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DEVELOPMENT OF A MODEL OF THE SERVICE SYSTEM OF BATCH ARRIVALS IN THE PASSENGERS FLOW OF PUBLIC TRANSPORT

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Запропоновано математичну модель системи масового обслуговування потоку пасажирів міського громадського транспорту. Отримана модель відрізняється від канонічних моделей теорії масового обслуговування урахуванням принципових особливостей реальних систем. По-перше, процес обслуговування розбитий на наступні один за одним різні сеанси обслуговування. По-друге, заявки на вході і на виході системи є груповими. По-третє, інтенсивності вхідного потоку в різних сеансах обслуговування різні. По-четверте, закони розподілу числа вимог в групових заявках для різних сеансів різні. По-п'яте, закони розподілу числа заявок в групах на вході системи і виході з неї також різні.

Розроблено критерій ефективності системи обслуговування. Критерій заснований на розрахунку розподілу ймовірностей станів системи обслуговування на вході системи і аналогічного розподілу на виході системи. Ці розподілу визначаються незалежно для кожного сеансу обслуговування, на які розбивається весь цикл обслуговування. Чисельне значення критерію задається відношенням середнього числа відмов в обслуговуванні до середнього числа вимог в сумарному входить потоці за весь цикл обслуговування. Його можна використовувати для оцінки ефективності системи обслуговування на будь-якому обраному часовому інтервалі протягом доби, т. я. значення запропонованого критерію залежить від довжини інтервалу між сеансами, яка визначається числом транспортних засобів на маршруті.

Отримані моделі адекватно відображають процес функціонування системи, що дає можливість прогнозувати безліч різних ситуацій і оцінювати наслідки пропонованих рішень. При цьому виникає можливість прогнозування транспортної забезпеченості населення та визначення кількісних значень ефективності функціонування системи міського громадського пасажирського транспорту

Ключові слова: система масового обслуговування, міський громадський транспорт, розподіл вимог в груповій заявці, розподіл числа відмов

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1. Introduction

Urban public transport is the industry that provides the city with services of delivering people on established routes, guaranteeing accessible, regular and mass traffic. The functioning of the transport system is one of the most important components of the city's life, affecting most of its inhabitants.

Passenger transport is one of the key sectors of the national economy. Due to the fact that many citizens do not have personal vehicles, timely and adequate satisfaction of transport demands develops from a purely transport problem into a social one. This problem determines the attitude of the population not only to the quality of transport services, but also to the processes that occur in the regions and the country as a whole.

The urban public transport system refers to open systems. According to the degree of abstraction, the methods used in the study of transport systems are arranged in the following order:

– observation and verbal description of processes;
– full-scale modeling;
– machine simulation;
– analytical models [1].

Observation of processes boils down to direct registration of transport phenomena: measuring the rate of passenger flows, passenger traffic, occupancy of rolling stock, etc. Full-scale modeling is testing the analyzed system or its parts in real or close conditions. In this case, the influence of traffic intervals on the occupancy of rolling stock, efficiency of various types of rolling stock on the route, etc. are taken into account. Machine simulation and analytical models represent the highest degree of formalization. Distinguishing between machine simulation and analytical models is sometimes difficult. They allow describing the urban public transport system as an interconnected set of numerical models. These models adequately reflect the functioning of the system, which makes it possible to predict many different situations and evaluate the consequences of proposed solutions. Thus, it becomes possible to predict the provision of the population with public transport and determine quantitative values of the efficiency of the urban public transport system. In this regard, the analysis of the organization of the urban public transport system is an urgent and important task.

2. Literature review and problem statement

From a functional point of view, urban public transport is a queueing system [2, 3]. However, the real system of servicing passenger flows by urban public transport has an important feature. The traditional canonical queuing theory assumes that arrivals are ordinary [4–8]. In [9], the problem of evaluating the efficiency of queuing systems with non-ordinary arrivals is posed and general recommendations on the analysis of such systems are given. In [10], the problem of serving batch arrivals in a system with refusals is considered for the case when the number of batch arrivals is fixed. In [11], the technology proposed in [10] is developed for infinite queuing systems. In [12], a single-channel batch arrival queueing system is considered. In [13], a multistage batch arrival queueing system is considered. In [14], a method for analyzing a queueing system with a random number of batch arrivals is proposed. The proposed approaches for analysis with batch arrivals for a number of fundamental reasons cannot be used in assessing the effectiveness of the service system of urban public transport passengers. Firstly, in a real system, not only arrivals but also departures are batch. Secondly, the above works assume that the arrival rate does not change during the entire service cycle. In reality, this rate is changed in each session of the service cycle. Thirdly, there are different distribution laws of jobs in the batch for different service sessions. Fourth, known techniques consider waiting systems, while in reality, these are systems with refusals. These circumstances determine the need to develop, in terms of queuing theory, a model of the system that takes into account the real features of a real service system of urban public transport passengers.

3. The aim and objectives of the study

The aim of the study is to develop a mathematical model and a method for assessing the efficiency of the service system of the passenger flow of urban public transport.

To achieve this aim, it is necessary to accomplish the following objectives:

– in terms of queuing theory, to develop a method for calculating the distribution of the number of busy channels after a batch arrival at the beginning of service session and the number of busy channels after the end of service session;

– to develop a method for calculating the distribution of the number of refusals;

– to develop a criterion for the efficiency of batch service.

4. Development of a mathematical model of a batch arrival queueing system

4. 1. Definitions and assumptions

We introduce the following definitions.

Service cycle – a set of all measures for servicing passengers of urban public transport, carried out from the moment the vehicle enters the route until the end of the working day.

Service session – a set of measures for servicing passengers starting from the moment the vehicle arrives at the next stop until it goes on the route.

Job – the next passenger arriving at the stop to enter the vehicle.

Service – passenger travel from the moment of entering the vehicle and occupying a free seat (channel) until the moment of exit.

Refusal – a situation that a passenger experiences if, by the time of entering the vehicle, all free seats (channels) are occupied.

We formulate the problem of forming a mathematical model of a real queuing system. We introduce the necessary notations:

– ζ – random number of jobs in the batch arrival;

– η – random number of jobs received by the system on free channels;

– ν – random number of jobs rejected due to the business of all channels;

– ξ – random number of jobs departing from the system upon completion of service;

– j – number of channels engaged in service.

Next, let:

– $\underline{\pi} = \{\underline{\pi}_0, \underline{\pi}_1, ..., \underline{\pi}_n\}$ – probability distribution of system states after the departure of served jobs;

– $\overline{\pi} = \{\overline{\pi}_0, \overline{\pi}_1, ..., \overline{\pi}_{j,...}, \overline{\pi}_n\}$ – probability distribution of system states after batch arrival of jobs to the system.

Let us assume that the flow of jobs entering the system, forming a batch arrival, is stationary, ordinary and has no aftereffect. Then the distribution of a random number of jobs ζ in the batch arrival during the time interval of length t is subject to Poisson law:

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \tag{1}$$

$P_k(t)$ – the probability that exactly k jobs arrive to the system during the time interval t; λ – arrival rate of jobs.

Let m channels be busy in the system by the end of the next service session. We introduce q – the probability that the service of each of the served jobs will be completed in the next session. Then the distribution of the number of jobs departing from the system at the end of the service is subject to Bernoulli law:

$$r_m^k = Ber\{\xi = k / j = m\} = C_m^k q^k (1-q)^{m-k}, \quad k = 0, 1, ..., m. \tag{2}$$

Let the distribution $\underline{\pi}$ of the number of busy channels after the end of the previous session be obtained by the time the next service session starts.

4. 2. Essence and justification of calculation methods

We determine the distribution of the number of busy channels after the next batch arrival. The probability of e channels business is equal to the sum of the products of $j \le e$ channels business probabilities multiplied by the conditional probability of arrival of the number of jobs transferring the system in a state with e busy channels. Then

$$\bar{\pi}_e = \underline{\pi}_0 P_e + \underline{\pi}_1 P_{e-1} + \ldots + \underline{\pi}_e P_0, \quad e = 0,1,\ldots,n. \tag{3}$$

Similarly, the probability

$$\underline{\pi}_e = \bar{\pi}_e r_e^0 + \bar{\pi}_{e+1} r_{e+1}^1 + \ldots + \bar{\pi}_{n-1} r_{n-1}^{n-e+1} + \bar{\pi}_n r_n^{n-e},$$

of e channels business after the end of the next service session is equal to the sum of the products of $j \ge e$ channels business probabilities multiplied by the conditional probability of the number of jobs departing from the system transferring the system into the state e. In this case, the distribution of the number of busy channels will be as follows:

$$e = 0, 1, 2, \ldots, n. \tag{4}$$

We now find the distribution of a random number of rejected jobs ν. In this case, the probability of rejection of exactly e jobs is equal to the product of the probability of j channels business multiplied by the probability that there will be exactly $n - j + e$ jobs in the batch arrival. Then

$$\nu_0 = Ber\{\nu = 0\} = \underline{\pi}_0 Ber(\xi \le n) +$$
$$+ \underline{\pi}_1 Ber(\xi \le n-1) + \ldots + \underline{\pi}_{n-1} Ber(\xi \le n+1) + \underline{\pi} \ \ Ber(\xi = 0);$$

$$\nu_1 = Ber\{\nu = 1\} = \underline{\pi}_0 Ber(\xi = n+1) + \underline{\pi}_1 Ber(\xi = n) +$$
$$+ \underline{\pi}_2 Ber(\xi = n-1) + \ldots + \underline{\pi}_{n-1} Ber(\xi = 2) + \underline{\pi} \ \ Ber(\xi = 1);$$

....

$$\nu_e = Ber\{\nu = e\} = \underline{\pi}_0 Ber(\xi = n+e) +$$
$$+ \underline{\pi}_1 Ber(\xi = n+e-1) +$$
$$+ \underline{\pi}_{n+1} Ber(\xi = e+1) + \ldots + \underline{\pi} \ \ Ber(\xi = e);$$

$$Ber\{\nu > n\} = \underline{\pi}_0 Ber(\xi \ge 2n) +$$
$$+ \underline{\pi}_1 Ber(\xi > 2n-1) + \underline{\pi}_n Ber(\xi \ge n). \tag{5}$$

To calculate distributions (3)–(5), it is necessary to know the values of the parameters of Poisson (1) and Bernoulli (2) laws. These parameters can be calculated from the results of statistical processing of experimental data. The resulting estimates of the parameters λ and q uniquely determine the analytical descriptions of the distribution laws (1) and (2). The method for calculating the required estimates is as follows [5, 6].

Suppose that, based on the results of direct observations, the set $(m_1, m_2, \ldots, m_n)$, specifying the arrival rate of a given number of jobs k during a given interval $(0, t)$ is formed. Here m_j is the arrival rate of exactly j jobs. In accordance with Poisson law, the probability of arrival of exactly k jobs on the interval of length t is determined by the formula

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \quad k = 0,1,2,\ldots$$

where λ is the desired value of the Poisson law parameter, which determines the arrival rate of jobs. We find this value by maximum likelihood estimation. Let us introduce the likelihood function

$$P = \prod_{k=0}^{n} \left[\frac{(\lambda t)^k}{k!} e^{-\lambda t} \right]^{m_k} = \prod_{k=1}^{n} \left(\frac{\theta^k}{k!} e^{-0} \right)^{m_k}. \tag{6}$$

We determine the value λ, maximizing (6). It is convenient to take the logarithm of the likelihood function (6) in advance. We have

$$L = \ln P = \sum_{k=0}^{n} m_k \ln \left[\frac{\theta^k}{k!} e^{-0} \right] = \sum_{k=1}^{n} m_k \left[\ln \frac{1}{k!} + k \ln \theta - \theta \right]. \tag{7}$$

Then

$$\frac{dL}{d\theta} = \sum_{k=1}^{n} m_k \left(\frac{k}{\theta} - 1 \right) = 0.$$

Hence

$$\frac{1}{\theta} \sum_{k=1}^{n} m_k k - n = 0, \quad \theta = \frac{n}{\sum_{k=1}^{n} m_k k}, \quad \hat{\lambda} = \frac{n}{t \sum_{k=1}^{n} m_k k}. \tag{8}$$

The resulting arrival rate of jobs is an unbiased, consistent and asymptotically normal estimate of the true value of the desired arrival rate.

In the real study (100 experiments), the number of experiments in which zero, one, two, ..., five jobs arrived within two minutes was respectively equal to (10; 17; 26; 25; 17; 5). The corresponding frequency distribution is as follows (0; 1; 0.17; 0.26; 0.25; 0.17; 0.05). Now, using (3), we obtain the most plausible estimate of the arrival rate of jobs:

$$\hat{\lambda} = \frac{5}{2(0.17 \cdot 1 + 0.25 \cdot 2 + 0.25 \cdot 3 + 0.17 \cdot 4 + 0.06 \cdot 5)} =$$
$$= 1.37 \frac{\text{req}}{\text{min}}.$$

The adequacy of the accepted hypothesis about the Poisson nature of the distribution law of a random number of jobs can be checked using the criterion χ^2. As is known, the Pearson χ^2 criterion allows checking the agreement of the empirical distribution function with a hypothetical function $f(x)$, selected from some set of traditionally used ones (indicative, binomial, normal, etc.). The range of possible values of the observed random variable is divided into a number of intervals of equal length. In relation to the study, it is convenient, instead of splitting, to record the observed value of the number of jobs, while determining the number of experiments in which the observed random variable took a specific value. Let the set $(n_0, n_1, n_2, \ldots, n_6)$ specify the resulting distribution. Here, n_i is the number of experiments in which the observed random variable took the value i, and

$$\sum_{i=1}^{k} i n_i = n,$$

where n is the total number of jobs.

Let us introduce the null hypothesis H_0: the chosen hypothetical distribution of the observed random variable does not contradict the experimental data. To validate the hypothesis H_0, the statistical criterion χ^2 is used, calculated by the formula:

$$\chi^2=\sum_{i=0}^{k}\frac{\left(n_i-np_i\right)^2}{np_i}.$$

Here p_i is the probability that the observed value will take the value *i*, which is determined in accordance with the hypothetical distribution law. If the null hypothesis assumes Poisson distribution of the random value of the number of jobs, then this probability is equal to

$$P_i=\frac{(\lambda t)^i}{i!}e^{-\lambda t},\quad i=0,1,2,...,k.$$

We calculate the values of P_i, $i=0,1,2,3,4,5$ for the given t=2 and the value of $\hat{\lambda}=1.37$ found in (4). We have

$$P_0=e^{-\lambda t}=e^{-2\cdot 1,37}=e^{-2,74}=0.065;$$

$$P_1=\lambda+e^{-\lambda t}=2,74\cdot e^{-2\cdot 74}=0.178;$$

$$P_2=\frac{(\lambda t)^2}{2!}e^{-\lambda t}=\frac{7,51}{2}e^{-2\cdot 74}=0.244;$$

$$P_3=\frac{(\lambda t)^3}{3!}e^{-\lambda t}=\frac{20.74}{6}e^{-2\cdot 74}=0.223;$$

$$P_4=\frac{(\lambda t)^4}{4!}e^{-\lambda t}=\frac{56.36}{24}e^{-2\cdot 74}=0.152;$$

$$P_5=\frac{(\lambda t)^5}{5!}e^{-\lambda t}=\frac{154.4}{120}e^{-2\cdot 74}=0.08;$$

$$P_6=\frac{(\lambda t)^6}{6!}e^{-\lambda t}=\frac{423.1}{720}e^{-2\cdot 74}=0.038.$$

The obtained set of probabilities of arrival of the given number of jobs to the service system allows calculating the expected distribution of the number of cases with a given number of jobs. Substituting this distribution in (5) determines the value of the Pearson criterion χ^2. We have

$$\begin{aligned}\chi^2&=\frac{(10-100\cdot 0.065)^2}{100\cdot 0.065}+\frac{(17-100\cdot 0.178)^2}{100\cdot 0.178}+\\&+\frac{(26-100\cdot 0.244)^2}{100\cdot 0.244}+\frac{(25-100\cdot 0.223)^2}{100\cdot 0.223}+\\&+\frac{(17-100\cdot 0.152)^2}{100\cdot 0.152}+\frac{(5-100\cdot 0.08)^2}{100\cdot 0.08}=\\&=\frac{(3.5)^2}{6.5}+\frac{(0.8)^2}{17.8}+\frac{(1.6)^2}{24.4}+\frac{(2.7)^2}{22.3}+\frac{(1.8)^2}{15.2}+\frac{3^2}{8}=\\&=1.888+0.04+0.105+0.327+\\&+0.213+1.125=3,69.\end{aligned}$$

The calculated value of the criterion χ^2 is now compared with the critical value χ^2_{cr} taken from the Student distribution table for the given value of the significance level $\alpha=0.05$ and the number of the degree of freedom $\nu=k-r-1$, where k is the number of intervals, r is the number of parameters of the hypothetical function. In our case $\nu=7-1-1=5$. Wherein $\chi^2_{cr}=11.07$. As a result of the comparison, one of two decisions is made:

– if $\chi^2\geq\chi^2_{cr}$, the null hypothesis H_0 is rejected;

– if $\chi^2<\chi^2_{cr}$, the hypothetical distribution function is considered consistent with the experimental results and there is no reason to reject the hypothesis H_0.

In the study, $\chi^2=3.69$, $\chi^2_{cr}=11.07$. Therefore, $\chi^2<\chi^2_{cr}$, so the hypothesis H_0 is accepted.

The Bernoulli distribution (2) parameter q is estimated similarly. Let n independent tests be carried out, each $i=0,1,2,...,n$ recording the number of jobs k_m, $i=0,1,2,...,m$, from the sample of the volume of m jobs departing from the system. As a result, we get the set $\left\{k_m^{(1)},\ k_m^{(2)},\ ...,\ k_m^{(n)}\right\}$. In accordance with Bernoulli law, the probability that exactly $k_m^{(i)}$ out of m jobs depart from the system is

$$P\left(k_i\right)=C_m^{k_m^{(i)}}q^{k_m^{(i)}}\left(1-q\right)^{m-k_m^{(i)}}.$$

Then the likelihood function is

$$P=\prod_{i=1}^{n}C_m^{k_m^{(i)}}q^{k_m^{(i)}}\left(1-q\right)^{m-k_m^{(i)}}=\prod_{i=1}^{n}C_m^{k_m^{(i)}}q^{\sum_{i=1}^{m}k_m^{(i)}}\left(1-q\right)^{\sum_{i=1}^{n}\left(m-k_m^{(i)}\right)}.$$

Then

$$L=\ln P=\ln\left(\prod_{i=1}^{n}C_m^{k_m^{(i)}}\right)+\sum_{i=1}^{m}k_m^{(i)}\ln q+\sum_{i=1}^{n}\left(m-k_m^{(i)}\right)\ln\left(1-q\right);$$

$$\frac{\mathrm{d}L}{\mathrm{d}q}=\frac{\sum_{i=1}^{m}k_m^{(i)}}{q}-\frac{\sum_{i=1}^{n}\left(m-k_m^{(i)}\right)}{1-q};$$

$$\frac{q}{1-q}=\frac{\sum_{i=1}^{m}k_m^{(i)}}{\sum_{i=1}^{n}\left(m-k_m^{(i)}\right)}.$$

Hence

$$\hat{q}=\frac{\sum_{i=1}^{m}k_m^{(i)}}{\sum_{i=1}^{m}k_m^{(i)}+\sum_{i=1}^{n}\left(m-k_m^{(i)}\right)}=\frac{\sum_{i=1}^{m}k_m^{(i)}}{m\cdot n}.$$

The resulting estimate of the probability of job departure is unbiased, consistent and asymptotically normal.

4. 3. Development of a criterion of efficiency of serving batch arrivals

Let us go back to the original problem. The above relations (3)–(5) specify the probability distributions of system states and the number of jobs rejected in each of the service sessions. This allows evaluating the effectiveness of the described service system as a whole. As an efficiency criterion of the system, it is natural to choose the ratio of the average number of rejected jobs during the entire service cycle to the average number of arrivals. Let us introduce

$$g=\frac{\sum_{s=1}\sum_{e=1}^{e^*}e\cdot\nu_e^s}{\sum_{s=1}\lambda_s T}.\tag{9}$$

Here s is the number of service session, $s=1,2,...$; ν_e^s – probability of rejection l of the job in the s-th session; $e=1,2,...,s^*$, λ_s – arrival rate of jobs in the s-th service session; T – duration of the interval between sessions.

The value e^* is determined from the inequality

$$\frac{(\lambda_s T)^{e^*}}{e^*!}e^{-\lambda_s T}=\varepsilon,$$

ε – a fairly small value (for example, $\varepsilon=10^{-2}$).

The value calculated in accordance with (9) gives an estimate of the efficiency of the passenger flow service system for the entire session. Of course, it can be used to evaluate the efficiency of this system at any selected time interval within 24 hours. It is clear that the value g given by (9) depends on the length of the interval T between sessions, which is determined by the number of vehicles on the route. If the obtained value of the share of rejected jobs to the total number of jobs exceeds the permissible threshold (for example, 0.2), then the number of vehicle units should be increased. The resulting integer optimization problem should be solved by successively increasing the number of vehicles per unit until the value of the criterion becomes acceptable.

5. Discussion of the results of the development of a model for serving batch arrivals

A mathematical model of the queuing system, taking into account the features of such a system in relation to servicing the passenger flow by urban transport is proposed. This model, in contrast to the known ones, takes into account the following fundamental features of real systems for servicing the flow of urban transport passengers. Firstly, the batch nature of arrivals and departures. Secondly, differences in the arrival rate for different service sessions. Thirdly, differences in the laws of distribution of the number of jobs in batch arrivals and departures for different service sessions.

The proposed model allows finding:

– the probability distribution of system states at the input at the beginning of service session (3);

– the probability distribution of states at the end of service session (4);

– the distribution of the number of rejections in each service session (5).

It should be noted that the values of the parameters of the Poisson and Bernoulli laws λ and q necessary for calculating the distributions (3)–(5) should be known. If they can be determined by the results of statistical processing of experimental data, the resulting parameter estimates uniquely specify analytical descriptions of the distribution laws (1) and (2).

According to the results of the study, the estimates of the arrival rate of jobs (λ) and the probability that serving of each of the jobs will be completed the next session (q) are unbiased, consistent and asymptotically normal. This allows assuming that there are no obstacles to the use of the proposed methods for calculating the distributions (3)–(5), since the parameter estimates uniquely specify analytical descriptions of the distribution laws (1) and (2).

However, it should be noted that in the development of the method, there was an assumption of stationarity, ordinariness and absence of aftereffect of jobs, forming a batch arrival. This can be considered one of the limitations of this study.

The service efficiency criterion taking into account the number of rejections during the entire service cycle is formulated. The proposed criterion (9) is the ratio of the average number of rejected jobs during the entire service cycle to the average number of arrivals. This idea is justified, since the value calculated in accordance with (9) gives an estimate of the efficiency of the passenger flow service system for the entire session. But, at the same time, it remains possible to use it to assess the efficiency of this system at any selected time interval within 24 hours. This is due to the fact that the value g given by (9) depends on the length of the interval T between sessions, which is determined by the number of vehicles on the route.

The practical use of the proposed criterion consists in the possibility of deciding whether to increase the number of vehicle units on the route. The principle of decision-making in this case consists in comparing the calculated value of the share of rejected jobs to the total number of jobs with an acceptable threshold (for example, 0.2). The resulting integer optimization problem should be solved by successively increasing the number of vehicles per unit until the value of the criterion becomes acceptable. The development of such an approach based on real operational observations may be of further theoretical and practical interest.

6. Conclusions

1. A method for calculating the number of busy channels in case of batch arrival is developed. It is based on the description of the distribution of the number of busy channels after the next batch arrival, taking into account the fact that the probability of business of e channels is equal to the sum of the products of the probabilities of $j \le e$ channels business multiplied by the conditional probability of arrival of the number of jobs transferring the system in the state with e busy channels.

2. A method for calculating the number of busy channels after the end of service is developed. It is based on the description of the distribution of the number of busy channels, taking into account the fact that the probability of business of e channels after the end of the next service session is equal to the sum of the products of the probability of $j \ge e$ channels business multiplied by the conditional probability of departure of the number of jobs transferring the system to the state e.

3. It is shown that in order to determine the distribution of a random number of rejected jobs ν, it is necessary to take into account that the probability of rejection of exactly e jobs is equal to the product of the probability of business of j channels multiplied by the probability that there will be exactly $n-j+e$ jobs in the batch arrival. It is shown that the estimates of the arrival rate of jobs (λ) and the probability that serving of each of the jobs will be completed in the next session (q) are unbiased, consistent and asymptotically normal. These estimates of the parameters of the Poisson and Bernoulli law should be known for using the proposed calculation methods.

4. It is proposed to choose the ratio of the average number of rejected jobs during the entire service cycle to the average

number of arrivals as a criterion of efficiency of the passenger flow service system. The value calculated in this case gives an estimate of efficiency of the passenger flow service system for the entire session. However, it can be used to evaluate the efficiency of this system at any selected time interval during the day, since the value of the proposed criterion depends on the length of the interval between sessions, determined by the number of vehicles on the route.

References

1. Vel'mozhin, A. V., Gudkov, V. A., Kulikov, A. V., Serikov, A. A. (2002). Effektivnost' gorodskogo passazhirskogo obshchestvennogo transporta. Volgograd, 256.
2. Venttsel', E. S. (2005). Teoriya veroyatnostey. Moscow: Vysshaya shkola, 576.
3. Glurman, V. E. (2006). Teoriya veroyatnostey i matematicheskaya statistika. Moscow: Vysshaya shkola, 404.
4. Borovkov, A. A. (1986). Teoriya veroyatnostey. Moscow: Nauka, 386.
5. Kramer, G. (1975). Matematicheskie metody statistiki. Moscow: Mir, 638.
6. Borovkov, A. A. (1984). Matematicheskaya statistika. Moscow: Nauka, 472.
7. Matveev, V. F., Ushakov, V. G. (1984). Sistemy massovogo obsluzhivaniya. Moscow: MGU, 240.
8. Kleynrok, L. (1979). Teoriya massovogo obsluzhivaniya. Moscow: Mashinostroenie, 432.
9. Afanas'eva, L. G., Rudenko, I. V. (2012). G|G|∞ queues and their applications to the transport models analysis. Theory of Probability and its Applications, 57 (3), 375–395. doi: https://doi.org/10.4213/tvp4460
10. Monsik, V. B., Skrynnikov, A. A., Fedotov, A. Y. (2010). Queuing system with group service of nonordinary flow of demands. Nauchniy vestnik Moskovskogo gosudarstvennogo tehnicheskogo universiteta grazhdanskoy aviatsii, 157, 42–50.
11. Monsik, V. B., Skrynnikov, A. A., Fedotov, A. J. (2012). Multichannel queuing system indivisible group of applications with a queue of unlimited length. Nauchniy vestnik Moskovskogo gosudarstvennogo tehnicheskogo universiteta grazhdanskoy aviatsii, 184, 108–112.
12. Banik, A. D. (2015). Single server queues with a batch Markovian arrival process and bulk renewal or non-renewal service. Journal of Systems Science and Systems Engineering, 24 (3), 337–363. doi: https://doi.org/10.1007/s11518-015-5268-y
13. Sharma, R. (2014). Mathematical Analysis of Queue with Phase Service: An Overview. Advances in Operations Research, 2014, 1–19. doi: https://doi.org/10.1155/2014/240926
14. Daw, A., Pender, J. (2019). On the distributions of infinite server queues with batch arrivals. Queueing Systems, 91 (3-4), 367–401. doi: https://doi.org/10.1007/s11134-019-09603-4