Оптимізація процесу управління запасами пов'язана з пошуком моделі прогнозування, методу побудови часового ряду прогнозування, моделі логістичної операції, визначення обґрунтованого рівня страхового запасу і визначення критерію оптимізації.

Успішного вирішення оптимізаційної задачі в цілому можна домогтися тільки в тому випадку, якщо буде успішно вирішене весь комплекс локальних задач управління. При цьому метод побудови коінтегрованого часового ряду прогнозування попиту є центральною ланкою технології оптимального управління запасами. Це пов'язано з тим, що імовірнісний характер попиту є основним фактором зниження ефективності управління в системах цього класу.

Показано, що запропонований метод підвищення ефективності управління може бути використаний в будь-яких економічних системах, зважаючи на можливість створення єдиної логістичної моделі onepauiï.

Пропонований підхід заснований на формуванні часового ряду, спеціально призначеного для вирішення завдання оперативного прогнозування попиту в системах буферизації запасів. Такий ряд містить у собі як інформацію про обсяги реалізованої продукиї, так і дані пов'язані зі споживчим попитом.

Оскільки споживча активність випереджає процес фізичного споживання продукції, з'являється можливість використання випереджальних маркерів в задачах прогнозування.

Дослідження операційних процесів з використанням веріфіцированного показника ефективності підтвердили гіпотезу наявності попереджувальних маркерів в рамках сформованого тимчасового ряду прогнозування.

Встановлено, що максимальна ефективність управління може досягатися в разі більш низьку точність побудови моделі прогнозу. Це пов'язано з тим, що логістична модель операції враховує витрати переміщення продукції та її вартісні оцінки на вході і виході операції

Ключові слова: прогнозування попиту, оперативне прогнозування, коінтегрований ряд попиту, ефективність використання ресурсів

UDC 007.5

DOI: 10.15587/1729-4061.2019.171260

IMPROVING CONTROL **EFFICIENCY IN BUFFERING SYSTEMS USING ANTICIPATORY** INDICATORS FOR **DEMAND FORECASTING**

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1. Introduction

Level of demand for a particular consumer product is one of the main factors determining the feasibility of business development and its scope [1]. Since the demand for a consumer product is closely connected with its price characteristic [2], in conditions of a saturated market, economic structures are faced with the problem of bettering efficiency of production and distribution of final products.

Effectiveness of the system process is influenced by many factors. They include the time of access to the sources of technological and energy products, their cost, the degree of development of the labor market, the degree of proximity of the final product to final consumers, availability of a developed infrastructure for product transportation, etc. All these factors are significant but stable enough making it possible to take into account their influence at the enterprise creation and development stages.

On the other hand, consumer demand is probabilistic in its nature which leads to a significant worsening of management efficiency and resource usage at the enterprise. Given that demand is probabilistic but not random, the increase in management efficiency is primarily achieved by using the methods increasing accuracy of demand forecasting.

In stationary demand systems, this requires solution of technological forecasting problems [3]. Such forecasting features by the fact that the technological parameters of intrasystem objects are the forecast objects.



In functional systems with their product demand of non-stationary nature, characteristics of external processes with respect to the system within which the forecast is made are the forecast objects [4]. Low quality of demand forecast in such systems is the main cause of low efficiency of management processes.

At the same time, an increase in forecast accuracy does not necessarily results in an increase in management efficiency [5].

Thus, one of the main ways to improved efficiency of resource usage consists in development of methods increasing efficiency of operational forecasting.

2. Literature review and problem statement

Despite long history of the issue, classical methods of processing retrospective data in the form of a moving average [6], exponential smoothing [7], hyperbolic-exponential smoothing [8], forecasting with exponential smooth moving [9, 10] are actively developing at present.

Active studies in these areas point to a series of problems in the field of forecasting, in particular to the fact that solution to the problem of management efficiency is insistently shifted into a plane of increasing forecasting accuracy. Therefore, the use of neural-network technologies [10] and methods for processing fuzzy information [12] expands methods of data processing but does not solve the problem of improving the management efficiency [13].

One of the factors hindering development of methods for bettering management efficiency is the use of technical criteria based on assessment of the forecast accuracy. This refers to a class of technical indicators, in particular, to such widely used indicator as root-mean-square deviation.

Let us consider the problem essence.

Let there be a time series of demand within the framework of which it is possible to distinguish a time interval for forecasting (AB), a unit time interval within which the forecast is generated (BC) and a planning horizon (CD) for which the forecast is actually made. According to the results of conditional forecasting, a deviation $(\sigma 1)$ of the forecast value from the actual value was obtained (Fig. 1, a).

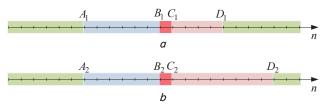


Fig. 1. Marked sections of the time series of demand for solving the forecasting problem: C_1D_1 planning horizon (a); C_2D_2 planning horizon, which is two times larger than C_1D_1 (b)

Naturally, when generating a forecast, one can change values of the *AB* and *CD* sections and if the planning horizon changes (*CD* section), the planned volume of sales will change, too.

Suppose a case when at a two-fold increase in the planning horizon, the value of root-mean-square deviation decreased by 2 %. Does this mean that the use of new model parameters will lead to higher management efficiency? Certainly not. An increase in of the planning horizon size

will necessitate an increase in the volume of purchased products almost twice which very likely reduce effectiveness of the planned operation. On the other hand, if an increase in the purchase volume will significantly reduce cost of a unit volume of the purchased product, effectiveness of such an operation may grow.

The above example shows that management efficiency is related to forecasting accuracy, however to make a weighted decision, it is necessary to use such an indicator as resource utilization efficiency [14] and special methods of operations research [15].

Analysis of [6–13, 16] also shows that development of demand forecasting methods is based on using time series of the demand itself as a time series of forecasting.

It is obvious, however, that this series contains information about seasonal demand fluctuations and allows one to make rather «rough» forecasts relying mainly on the market inertance. However, this does not mean that it is unnecessary to strive for bettering the forecasting accuracy. It must be done. But the increase in forecasting accuracy in conjunction with pricing policy should affect improvement of management efficiency.

Thus, enhancing management efficiency by means of improvement of forecasting accuracy is an important scientific task.

- 1. It has been established that the improved efficiency of logistic system management is ensured by the method of increasing efficiency of demand forecasting.
- 2. The proposed method for determining optimal management in logistic systems is based on determination of such operation model parameters as scaling interval, forecast interval, product delivery time, planning horizon. In this case, choice of the best parameters of the operation model is determined by the criterion of resource usage efficiency based on the cost parameters of the product and the costs associated with its movement.
- 3. A method for formation of a cointegrated time series of demand forecasting was proposed. It is based on the use of the data structure of the number-of-consumers time series and the corresponding quantitative parameters of volumes of the product consumed. Such a series contains both information on sales volumes and that related to the consumer demand. Compared with the use as a series for forecasting sales volumes, the proposed cointegrated series contains anticipatory indicator signals. This allows one to forecast future variations in sales dynamics before its appearance in a trend.

3. The aim and objectives of the study

The study objective was to improve management efficiency by developing a method for generating a time series with anticipatory capabilities to solve the problem of raising efficiency of demand forecasting.

To achieve the objective, the following tasks were set:

- develop a method of anticipatory diagnosis of changes in the sales trend using the data of the factor marker;
- develop a method of forming an optimal management based on the developed model of logistic operation which takes into account features of the method of anticipatory diagnosis of changes in the demand dynamics, the costs of movement of the product and its valuation at the operation input and output.

4. The model of operation of a logistic system

Basis of any enterprise is formed by functional systems of three classes: moving, transforming, and buffering systems. If a trading system is considered, its typical architecture is shown in Fig. 2.

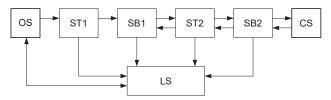


Fig. 2. OS is the external product delivery system; ST1 is the system for moving products from an external supplier; SB1 is the input product buffering system; ST2 is the buffering system in the area accessible to the consumer; SB2 is the buffering system in the customer service area; CS is the product consumers; LS is the logistic system

Here, stocks move from the supplier to the consumer through the use of two movement systems (Fig. 3).

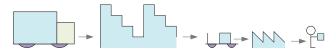


Fig. 3. The process of moving products from a supplier to a consumer

In order to manage the products movement from a supplier to an enterprise, it is necessary to have an idea about the level of demand for the enterprise product, the general stocks level of the buffering systems, transportation costs and the supplier's price policy.

Fulfillment of this task requires special information from the functional systems of the enterprise, suppliers of the product, its processing and formation of management.

Such system of a conversion class was defined by the concept of «logistic system».

An objective need to use such a system relates to the fact that the moving systems, the internal processes of the product supplier and its location affect the system process in the form of delays and costs (Fig. 4).

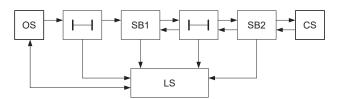


Fig. 4. Logistic system in the structure of the linear part of the commercial enterprise

In addition, each supplier has a certain pricing policy and organization of movement processes requires costs.

Management in production systems is realized in a somewhat different way (Fig. 5).

Knowing percentage of products from supplier 1 and supplier 2 in the enterprise output, the structure shown in Fig. 5 can be converted to two structures equivalent to that shown in Fig. 4.

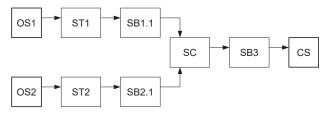


Fig. 5. Structure of a two-product production system

Thus, approach to stocks management in trading and production systems has a single logistic model of the corresponding functional system.

In order to form a management signal (U) for replenishment of stocks, it is necessary to obtain value of the product remainder at the time of management formation (R), generate sales forecast relative to the sampling interval (PF), have information about the time of delivery of the production batch (T_D), determine value of the planning horizon (T_G) and amount of reserve stocks (PQ).

Then, the stocks remaining at the time of receipt of the new batch (R') can be determined as a difference between the stocks remaining at the time of management formation and the amount of the planned product sales in the time interval of delivery (TT):

$$R' = R - PF \cdot TT.$$

Replenishment is unnecessary if:

$$R' \ge (PF \cdot GP + SZ).$$

Otherwise, amount of the batch of stocks replenishment is determined as the difference between the sales forecast on the planning horizon taking into account the reserve stocks and the remaining stocks at the time of receipt of a new product batch (Fig. 6).

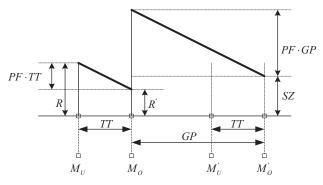


Fig. 6. Explanation of quantitative determination of the size of the batch of stocks replenishment

In a general case, the estimated value of the magnitude of stocks replenishment is determined from the expression:

$$RQ' = \begin{cases} PF \cdot GP + SZ - R', & \text{if } R' \leq \left(PF \cdot GP + SZ\right), \\ 0, & \text{if } R' \geq \left(PF \cdot GP + SZ\right). \end{cases}$$

As can be seen, this requires information about the delivery time. All other parameters (*PF*, *PH*, *SZ*) are determined by examining simulated operations using the forecasting time series.

However, this is not enough to form management since the planning horizon size can change. Value of the volume of external replenishment can affect valuation of a unit input product batch and, consequently, the operation efficiency.

Thus, to assess effectiveness of a logistic operation, data is needed not only on the quantitative parameters of the input and output product flows but also on the cost parameters (Fig. 7).

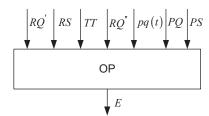


Fig. 7. Conceptual model of the logistic operation: OP is the operation process; RQ' is the volume of the product replenishment batch; RS is the unit cost estimate for the input batch of replenishment products; TT is the time interval for delivery of the input batch of products; RQ'' is the carry-over stocks batch; pq(t) is the time series of sales volumes; PQ is the size of the reserve stocks batch; PS is the cost estimate of a unit of the realized output product batch

Without exception, all parameters at the input of the logistic operation are determined based on the forecast time series, that is, the higher forecast capabilities of the method used the forecast generation, the more effective the management.

5. The time series of the demand forecast

The principle of generation of any forecast is reduced to choosing a method for processing the nearest section of retrospective data (of the forecast interval) and using the possibility of extrapolation of the resulting trend for the future time period (of the planning horizon). At the same time, it is necessary to determine optimal parameters of the forecast interval itself, the planning horizon and the choice of a model for generation of the forecast.

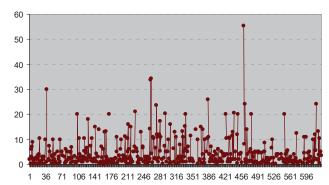
However, all these methods are well developed and have exhausted themselves as a source of increasing management efficiency.

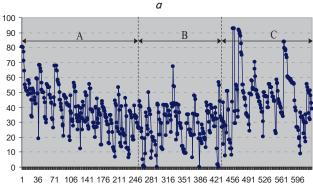
It is obvious that the maximum management efficiency can be obtained only if the forecast itself is absolutely accurate.

It is clear that it is impossible to achieve absolute accuracy since the cause of reduction or demand growth is beyond the scope of the functional system. However, it is quite possible to further improve management efficiency if not only capabilities of the time series of sales volumes but also the time series of the number of buyers are used to generate the forecast.

For example, sales volumes can be used as a time series of demand only if the level of reserve stocks has not dropped to a critical level. Thus, judgment of the past level of demand for products (Fig. 8) can be expressed at the A and C sections while the level of reserves has decreased to zero at B section.

On the other hand, it is obvious that it is impossible to explain the cause of change in demand in the C section based on the data of sales in the B section. This is due to the fact that the sales data do not contain indicators that can be used to forecast the future demands trend.





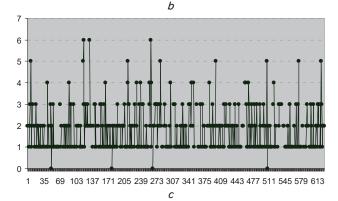


Fig. 8. Retrospective data on products movement: sales volume (a); stock remainder (b); the number of buyers (c)

Here, it is possible to assess trends associated with seasonality of demand, the influence of weather conditions or holiday dates. However, the forecast should take into account not only influence of natural factors on the overall trend but also changes in demand within the trend of natural changes.

Let us express a hypothesis that the data on time series of the number of product buyers can be used as markers of future changes in demand (Fig. 8, c). Then, using data on the time series of sales volumes and the number of product consumers, one can form a cointegrated time series. Its data can be used to generate more accurate forecasts. The use of more accurate forecasts will improve management efficiency in logistic systems.

This hypothesis cannot be substantiated theoretically since the demand for products is the result of functioning of external systems whose operation principles and parameters are inaccessible to the logistic system. Therefore, the method can be verified only indirectly by evaluating the results of effectiveness of the operational process based on the use of synthesized time series.

The hypothesis can be considered confirmed in the event that the management efficiency increases when using the synthesized time series as the time series of demand.

The time series obtained by combining the data of the time series of product sales and the time series of the number of consumers can be defined as a time series of forecasting or a forecasting function.

The idea that behavior of externally weakly interconnected economic variables leads to some interconnected change has led to the development of a theory of cointegrated time series in order to construct an error correction model [17]. Therefore, the time series of demand forecasting is also defined further in the text as a cointegrated time series.

In order to use forecasting capabilities of the time series of the number of consumers and gain access to quantitative parameters of production volumes, it is necessary to construct a time series combining these data.

The method's essence consists in generation of a time series whose properties include structural data related to purchase requirement and quantitative data related to sales volumes.

So, the number of buyers is determined in the section [*ab*] according to the data of the *A* series (Fig. 9):

$$CQ_j = \sum_{a}^{b} cq_i, \quad i = \overline{a...b},$$

and the consumption volume is determined according to the *B* series:

$$PQ_j = \sum_{a}^{b} pq_i, \quad i = \overline{a...b},$$

where [a, b] is the scaling interval (SI); CQ_j is the number of buyers in the scaling interval; cq_i is the number of customers in a unit sampling interval; i is the current sampling interval; PQ_j is the volume of the product consumed in the scaling interval; pq_i is the volume of the product consumed in a unit sampling interval; j is the unit sampling interval that immediately follows the scaling interval.

Next, average sales per customer in a unit sampling interval, $j(pq_j^*)$ are determined:

$$pq_j^* = \frac{PQ_j}{CQ_i},$$

and the value of the forecasting time series in a unit sampling interval:

$$pf_j = cq_j \frac{PQ_j}{CQ_i},$$

where cq_j is the number of buyers in the unit sampling interval j, pf_j is the value of the demand forecasting series in the unit sampling interval j (Fig. 4).

$$j = \overline{k...m}$$
.

Fig. 10 shows an example of converting a time series of the number of buyers (Fig. 10, a) and a time series of volumes of consumed products (Fig. 10, b) into a time series of demand forecast (Fig. 10, c).

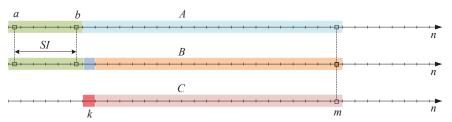
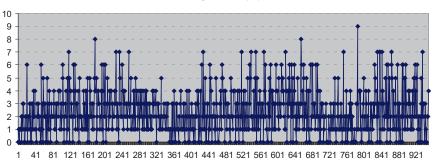
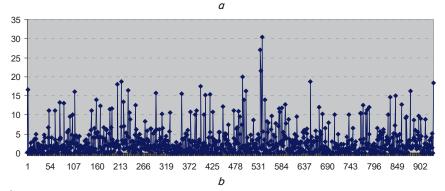
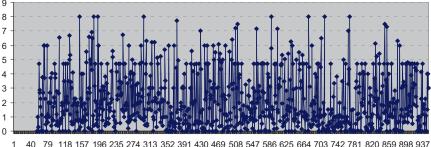


Fig. 9. The principle of forming a time series of demand forecast (C) based on the time series of the product consumption volume (A) and the number of buyers of this product (B)







1 40 79 118 157 196 235 274 313 352 391 430 469 508 547 586 625 664 703 742 781 820 859 898 937

Fig. 10. Converting the data of sales volumes and the number of buyers into a time series of demand forecast: sales volumes (a); the number of buyers (b); cointegrated demand forecasting series (c)

Thus, the cointegrated time series (Fig. 4, c) obtained by processing time series (Fig. 4, a, b) will be used as a time series for forecast generation but not the series of volumes of products consumed daily.

6. The method and results of comparative study of forecasting effectiveness

In order to assess the possibility of using the cointegrated time series in the demand forecast problems, it is necessary to compare the forecast results based on the use of the original and synthesized series.

The process of management formation is reduced to determination of the reference forecast interval and the planning horizon. What concerns the synthesized series, determination of the scaling interval magnitude is also required.

Studies have shown (Fig. 11) that with a change in the reference forecast interval (*RFI*), the maximum forecast efficiency is achieved at *RFI*=21.

Fig. 12 shows results of the study when the size of the planning horizon changes.

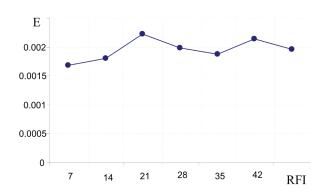


Fig. 11. Dependence of forecast effectiveness on change in the reference forecast interval

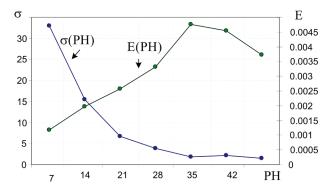


Fig. 12. Dependence of efficiency and root-mean-square deviation of the forecast value on the actual demand when the planning horizon (*PH*) changes

Studies have shown that efficiency of the operational process for a given product is maximum if the planning horizon value PH=35 is adopted. In parallel, such a parameter as the root-mean-square deviation was assessed.

One can see that the forecast accuracy increases with an increase in PH. However, the value of PH=35 corresponds to the maximum efficiency.

Using the selected *RFI* and *PH* values, efficiency of operational processes was determined when the size of the reserve stocks (SI) changed (Fig. 13).

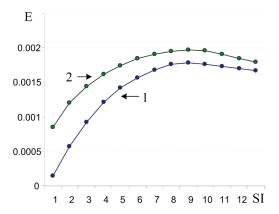


Fig. 13. Dependence of effectiveness of the demand forecasting process on the use of the sales volume (1) and cointegrated series (2) as forecast series

One can see that maximum efficiency of the operational process of assessing the management parameters in the case of using the cointegrated series is $10\,\%$ higher for the reserve stocks of 9 units.

7. Discussion of the results obtained in the studies related to assessment of the expected effectiveness of parameters

A typical requirement to the forecast result is its accuracy. Such an opinion can be considered an axiom when it comes to environmental, scientific, and technical forecast or weather forecast.

The demand forecast falls into the category of forecasts where accuracy gives way to efficiency.

For example, if growth of the planning horizon raises the forecast accuracy by 1 % but requires a two-fold increase in the volume of purchases, a less accurate forecast will be preferred since efficiency of the generated operations is higher in the first case. Certainly, in this case, the indicator of resource usage efficiency takes into account accuracy of forecasting as well but with taking into account resource consumption and pricing policy.

At the same time, an increase in forecasting accuracy can be obtained through the use of factor criteria.

In this case, the universal anticipatory index was used: the number of purchases.

The fact that management efficiency increases with cointegrated forecasting series taken as a basis is clearly shown graphically in Fig. 13. At the same time, to use the proposed method, there is no need to collect additional specific data.

The fact of improving management efficiency with the use of combined series indicates that before increasing consumption of a particular product, the number of small purchases first increases.

Definitely, the proposed method will work reliably only if there were no noticeable areas of product shortage or hidden deficit during functioning of the enterprise. Fig. 14 shows how the hidden deficit in location b leads to a change in demand relative to location a.

This means that availability of a non-zero stocks level does not guarantee working quality of the proposed method. It is necessary to conduct additional studies to determine value of the hidden deficit level and apply a method of constructing the demand time series in conditions of presence of this deficit.

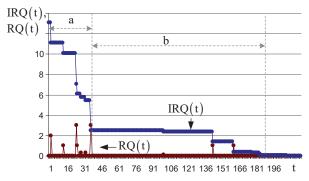


Fig. 14. Change in the volume of products consumed with a change in stocks levels

8. Conclusions

- 1. It has been established that the increase in management efficiency in logistic systems is provided by the method of improving efficiency of demand forecasting.
- 2. The proposed method of determining optimal management in logistic systems is based on determination of such parameters of the operation model as scaling interval, forecast interval, product delivery time, planning horizon. In this case, choice of the best parameters of the operation model is determined by the criterion of resource usage efficiency based on the product cost parameters and the costs associated with the product movement.
- 3. A method of formation of a cointegrated time series of demand forecasting was proposed. It is based on using the structure of data of the time series of the number of consumers and corresponding quantitative parameters of volumes of products consumed. Such a series contains both information on sales volumes and data related to consumer demand. Compared with the use as a series for forecasting sales volumes, the proposed cointegrated series contains anticipatory indicator signals. This makes it possible to forecast future change in sales dynamics before its appearance in a trend.

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