces at MAS due to the ADT forces of the enemy.

Under an enemy's MAS, supporting fighters, at probability P_{1ls} ($l=1, s=1$), will destroy our fighters. The same applies to MAS by our forces. Mathematical expectation for the magnitude of losses in combat potential by our fighter aviation due to activities of the enemy's supporting fighters is

$$\Delta CP_{si} = C_l \frac{CP_{si}}{C_s z} \cdot P_{1ls}. \quad (14)$$

In the course of a military action, MAS will attack the means in the subsystem for defeating the means at sites of

their bases and on positions, the means in the subsystem for defeating the means in the air, as well as other military facilities. Distribution of means of destruction is carried out when planning a MAS. The procedure under consideration implies the means of destruction at MAS are distributed using coefficients F_{rm} and F_{rl} (under an enemy's MAS) and F_{mr} and F_{ms} (under a MAS by our forces). In this case, the following conditions must be satisfied:

$$\sum_m F_{rm} + \sum_l F_{rl} \leq 1,$$

$$\sum_r F_{mr} + \sum_s F_{ms} \leq 1. \tag{15}$$

Planning MAS typically implies determining the objects and ground orders for means that ensure their destruction [3].

When determining the ground orders, the safety and masking of objects to be destroyed are taken into account.

The average number of means of destruction of the r -th component of the enemy forces at MAS, assigned to defeat a single means of the m -th or l -th components of our forces, is

$$v_{rm} = \frac{N_{rm}}{q_{rm}}; v_{rl} = \frac{N_{rl}}{q_{rl}}, \tag{16}$$

where N_{rm}, N_{rl} is the number of means of destruction of the r -th component of enemy forces at MAS in the ground orders for defeating the means of the m -th or l -th component of our forces, respectively; q_{rm}, q_{rl} is number of means of the m -th or l -th components of our forces, which the enemy plans to destroy by the r -th component of its forces.

Mathematical expectations for the magnitudes of combat potentials of the m -th or l -th components of our forces, destroyed by the r -th component of enemy forces at MAS, are determined from formulae:

$$\Delta CP_{mri} = \frac{\left(CP_{ri} - \sum_l \Delta CP_{rli} \right) \cdot C_m \cdot F_{rm}}{v_{rm} \cdot C_r};$$

$$\Delta CP_{lri} = \frac{\left(CP_{ri} - \sum_l \Delta CP_{rli} \right) \cdot C_l \cdot F_{rl}}{v_{rm} \cdot C_r}. \tag{17}$$

Mathematical expectations for the magnitudes of combat capabilities in the r -th or s -th components of enemy forces, destroyed at a MAS by our forces, are calculated based on similar dependences.

The above dependences make it possible to calculate the saved combat potentials of components in the forces of opposing sides at the end of the i -th stage of military action (Table 1).

In Table 1, in the notation of mathematical expectations for losses in the magnitudes of combat potentials ΔCP , the first digit indicates the number of a component, which suffers losses of combat potential, the second is the number of a components, which causes losses to combat potential.

Using the expressions that are listed in Table 1, the ratio of combat potentials by opposing sides (C^{op}) at the end of the i -th stage of military action is determined as follows

$$C_i^{op} = \frac{\sum_r CP_{ri}^{sv} + \sum_s CP_{si}^{sv}}{\sum_m CP_{mi}^{sv} + \sum_l CP_{li}^{sv}};$$

$$r = \overline{1, R}; \quad s = \overline{1, S}; \quad m = \overline{1, M}; \quad l = \overline{1, L}. \tag{18}$$

Table 1

Expressions for calculating the saved combat potentials of means in the components of opposing forces at the stages of military action

Parameters (specifications)	Dependences to calculate the saved combat potentials
Strike by an enemy	
Saved combat potentials of means of the r -th components of enemy forces ($r = \overline{1, R}$)	$CP_{li}^{sv} = CP_{l(i-1)}^{sv} - \sum_l \Delta CP_{li}, \quad l = \overline{1, L}$
	$CP_{2i}^{sv} = CP_{2(i-1)}^{sv} \left[1 - K_{R2i} (1 - K_{p2i}) \right]$
	$CP_{3i}^{sv} = CP_{3(i-1)}^{sv} \left[1 - K_{R3i} (1 - K_{p3i}) \right]$
	$CP_{4i}^{sv} = CP_{4(i-1)}^{sv} - \sum_l \Delta CP_{4i}, \quad l = \overline{1, L}$
Saved combat potentials of means of the s -th components of enemy forces ($s = \overline{1, S}$)	$CP_{li}^{sv} = CP_{l(i-1)}^{sv} - \sum_l \Delta CP_{li}, \quad l = \overline{1, L}$
	$CP_{2i}^{sv} = CP_{2(i-1)}^{sv}; \quad CP_{3i}^{sv} = CP_{3(i-1)}^{sv};$ $CP_{4i}^{sv} = CP_{4(i-1)}^{sv}$
Saved combat potentials of means of the m -th components of our forces ($m = \overline{1, M}$)	$CP_{mi}^{sv} = CP_{m(i-1)}^{sv} - \sum_r \Delta CP_{mri}, \quad r = \overline{1, R}$
Saved combat potentials of means of the l -th components of our forces ($l = \overline{1, L}$)	$CP_{li}^{sv} = CP_{l(i-1)}^{sv} - \sum_r \Delta CP_{lri}, \quad r = \overline{1, R}$
Strike by our forces	
Saved combat potentials of means of the m -th components of our forces ($m = \overline{1, M}$)	$CP_{1i}^{sv} = CP_{1(i-1)}^{sv} - \sum_s \Delta CP_{1si}, \quad s = \overline{1, S}$
	$CP_{2i}^{sv} = CP_{2(i-1)}^{sv} \left[1 - K_{R2i} (1 - K_{p2i}) \right]$
	$CP_{3i}^{sv} = CP_{3(i-1)}^{sv} \left[1 - K_{R3i} (1 - K_{p3i}) \right]$
	$CP_{4i}^{sv} = CP_{4(i-1)}^{sv} - \sum_s \Delta CP_{4si}, \quad s = \overline{1, S}$
Saved combat potentials of means of the l -th components of our forces ($l = \overline{1, L}$)	$CP_{li}^{sv} = CP_{l(i-1)}^{sv} - \sum_s \Delta CP_{lsi}, \quad s = \overline{1, S}$
	$CP_{2i}^{sv} = CP_{2(i-1)}^{sv}; \quad CP_{3i}^{sv} = CP_{3(i-1)}^{sv};$ $CP_{4i}^{sv} = CP_{4(i-1)}^{sv}$
Saved combat potentials of means of the r -th components of enemy forces ($r = \overline{1, R}$)	$CP_{ri}^{sv} = CP_{r(i-1)}^{sv} - \sum_m \Delta CP_{rmi}, \quad m = \overline{1, M}$
Saved combat potentials of means of the s -th components of enemy forces ($s = \overline{1, S}$)	$CP_{si}^{sv} = CP_{s(i-1)}^{sv} - \sum_m \Delta CP_{smi}, \quad m = \overline{1, M}$

The ratio of combat potentials by the aviation forces of sides (C^{av}), similar to (4) at the end of the i -th stage of military action, is determined from formula

$$C_i^{av} = \frac{\sum_r CP_{ri}^{sv} + \sum_s CP_{si}^{sv}}{\sum_m CP_{mi}^{sv} + \sum_l CP_{li}^{sv}}; \quad r = m = 1; \quad 4; \quad s = l = 1. \tag{19}$$

In order to prevent the enemy's superiority, or its strengthening, in the air, it is necessary to ensure that the ratio of combat potentials by the aviation forces from opposing sides at the end of military action does not exceed the assigned one, which can be equal to $C_{adj}^{av} = 1.5 - 2.0$ [24, 25].

The criterion C_{adj}^{av} can be assigned to be even smaller. This condition can be satisfied by increasing the appropriate forces at the threatening direction of military action or by changing, to more appropriate, the sequence of MAS against enemy's troops and facilities.

According to the structural diagram of the procedure shown in Fig. 3, upon defining such measures, the calculations are repeated, which can be performed many times (the method of iterations is used). Upon meeting the condition for preventing the enemy's superiority, or its strengthening, in the air, one determines the required combat structure of our forces to perform this task.

The application of principles of systems analysis when devising methodological provisions for the substantiation of combat structure of forces for activities in airspace has made it possible to account for a joint influence by components of the opposing sides on the loss of their combat capabilities in the course of warfare and thereby correctly identify the required combat structure of our forces in order to prevent the enemy's superiority, or its strengthening, in the air.

Scientific novelty of the obtained results is in considering an integrated staged application of different means of destruction when evaluating losses of forces by opposing sides in the course of military action for superiority in the air.

That has made it possible to devise a coherent procedure for the substantiation of the required combat structure of forces in order to prevent the enemy's superiority, or its strengthening, in the air.

5. The procedure for determining the required combat structure of forces for activities in airspace using an example of strike aircraft

Two stages of military action are considered: at the first stage, MAS is performed by an enemy, the second by our forces. The initial combat potentials of components by our forces and the enemy (variant 1) are given in conditional units in Table 2.

When carrying out calculations, coefficients of combat readiness, reserve, participation of means in military action, etc. are taken in accordance to acting regulations and based on the results from exercises. The capabilities of combat means in terms of destruction aerial targets are taken according to their tactical-technical characteristics. Polygon orders of BM and aviation for defeating the objects meet the standards that were used previously when performing operational-tactical calculations.

The purpose of calculations using the developed procedure is to determine the required combat capacity of our strike aviation: first, to perform a task on preventing an enemy from strengthening superiority in the air and, second, to ensure the equality of combat potentials by our aviation troops and those by an enemy after two stages of military action.

Table 2

Initial combat potentials of components by our forces and the enemy

Names of samples of weapons in components of sides' forces	Enemy forces	Our forces		
		Variant 1	Variant 2	Variant 3
Strike aircraft	1500	900	1050	1200
Tactical ballistic missiles	100	200	200	200
Helicopters	300	160	160	160
Fighter aircraft	240	490	490	490
Anti-aircraft missile complexes	60	90	90	90
Anti-aircraft systems ADT GF	20	60	60	60
Original ratio between combat potentials by forces from opposing sides, C_0		1.17	1.08	1.00
Original ratio between combat potentials by aviation from opposing sides, C_0^{av}		1.29	1.20	1.10

The original variant for calculation is the first variant (Table 2) of initial combat capabilities of the components by our forces. To perform the first task, the criterion $C_{adj}^{av} = 1.29$ for the second task, $C_{adj}^{av} \approx 1$.

Results of determining the saved combat capabilities of components by our forces and by an enemy based on stages of military action are given in Table 3.

Table 3

Saved combat potentials of components by opposing sides

Indicators	Names of samples of weapons of components of sides' forces	Variant 1		Variant 2		Variant 3	
		Stages of military action		Stages of military action		Stages of military action	
		1 – MAS by enemy	2 – MAS by our forces	1 – MAS by enemy	2 – MAS by our forces	1 – MAS by enemy	2 – MAS by our forces
Saved combat potentials of components of enemy's striking forces	Strike aircraft	1038	672	1038	632	1038	594
	Tactical ballistic missiles	46	4	46	4	46	4
	Helicopters	215	183	215	182	215	181
Saved combat potentials of components of enemy's ADT forces	Fighter aircraft	228	174	228	154	228	131
	Anti-aircraft missile complexes AMC	60	39	60	39	60	39
	Anti-aircraft systems ADT GF	20	13	20	13	20	13
Saved combat potentials of components of our striking forces	Strike aircraft	398	262	548	377	698	489
	Tactical ballistic missiles	177	81	177	81	177	81
	Helicopters	97	70	97	71	93	68
Saved combat potentials of components of ADT by our forces	Fighter aircraft	324	306	324	307	324	308
	Anti-aircraft missile complexes AMC	78	78	78	78	78	78
	Anti-aircraft systems ADT GF	44	44	44	44	44	44

Results from calculating the ratios of fighting potentials of opposed forces at the stages of military activities are shown in Fig. 4.

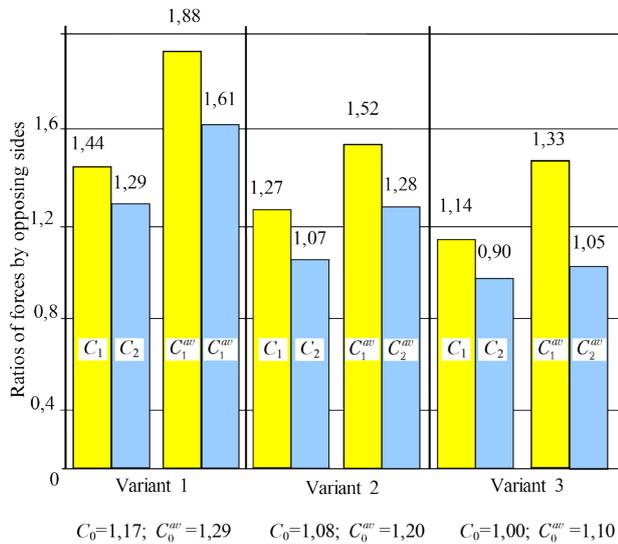


Fig. 4. Change in the ratio of combat potentials by opposing sides for variants considered in the example

The results shown in Fig. 4 illustrate the dependence of the total ratio and the ratio of aviation forces from opposing sides on their initial combat structure.

This, in turn, makes it possible to evaluate the effectiveness of activities that are carried out by bodies of military administration in order to prevent the enemy's superiority, or its strengthening, in the air.

6. Discussion of results of studying the combat structure of forces engaged in the airspace

It follows from the results for the above example that combat potentials of components of our forces, which are original (variant 1), do not make it possible to fulfil the task on preventing an enemy from strengthening its superiority in the air (following a MAS by our forces, $C_2^{av} = 1.61$ which substantially exceeds the initial ratio $C_0^{av} = 1.29$).

This is explained by the insufficient combat potential of our strike aviation to defeat an enemy's means at their home bases and on positions.

To perform the set task, it is necessary to increase combat potential (structure) of our strike aviation by 150 conditional units (by 16 %) (Table 2, variant 2). In this case, the ratio of aviation forces after our MAS is $C_2^{av} = 1.28$. To ensure equality of aviation forces by opposing sides, the combat potential of our strike aviation must be increased by 33 %, to 1,200 conditional units (Table 2, variant 3).

By using an example of calculating the structure of strike aviation, we have shown the efficiency of the devised procedure for substantiating the required combat structure of forces in order to prevent the enemy's superiority, or its strengthening, in the air.

The developed methodological positions can be used to determine the required combat potential (structure) not only of strike aviation, but also of all the components in the system for defeating enemy's troops and facilities in order to prevent the enemy's superiority, or its strengthening, in the air.

A special feature of the proposed method, compared to others, is the stage-wise consideration of the integrated application of all different forces by an enemy, as well as by our troops, engaged in a fight for air supremacy.

When conducting a research using the devised procedure, the limitation is the existence in the armed forces of weapons to perform a task on preventing the enemy's superiority, or its strengthening, in the air.

The procedure does not imply using the means of destruction, designed using new physical principles.

The developed methodical provisions for the substantiation of the combat structure of forces for activities in airspace should be used when developing the appropriate specialized mathematical software for bodies of military administration.

The disadvantage of the proposed procedure is incomplete accounting of features in the process of applying dissimilar forces by opposing sides during military action (fight) that could be eliminated in the future by using a mathematical model of bilateral military activities. Therefore, constructing such a model for the substantiation of a combat structure of forces that must act to accomplish superiority in the air is a promising direction in the advancement of this study.

7. Conclusions

1. We have shown the expediency of considering the totality of disparate forces that are engaged in the struggle for superiority in the airspace as a complex organizational-technical system for military purposes – a system for defeating troops and facilities. The morphological pattern, derived though its decomposition, has made it possible to determine the impact of the application of system components on the superiority of enemy's aviation during military action.

2. It is accepted to consider, as a criterion for enemy's aviation superiority, the ratio of combat potentials by aviation forces from opposing sides at the end of military action, which is equal to 1.5–2.0. According to the proposed methodological approach, determining the required structure of forces to prevent an enemy's superiority, or its strengthening, is carried out on the basis of a comparison of the values for the estimated and assigned criteria. The measures to prevent the superiority of enemy's aviation are defined using the method of iteration.

3. For the calculation of ratios between the forces from opposing sides, we have derived mathematical expressions for determining the saved combat potentials of components by opposing sides at each stage of military action, which take into consideration the mathematical expectations of their losses when exchanging missile and air strikes.

4. Using an example, we determined the required structure of our forces in order to prevent the enemy's superiority in the air. According to the initial data for the given example, the ratio of combat potentials in the aviation forces by opposing sides at the beginning of military action is 1.29, and after MAS exchange, it is 1.61, indicating the achievement of superiority in the air by the enemy. It is shown that in order to prevent the superiority of aviation forces by an enemy, it is expedient, when considering the integrated application of all opposing forces, to increase the combat potential (structure) of strike aviation by 16 %. In this case, the ratio of combat potentials in the aviation forces by opposing sides after MAS exchange would make up 1.28.

References

1. Tomashev V. N. O sovershenstvovanii metodov ocenki boevykh vozmozhnostey voysk // *Nauka i voennaya bezopasnost'*. 2006. Issue 2. P. 18–22.
2. Zahorka O. M., Perepelytsia V. A., Zaplishna A. I. Metodychni pidkhody do vyznachennia boiovykh potentsialiv i koefitsientiv porivniannia zrazkiv ozbroiennia ta viyskovoï tekhniky // *Zbirnyk naukovykh prats TsNDI OVT Zbroinykh Syl Ukrainy*. 2008. Issue 19. P. 32–43.
3. Prognostication of troops and objects losses from the aviation shots of opponent / Onischenko S. I., Zagorka O. M., Koval V. V., Tyurin V. V. // *Systemy ozbroiennia i viyskova tekhnika*. 2011. Issue 2. P. 2–8.
4. Prognostication of correlation of forces and facilities of counteractive sides is during an air operation / Onischenko S. I., Zagorka O. M., Koval V. V., Tyurin V. V. // *Systemy ozbroiennia i viyskova tekhnika*. 2011. Issue 1. P. 2–7.
5. Onyshchenko S. I., Zahorka O. M., Koval V. V. Do pytannia rozpodilennia syl ta vyznachennia poslidovnosti zavdannia raketno-avitsiynykh udariv u povitrianiy operatsiyi // *Nauka i oborona*. 2012. Issue 1. P. 39–44.
6. Mozharovski V. M., Hodz' S. V. Methodical aspects of the substantiation of a staff of the armed forces of a state from the standpoint of prevented damage theory // *Kybernetyka i systemnyi analiz*. 2018. Vol. 54, Issue 1. P. 154–167.
7. Sofranov S. P., Shishkin V. S. To the question of quantitative-qualite relation determination of aviations groupment forces // *Nauka i tekhnika Povitrianykh Syl Zbroinykh Syl Ukrainy*. 2011. Issue 2 (6). P. 10–11.
8. Bonin A. S. Substantiation technique composition of aviation grouping on a strategic direction, in operations of an initial stage of war // *Vestnik Akademii voennykh nauk*. 2014. Issue 3 (48). P. 39–42.
9. Drozdov S., Leontyev O. The methodology of setting and solving the inverse problem of optimization of the combat (quantitative and qualitative) structure of tactical aviation and the anti-aircraft missile forces of perspective air forces // *Nauka i tekhnika Povitrianykh Syl Zbroinykh Syl Ukrainy*. 2017. Issue 2 (27). P. 7–14.
10. Development of research on methodological preliminary rationale required quantitative and qualitative composition of the anti-aircraft missile armed group anti-aircraft missile troops / Laneckij B. N., Lukyanchuk V. V., Vasilyev V. A., Koval I. V. // *Nauka i tekhnika Povitrianykh Syl Zbroinykh Syl Ukrainy*. 2012. Issue 3 (9). P. 58–60.
11. Faber P. Competing Theories of Airpower: a Language for Analysis. URL: <http://www.au.af.mil/au/awc/awcgate/au/faber.htm>
12. Grunwald Jr. M. Transforming Air Force ISR for the long war and beyond. 2009. URL: <https://apps.dtic.mil/dtic/tr/fulltext/u2/a495105.pdf>
13. Speed J., Panagiotis S. SEAD Operations of the Future. The Necessity of Jointness // *The Journal of the JAPCC*. 2018. URL: <https://www.japcc.org/sead-operations-of-the-future/>
14. Liu Y., A. Simaan M., B. Cruz Jr J. An application of dynamic Nash task assignment strategies to multi-team military air operations // *Automatica*. 2003. Vol. 39, Issue 8. P. 1469–1478. doi: [https://doi.org/10.1016/s0005-1098\(03\)00122-5](https://doi.org/10.1016/s0005-1098(03)00122-5)
15. Ryan A. J. Military Applications of Complex Systems // *Philosophy of Complex Systems*. 2011. P. 723–780. doi: <https://doi.org/10.1016/b978-0-444-52076-0.50024-9>
16. Weapon System Capability Assessment under uncertainty based on the evidential reasoning approach / Jiang J., Li X., Zhou Z., Xu D., Chen Y. // *Expert Systems with Applications*. 2011. Vol. 38, Issue 11. P. 13773–13784. doi: <https://doi.org/10.1016/j.eswa.2011.04.179>
17. Barszcz P., Zieja M. Method of Choosing the Information Technology System Supporting Management of the Military Aircraft Operation // *Research Works of Air Force Institute of Technology*. 2014. Vol. 35, Issue 1. P. 141–154. doi: <https://doi.org/10.1515/afit-2015-0010>
18. Nikolaev V. I., Bruk V. M. *Sistemotekhnika: metody i prilozheniya*. Leningrad: Mashinostroenie, 1985. 199 p.
19. *Elementy doslidzhennia skladnykh system viyskovoho pryznachennia* / Zahorka O. M., Mosov S. P., Sbitniev A. I., Stuzhuk P. I. Kyiv: NAOU, 2005. 100 p.
20. Baskakov A. Ya., Tulenkov N. V. *Metodologiya nauchnogo issledovaniya: ucheb. pos.* Kyiv: MAUP, 2002. 216 p.
21. Petuhov S. I., Stepanov A. N. *Effektivnost' raketnykh sredstv PVO*. Moscow: Voenizdat, 1976. 104 p.
22. Ventcel' E. S. *Issledovanie operatsiy*. Moscow: "Sovetskoe radio", 1972. 552 p.
23. *Modeliuvannia boiovykh diy viysk (syl) protypovitrianoi oborony ta informatsiynе zabezpechennia protsesiv upravlinnia nymy (teoriya, praktyka, istoriya rozvytku): monohrafiya* / Horodnov V. P., Drobakha H. A., Yermoshyn M. O., Smirnov Ye. B., Tkachenko V. I. Kharkiv: KhVU, 2004. 410 p.
24. Rudnenko A. V. O kriteriyah dostizheniya celey vozdushnykh operatsiy i operatsiy ob'edineniy VVS // *Voennaya mysl'*. 1999. Issue 2. P. 46–52.
25. Zagorka A. N., Koval V. V., Zharik O. M. To the question of ground of indexes and criteria of efficiency of air defensive // *Nauka i tekhnika Povitrianykh Syl Zbroinykh Syl Ukrainy*. 2013. Issue 2 (11). P. 35–40.