1. Introduction

Due to a series of their operational advantages, the tumbling type mills remain to be the main equipment in many industries for small- and large-tonnage fine grinding of solid materials.

Replacement of the conventional steady-state grinding process with a novel self-oscillating process improves existing equipment of relatively low power efficiency. Use of the phenomenon of excitation of self-oscillations makes it possible to apply conventional solutions to designing the tumbling mills with a smooth working chamber surface without additional activating elevators in a form of protruding elements which undergo rapid abrasive wear.

On the other hand, significant variability of the self-excited pulsation behavior of the rotating chamber fill de-
The DEM method was applied in [10] to numerical modeling of tumbling mill processes. Effects of discrete values of the chamber filling degree \( \kappa \) on power intensity of the grinding process in the tumbling mill seem rather relevant. A three-dimensional numerical DEM modeling of the effect of the chamber filling degree \( \kappa \) in a semi-autogenous-grinding (SAG) mill on the impact action of the grinding fill was performed in [11]. Three-dimensional DEM modeling of the effect of the chamber filling degree \( \kappa \) on power, and operational parameters of the SAG mills was carried out in [12]. The DEM method was used in [13] for numerical evaluation of the effect of the chamber filling degree on power intensity of SAG mills. Experimental analysis of the effect of discrete values of the chamber filling degree \( \kappa = 0.25, 0.35 \) and 0.45 on efficiency of the gypsum grinding process in a mill was performed in [14]. Analysis of influence of the chamber filling degree \( \kappa \) on power efficiency of the process of grinding various materials in a tumbling mill based on numerous experimental data is given in [15]. Effect of the chamber filling degree \( \kappa \) on technological efficiency of the processes of clinker and quartz grinding in a tumbling mill was experimentally studied in [16]. Experimental study of the effect of the charge filling degree \( \kappa \) on efficiency of the process of ultrafine grinding of gypsum in a tumbling mill was performed in [17].

Rational values of degree of filling the chamber with fill were established numerically and experimentally to optimize parameters of the conventional grinding process. Effect of discrete values of the chamber filling degree \( \kappa = 0.05, 0.1, 0.2, 0.3, 0.4 \) and 0.5 on power intensity of the quartz grinding process using a tumbling mill was determined numerically in [18] with further experimental verification. Rational value of \( \kappa = 0.3 \) was established for maximum grinding performance. Effect of the chamber filling degree \( \kappa = 0.15, 0.2, 0.25, 0.3, 0.35 \) and 0.4 on power intensity of the process of grinding in a tumbling mill was performed in [19] by numerical DEM method with experimental verification. Rational value of \( \kappa = 0.35 \) was found for the maximum rotary drive power.

The effect of discrete values of the chamber filling degree \( \kappa = 0.15, 0.2 \) and 0.25 on specific power intensity of the process of fine grinding cement in tumbling mills was experimentally studied in [20]. It was established that rational value of \( \kappa \) depends on conditions of realization of the grinding process. The results of an experimental study of the effect of the chamber filling degree \( \kappa \) on parameters of barite grinding in a tumbling mill were given in [21]. The value of \( \kappa = 0.35 \) was experimentally determined for maximum grinding performance. Influence of values of the chamber filling degree \( \kappa = 0.2, 0.3, 0.4 \) and 0.5 on parameters of silica grinding process in a tumbling mill was established in [22]. It was found that specific power intensity of grinding was minimal at \( \kappa = 0.3 \). Influence of discrete values of the chamber filling degree \( \kappa = 0.2, 0.3, 0.35, 0.4 \) and 0.45 on technological parameters of the process of dry grinding of calcite in a tumbling mill was experimentally investigated in [23]. A rational value of \( \kappa = 0.35 \) was established for maximum efficiency of the grinding process. Effect of the chamber filling degree on dynamic parameters of intrachamber fill of a tumbling mill during dry and wet grinding was experimentally studied in [24] for values of \( \kappa = 0.2 \) and 0.3, and 0.4. An increase in frequency and energy of impact action of the fill with increasing \( \kappa \) was observed. It was established that the lowest specific power intensity of the dry grinding process is achieved at \( \kappa = 0.2 \).

Numerical and experimental methods were applied in determining influence of the degree of chamber filling with fill on parameters of conventional grinding process in tumbling mills.
Influence of the chamber filling degree $\kappa$ on productivity and power intensity of the process of grinding brittle materials in tumbling mills was investigated numerically in [25] by means of the DEM method. It was found that when $\kappa$ increases, kinetic energy of intrachamber and productivity increase. However, a significant increase in $\kappa$ results in dissipation of energy of the grinding bodies as a result of increase in colliding frequency and a lower productivity and power efficiency of grinding. Influence of the chamber filling degree $\kappa$ on interaction of the fill elements in the SAG mill chamber was studied in [26] by numerical DEM modeling. It was found that an increase in $\kappa$ causes a decrease in impact action and an increase in abrasion effect of the fill.

Effect of the chamber filling degree $\kappa$ on power of the SAG mill drive and impact action of the grinding fill on particles of the crushed material were experimentally studied in [27]. It was found that a significant decrease in $\kappa$ increases magnitude and frequency of impacts and productivity of the grinding process. Effect of values of the chamber filling degree $\kappa = 0.12, 0.18, 0.24, 0.3$ and 0.36 on the tumbler mill drive power during wet grinding of copper ore was experimentally studied in [28]. A significant increase in power was observed with an increase in $\kappa$. Influence of the chamber filling degree in the range of $\kappa = 0.2 \ldots 0.4$ on technological parameters of the process of wet grinding of platinum ore in a tumbling mill was experimentally studied in [29]. An increase in grinding fineness with an increase in $\kappa$ was established. Influence of the chamber filling degree $\kappa = 0.12, 0.18, 0.24, 0.3$ and 0.36 on the impact action of wet grinding of copper ore in a tumbling mill was experimentally studied in [30]. A decrease in the values of impact forces of grinding bodies with an increase in $\kappa$ was found. The effect of discrete values of the chamber filling degree $\kappa = 0.05, 0.1, 0.15, 0.2, 0.25$ and 0.3 on impact action of the grinding fill and the mill drive power in grinding iron ore was experimentally studied in [31]. It was found that the drive power increases with an increase in $\kappa$. Forces of impact action of the ball grinding bodies of the fill attain maximum values at $\kappa = 0.2$.

Thus, the data obtained from numerical simulation and experiments have shown a significant influence of the chamber filling degree $\kappa$ on parameters of the grinding process occurring in the tumbling mills. This effect consists in a growth of the impact action of the grinding fill with a decrease in $\kappa$. Instead, with an increase in $\kappa$, impact action is reduced and the abrasion effect increases, dissipation of kinetic energy of the grinding bodies increases because of collision and the grinding power efficiency is reduced. However, these results relate just to a conventional grinding process at simple conditions of steady-state intrachamber fill motion.

No models have been constructed to date to determine effect of the chamber filling degree $\kappa$ on grinding parameters in self-excitation of a complex mode of transient motion of the pulsating fill. Absence of such models is particularly negative in the case of realization of the innovative self-oscillating grinding process in tumbling mills.

### 3. The aim and objectives of the study

The study objective consists in establishing the effect of chamber filling degree on characteristics of dynamic action of the fill and technological and power parameters of conventional steady-state and innovative self-oscillating processes of grinding in a tumbling mill. It would make it possible to predict efficiency of realization of self-oscillating grinding processes at a varying degree of filling the chamber with fill.

To achieve this objective, the following tasks were set:

- to perform analytical and experimental modeling of dynamic action of the tumbling mill fill under steady-state and self-oscillating conditions of motion for various chamber filling degrees;
- to determine and compare magnitudes of parameters of the collision momenta of the fill particles for various conditions of motion and the chamber filling degrees;
- to perform experimental modeling of technological and power parameters of the conventional steady-state and the innovative self-oscillating grinding processes in a tumbling mill for various degrees of chamber filling with the fill;
- to determine and compare magnitudes of grinding productivity and power intensity for various processes and the chamber filling degrees.

### 4. The procedure of studying the effect of degree of chamber filling with the fill on the process of grinding in a tumbling mill

It is convenient to use dimensionless criterial parameters for modelling the grinding process in a tumbling mill. The drum rotation is defined by relative velocity:

$$\psi_w = \frac{\omega R}{R}$$

where $\omega$ is the angular velocity of rotation; $R$ is the radius of the drum chamber; $g$ is gravitational acceleration.

Useful filling of the drum is defined by the degree of filling the chamber with the fill:

$$\kappa = \frac{w}{\pi R^2 L},$$

where $w$ is the volume of the charge at rest; $L$ is the length of the drum chamber.

Conventional steady-state grinding process in a tumbling mill is realized at a moderate relative rotational velocity $\psi_w = 0.7 \ldots 0.9$. At the chamber filling degree $\kappa = 0.25 \ldots 0.45$, the fill circulates in a three-phase mode with formation of three flow zones in the chamber cross-section (Fig. 1, 2, [9]).

**Fig. 1.** A pattern of steady-state fill motion with absolute $d$ and relative $d/(2R) = 0.01 \ldots 0.03$ particle size at the chamber filling degree $\kappa = 0.35$; the chamber rotates with relative velocity $\psi_w = 0.75$.
The innovative self-oscillating grinding process in a tumbling mill is realized in the case of increased rotational velocity $\psi_w \approx 1...1.2$. At $\kappa = 0.25...0.45$, a pulsating zone of oscillating fill flow appears due to self-excitation. The creeping layer and the restricted fall zone partially (Fig. 5, [9]) or completely (Fig. 6) transform into this zone.

The self-oscillating grinding (Fig. 7, 8, [9]) is realized by an impact action on the BCE boundary including sections BC and CE. The BC boundary corresponds to the transition of the restricted fall zone 2 (Fig. 7 and [9]) or the pulsation zone 4 (Fig. 8) to the solid state zone 1. The CE section corresponds to the contact of the pulsation zone 4 with the chamber surface.

Conventionally, grinding (Fig. 3, 4, [9]) is performed mainly by an impact action on the boundary BC of transition from the zone of restricted fall 2 to the zone of the creeping layer 3 and the abrasion action in the creeping layer 3. Technological results of grinding resulting from such actions are commensurate.

**Fig. 2.** A pattern of steady-state fill motion with relative particle size $d/(2R) = 0.01...0.03$ at the chamber filling degree $\kappa = 0.25$; the chamber rotates with relative velocity $\psi_w = 0.75$

**Fig. 3.** Design diagram for determining the fill impact in a steady motion at $d/(2R) = 0.01...0.03$, $\kappa = 0.35$ and $\psi_w = 0.75$ (according to the motion pattern in Fig. 1): 1 — solid state zone; 2 — restricted fall zone; 3 — creeping zone

**Fig. 4.** Design diagram for determining the impact action of the fill in a steady-state motion at $d/(2R) = 0.01...0.03$, $\kappa = 0.25$ and $\psi_w = 0.75$ (according to the motion pattern in Fig. 2): 1 — solid state zone; 2 — restricted fall zone; 3 — creeping zone

**Fig. 5.** Consecutive patterns of the fill motion over time $t$ for one period $T_{op}$ of self-oscillations with a maximum swing at $d/(2R) = 0.01...0.03$, $\kappa = 0.35$ and $\psi_w = 1.075$:
- $a - t = 0$; $b - t = T_{op}/12$; $c - t = 2T_{op}/12$; $d - t = 3T_{op}/12$; $e - t = 4T_{op}/12$; $f - t = 5T_{op}/12$; $g - t = 6T_{op}/12$; $h - t = 7T_{op}/12$; $i - t = 8T_{op}/12$; $j - t = 9T_{op}/12$; $k - t = 10T_{op}/12$; $l - t = 11T_{op}/12$; $m - t = T_{op}$
a                               b                                c

d                               e                                f

g                               h                                i

j                               k                                l

m

Fig. 6. Consecutive patterns of the fill motion over time $t$ for one period $T_{op}$ of self-oscillations with a maximum swing at $d/(2R) = 0.01...0.03$, $\kappa = 0.25$ and $\psi_{w} = 1.05$: $a - t = 0$; $b - t = T_{op}/12$; $c - t = 2T_{op}/12$; $d - t = 3T_{op}/12$; $e - t = 4T_{op}/12$; $f - t = 5T_{op}/12$; $g - t = 6T_{op}/12$; $h - t = 7T_{op}/12$; $i - t = 8T_{op}/12$; $j - t = 9T_{op}/12$; $k - t = 10T_{op}/12$; $l - t = 11T_{op}/12$; $m - t = T_{op}$

Fig. 7. Calculation diagram for determining impact of the fill in a self-oscillating motion at $d/(2R) = 0.01...0.03$, $\kappa = 0.35$ and $\psi_{w} = 1.075$ (according to the motion patterns in Fig. 5): $1$ — solid state zone; $2$ — restricted fall zone; $4$ — pulsation zone

Fig. 8. Design diagram for determining the impact of fill in self-oscillating motion at $d/(2R) = 0.01...0.03$, $\kappa = 0.25$ and $\psi_{w} = 1.05$ (according to the motion patterns in Fig. 6): $1$ — solid state zone; $4$ — pulsation zone

5. Analytical modeling of impact action of the grinding fill on the ground material

Technical results of the grinding process in conventional and self-oscillating modes of the mill operation can be conveniently evaluated by means of a comparative analysis of parameters of the fill impact action only.

Expression for the ratio of average sums of vertical components of collision momenta of intrachamber fill of a rotating drum in auto-oscillatory and steady-state motion modes per unit of time is as follows [9]:

$$\frac{S_{st}^{s}}{S_{st}^{a}} = \frac{K_{a}(1-\kappa_{op})\psi_{w}K_{ch}\sqrt{h_{op}}}{} + \kappa_{op}\psi_{w}K_{ch}\sqrt{h_{op}},$$  (1)

where $\kappa_{op}$ is the mass fraction of the pulsation zone of motion in the mass of the entire fill in the auto-oscillatory mode; $K_{st}$ is the fill turnover in a steady-state mode which determines number of the circulation periods in a single turn of the drum; $t_{st}$ is the duration of a period of fill circulation in the drum chamber in a steady-state mode; $\omega_{t}$ is the angular velocity of the drum rotation in a steady-state mode;

$$\omega_{st} = \frac{2\pi}{t_{st}\omega_{t}}$$

is the relative velocity of rotation of the drum in a steady-state mode;

$$K_{a} = \frac{2\pi}{t_{a}\omega_{t}}$$

is the fill turnover in a self-oscillating mode; $t_{a}$ is the duration of a period of fill circulation in the chamber in the self-oscillating mode; $\omega_{a}$ is the angular velocity of the drum in a self-oscillating mode;
\[ \psi_{\text{w}} = \omega \sqrt{\frac{R}{g}} \]

is the relative velocity of the drum rotation in an oscillating mode;

\[ \psi_{\text{exp}} = 2\pi \sqrt{\frac{R}{g} f_{\text{op}}} \]

is the relative angular frequency of the fill self-oscillations; \( f_{\text{op}} \) is the cyclic frequency of the fill self-oscillations; \( K_{\text{chf}} \) is the average coefficient of loss of the vertical velocity component of motion of the particles in the restricted fall zone in a steady-state mode;

\[ h_{j} = \cos \varphi_{j} \sum_{i=1}^{n} \frac{h_{i}}{k} \]

is the average height of falling of \( n \) particles in the restricted fall zone in a steady-state mode; \( h_{j} \) is the elementary linearized trajectory of falling of a single particle in the restricted fall zone in a steady-state mode; \( \varphi_{j} \) is the angle of inclination to the vertical of averaged linearized trajectories of the particle fall in the restricted fall zone in a steady-state mode; \( K_{\text{chf}} \) is the average coefficient of loss of vertical component of velocity of the particles in the restricted fall zone in a self-oscillating mode;

\[ \frac{S_{k}^{e}}{S_{k}^{o}} \]

is the average angle of inclination to the vertical of averaged linearized trajectories of the particle fall in the restricted fall zone in a self-oscillating mode; \( h_{j} \) is the elementary linearized trajectory of falling of a single particle in the restricted fall zone in a self-oscillating mode; \( \varphi_{j} \) is the angle of inclination to the vertical of averaged linearized trajectories of the particle fall in the restricted fall zone in a self-oscillating mode; \( K_{\text{chf}} \) is the average coefficient of loss of vertical component of velocity of the particles in the restricted fall zone in a self-oscillating mode;

\[ \frac{S_{k}^{e}}{S_{k}^{o}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

is the ratio of the average values of sums of powers of vertical components of the collision momenta of the intrachamber fall of the rotating drum in self-oscillating and steady-state motion modes per unit of time as follows [9]:

\[ \left( \frac{P_{a}}{P_{s}} \right)_{\text{exp}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

The expression for the ratio of the average values of sums of powers of vertical components of the collision momenta of the intrachamber fall of the rotating drum in self-oscillating and steady-state motion modes per unit of time is as follows [9]:

\[ \left( \frac{P_{a}}{P_{s}} \right)_{\text{st}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}} \psi_{\text{exp}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}} \psi_{\text{exp}}} \left| \frac{h_{j}}{h_{j}} \right| \]

The relation (4) varies during the self-oscillation period from the minimum value:

\[ \left( \frac{P_{a}}{P_{s}} \right)_{\text{st}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

the relative coefficient of loss of vertical component in the restricted fall in a self-oscillating mode; \( h_{j} \) is the relative velocity of the drum rotation in an oscillating mode; \( f_{\text{op}} \) is the cyclic frequency of the fill self-oscillations; \( K_{\text{chf}} \) is the average coefficient of loss of the vertical velocity component of motion of the particles in the restricted fall zone in a steady-state mode;

\[ h_{j} = \cos \varphi_{j} \sum_{i=1}^{n} \frac{h_{i}}{k} \]

The ratio (1) changes during the self-oscillation period from the minimum value:

\[ \frac{S_{k}^{e}}{S_{k}^{o}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

The relation (4) varies during the self-oscillation period from the minimum value:

\[ \left( \frac{P_{a}}{P_{s}} \right)_{\text{st}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

The ratio (4) varies during the self-oscillation period from the minimum value:

\[ \left( \frac{P_{a}}{P_{s}} \right)_{\text{st}} = \frac{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}}{K_{n} \psi_{\text{w}} \psi_{\text{exp}} K_{\text{chf}}} \left| \frac{h_{j}}{h_{j}} \right| \]

Values of the variables \( \psi_{\text{w}} \), \( \psi_{\text{exp}} \), \( K_{\text{chf}} \), \( h_{j} \), \( h_{j} \) and \( h_{j} \) in expressions (1)–(6) can be determined by visual analysis of the fill motion patterns. Values of variables \( \psi_{\text{w}} \) and \( \psi_{\text{exp}} \) can be determined by visual analysis of transient self-oscillating modes of the fill motion. The value of \( \psi_{\text{w}} \) is specified.

Fig. 9 shows the obtained quasi-static dependences of the \( K_{\text{t}} \) turnover on the relative rotational velocity \( \psi_{\text{w}} \) for a discrete charge with a relative particle size \( d/(2R) = 0.01...0.03 \).

The ratio of the collision parameters for self-oscillating and conventional steady-state operation modes (1) to (6) is convenient to use for comparative assessment of the impact of the grinding fill at different of the chamber filling degrees \( \kappa \).

6. Results obtained in determining the effect of the chamber filling degree on parameters of the fill impact action

According to the experimental data obtained, geometric and kinematic parameters of the fill motion at the chamber filling degree \( \kappa = 0.35 \) had the following values: \( K_{\text{chf}} = 0.3 \), \( K_{\text{chf}} = 0.75 \), \( K_{\text{chf}} = 1 \), \( h_{j}/h_{j} = 1.48 \), \( h_{j}/h_{j} = 1.6 \), \( h_{j}/h_{j} = 1.05 \), \( h_{j}/h_{j} = 1.075 \), \( h_{j}/h_{j} = 1.31 \) (at \( f_{\text{op}} = 2 \) Hz), \( K_{\text{chf}} = 1.82 \) (at \( f_{\text{op}} = 2 \) Hz), \( K_{\text{chf}} = 1.19 \) (at \( f_{\text{op}} = 2 \) Hz), \( K_{\text{chf}} = 1.05 \). Parameters of the fill motion at the chamber filling degree \( \kappa = 0.25 \) acquired the following values: \( K_{\text{chf}} = 0.2 \), \( K_{\text{chf}} = 0.0 \), \( K_{\text{chf}} = 1 \), \( h_{j}/h_{j} = 0 \), \( h_{j}/h_{j} = 2.46 \), \( h_{j}/h_{j} = 0.75 \), \( h_{j}/h_{j} = 1.05 \), \( h_{j}/h_{j} = 1.31 \) (at \( f_{\text{op}} = 2 \) Hz), \( K_{\text{chf}} = 2.01 \) (at \( f_{\text{op}} = 0.75 \)), \( K_{\text{chf}} = 1.29 \) (at \( f_{\text{op}} = 0.75 \)). \( \psi_{\text{max}} = 0.25 \).
Relation of average sums of vertical components of the collision momenta and the power of these momenta for self-oscillating and steady-state modes per unit time were determined. According to (2), (3), (5) and (6), extreme values of these relations at the chamber filling degree $\kappa = 0.35$ had the following values:

$$
\left( \frac{S_{aw}}{S_{aw}} \right)_{max} = 2.96, \quad \left( \frac{S_{sw}}{S_{sw}} \right)_{max} = 3.07,
$$

$$
(P_{sh}/P_{sh})_{max} = 9.35, \quad (P_{oh}/P_{oh})_{max} = 9.57.
$$

Extreme values of parameters of the collision momenta at the chamber filling degree $\kappa = 0.25$ acquired the following values:

$$
\left( \frac{S_{aw}}{S_{aw}} \right)_{max} = 0, \quad \left( \frac{S_{sw}}{S_{sw}} \right)_{max} = 5.79,
$$

$$
(P_{sh}/P_{sh})_{max} = 0, \quad (P_{oh}/P_{oh})_{max} = 45.5.
$$

Parameters of the collision momenta at $\kappa = 0.45$ were determined in [9]:

$$
\left( \frac{S_{aw}}{S_{aw}} \right)_{max} = 2.32, \quad \left( \frac{S_{sw}}{S_{sw}} \right)_{max} = 2.39,
$$

$$
(P_{sh}/P_{sh})_{max} = 5.41, \quad (P_{oh}/P_{oh})_{max} = 5.7.
$$

The graphs of dependences $S_{aw}/S_{aw}$ for variable values of numerators $S_{aw}$ and $S_{sw}$ for one period of self-oscillations at the degree of chamber filling with the fill $\kappa = 0.25, 0.35$ and 0.45 are shown in Fig. 10.

Fig. 10. Dependence of relations of average sums of vertical components of the collision momenta for steady-state modes per unit time $S_{sh}/S_{sh}$ and $S_{oh}/S_{oh}$ on the period of self-oscillations $T_{up}$ at $d/(2R) = 0.01...0.03, \kappa = 0.25, 0.35$ and 0.45.

The graphs of dependences $P/P_{sh}$ for the variable values of numerators $P_{sh}$ and $P_{oh}$ during the self-oscillation period at the chamber filling degree $\kappa = 0.25, 0.35$ and 0.45 are shown in Fig. 11.

Numerical values of dynamic parameters of the fill impact action indirectly characterize influence of the chamber filling degree $\kappa$ on the grinding process course.

7. Experimental modeling of effect of the degree of the chamber filling with the fill on the grinding process

Effect of the degree of the chamber filling with the fill on efficiency of the self-oscillating grinding process in a tumbling mill was evaluated for the case of grinding cement clinker.

Particles of pre-crushed clinker with relative size $d_{50}/(2R) < 0.0039$ completely filled in rest condition the gaps between steel ball grinding bodies with relative size $d_{50}/(2R) = 0.026$. Productivity of the grinding process with duration of 30 min was determined by sieving through a 0.08 mm mesh sieve.

Technological efficiency of the self-oscillating grinding process was evaluated by relative productivity:

$$
C_s = \frac{1 - m_{rs}/m_o}{1 - m_{rs}/m_o},
$$

where $C_s$ is productivity of the self-oscillating process; $C_o$ is productivity of the conventional steady-state process; $m_o$ is weight of the sieve residue of the crushed material after sieving during the self-oscillating process; $m_{rs}$ is weight of the sieve residue in the conventional steady-state process; $m_o$ is total weight of a portion of ground material prior to sieving.

Power efficiency of the self-oscillating grinding process was evaluated by relative power intensity:

$$
P_{sh}/P_o = \psi_{sh} \psi_{o5} \psi_{w} \frac{P_{sh}}{P_o} = \psi_{sh} \psi_{o5} \psi_{w} \frac{P_{sh}}{P_o},
$$

and relative specific power intensity:

$$
E_s = \frac{P_{sh}}{C_s} = \frac{P_{sh}}{C_s} / \frac{E_o}{C_o},
$$

where $P_{sh}$ is the power of the drive rotating the filled drum in the self-oscillating process; $P_o$ is the power of the drive in the conventional steady-state process; $E_o = P_o/C_o$ is the specific power intensity of the self-oscillating process; $E_s = P_{sh}/C_s$ is the specific power intensity of the conventional steady-state process; $\psi_{sh} = \psi_{o5} \psi_{w} \psi_{w}$ is the relative power of the drive in the self-oscillating process (Fig. 12); $\psi_{o5} = \psi_{o5} \psi_{w}$ is the relative power of the drive in the conventional steady-state process.
process (Fig. 12); \( \psi_{W0.5} = M_0/M_{max0.5} \) is the relative torque of the drive in the self-oscillating process (Fig. 13); \( \psi_{W0.5} = M_0/M_{max0.5} \) is the relative torque of the drive in the conventional steady-state process (Fig. 13); \( M_0 \) is the absolute torque of the drive in the self-oscillating process; \( M_0 \) is the absolute torque of the drive in the conventional steady-state process; \( M_{max0.5} \) is the absolute value of the conditional maximum antitorque for the half-filled chamber (\( \kappa = 0.5 \)) which corresponds to the apparent load distribution over cross section of the drum chamber in a form of an ideal solid-state segment rotated at a right angle relative to the initial position; \( \psi_{W0} \) is the relative velocity of the drum rotation during the self-oscillating process; \( \psi_{W0} \) is the relative velocity of the drum rotation in the conventional steady-state process.

\[
\kappa = 0.5
\]


d/\(2R\) = 0.01...0.03, \( \kappa = 0.25, 0.35 \) and 0.45

![Fig. 12. Dependence of relative power of the drive in rotation of the filled drum, \( \psi_{W0.5} \), on relative rotational velocity, \( \psi_{W0} \), at \( d/\(2R\) = 0.01...0.03, \kappa = 0.25, 0.35 \) and 0.45](image)

The obtained quasi-static dependences of dynamic parameters (Fig. 12, 13) are suitable for quantitative evaluation of force and power characteristics of the filled drum drive.

8. Results of determining the effect of the degree of filling the chamber with the fill on the grinding process

According to the obtained experimental data, technological and power parameters of the process of grinding in a tumbling mill with the chamber filling degree \( \kappa = 0.35 \) had the following values: \( C_e = 0.373 \), \( C_o = 0.485 \), \( P_{do} = P_{po5.5} = 0.501 \) (at \( \psi_{fo5} = 0.75 \)), \( P_{do} = P_{po5.5} = 0.375 \) (at \( \psi_{fo5} = 1.075 \)).

Parameters of the grinding process at the chamber filling degree \( \kappa = 0.25 \) acquired the following values: \( C_e = 0.323 \), \( C_o = 0.472 \), \( P_{do} = \psi_{fo5.5} = 0.424 \) (at \( \psi_{fo5} = 0.75 \)), \( P_{do} = \psi_{fo5.5} = 0.282 \) (at \( \psi_{fo5} = 1.05 \)).

Then, according to (7) to (9), relative productivity, power intensity and specific power intensity of the self-oscillating grinding process at the chamber filling degree \( \kappa = 0.35 \) have the following values: \( C_o/C_e = 1.3, P_{do}/P_{ds} = 0.749, E_o/E_s = 0.576 \).

Relative characteristics of the self-oscillatory process at the chamber filling degree \( \kappa = 0.25 \) acquired the following values: \( C_o/C_e = 1.46, P_{do}/P_{ds} = 0.665, E_o/E_s = 0.453 \).

The process characteristics at \( \kappa = 0.45 \) were determined in [9]: \( C_o/C_e = 1.067, P_{do}/P_{ds} = 0.776, E_o/E_s = 0.728 \).

Graphs of the obtained dependences of \( C_o/C_e, P_{do}/P_{ds} \) and \( E_o/E_s \) on the chamber filling degree \( \kappa \) are shown in Fig. 14.

![Fig. 14. Dependences of relative productivity \( C_o/C_e \), power intensity \( E_o/E_s \) and specific power intensity \( P_{do}/P_{ds} \) of the self-oscillating process of grinding cement clinker in a tumbling mill at \( d/\(2R\) = 0.026 and \( d_{fo}/(2R) < 0.0059 \) on the degree of chamber filling with fill, \( \kappa \)](image)

Dependences of numerical values of relative productivity, power intensity and specific power intensity characterize influence of the chamber filling degree on efficiency of the self-oscillating process of grinding in a tumbling mill.

9. Discussion of results obtained in the study of effect of the degree of chamber filling with the fill on parameters of the self-oscillating grinding process

The numerical results obtained have allowed us to quantify effect of the chamber filling degree \( \kappa \) on magnitude of the self-oscillating impact action of the ground fill in comparison with the conventional steady-state motion mode. As it was found, maximum value of the average sum of vertical components of the collision momenta of the fill particles per unit time (3) increases significantly in self-oscillation. This value increases approximately 2.4 times at the chamber filling degree \( \kappa = 0.45 \), 3.1 times at \( \kappa = 0.35 \) and 5.8 times at \( \kappa = 0.25 \) (Fig. 10). In addition, maximum value of the average sum of power of such momenta (6) increases approximately 5.7 times at \( \kappa = 0.45 \), 9.6 times at \( \kappa = 0.35 \) and 45.5 times at \( \kappa = 0.25 \) (Fig. 11). This indicates a significant increase in self-oscillatory impact action due to a decrease in the fraction of passive quasi-solid zone and a significant increase in the proportion of active pulsating zone of the fill movement in the chamber cross section with a decrease in \( \kappa \). This is determined by manifestation of the established dynamic effect of a significant increase in the self-oscillations swing with
a decrease in the degree of filling of the rotating chamber with a discrete fill (9), Fig. 5, 6).

Influence of the degree of chamber filling with the fill, \( \kappa \), on power intensity and productivity of the process of self-oscillating grinding in the tumbling mill was experimentally quantified. It was found that self-oscillation reduced relative specific power intensity of grinding (9) by approximately 27% at \( \kappa = 0.45 \), by 42% at \( \kappa = 0.35 \) and by 55% at \( \kappa = 0.25 \) (Fig. 14). Relative productivity of the process (7) increased by about 7% at \( \kappa = 0.45 \), by 30% at \( \kappa = 0.35 \) and by 46% at \( \kappa = 0.25 \). This indicates a significant decrease in specific power intensity and increase in productivity of the self-oscillating process due to a significant increase in the impact action of the fill (Fig. 10, 11) and some decrease in power for driving the filled drum (Fig. 12) with a decrease in \( \kappa \).

Such results of dynamic and technological studies were obtained for the chamber filling degree \( \kappa = 0.25, 0.35 \) and 0.45, relative size of the ball elements 0.026 and complete filling of the gaps between the grinding bodies with the ground material. Two modes of grinding cement clinker in a tumbling mill were assessed: the conventional steady-state mode at a relative rotational velocity \( \psi_w = 0.75 \) (9), Fig. 1, 2) and the innovative self-oscillating mode at \( \psi_w = 1.05...1.1 \) (9), Fig. 5, 6).

Disadvantages of the applied approach to assessment of influence of self-oscillations on the workflow include the failure to account for the structure of the discrete fill of the drum chamber.

In the future, it is expedient to find qualitative and quantitative influence of presence of crushed material particles between the grinding bodies on dynamic and technological parameters. It will allow us to establish rational conditions of self-excitation of the fill pulsations for realization of the self-oscillating process of grinding in tumbling mills at a varying degree of filling of the internal chamber with the ground material fill.

### 10. Conclusions

1. Ratio of the average sums of the vertical components of the collision momenta per unit time for self-oscillating and steady-state motion modes was approximately determined numerically. For the relative size of the ball elements of the fill of 0.01...0.03, this ratio was approximately 2.32...2.39 at the chamber filling degree \( \kappa = 0.45 \), 2.96...3.07 at \( \kappa = 0.35 \) and 0...5.79 at \( \kappa = 0.25 \). Ratio of the average sums of powers of such impacts per unit of time under such conditions was 5.41...5.7 at \( \kappa = 0.45 \), 9.35...9.57 at \( \kappa = 0.35 \) and 0...45.5 at \( \kappa = 0.25 \).

2. It was found that with a decrease in the chamber filling degree, \( \kappa \), the self-oscillating impact action increases significantly due to the decrease in the passive quasi-solid zone and a significant increase in the active pulsation zone of the charge motion in cross section of the rotating chamber. This is due to manifestation of the dynamic effect of significant increase in the self-oscillations swing with a decrease in \( \kappa \).

3. A decrease in relative specific power intensity in grinding cement clinker was established experimentally for an innovative self-oscillating process compared to the conventional process. For the relative size of the fill particles of 0.026 and complete filling of the gaps between the grinding bodies, this reduction was approximately 27.2% at \( \kappa = 0.45 \), 42.4% at \( \kappa = 0.35 \) and 54.5% at \( \kappa = 0.25 \). Under these conditions, relative productivity of the self-oscillating process has increased by about 6.7% at \( \kappa = 0.45 \), by 30% at \( \kappa = 0.35 \) and by 46% at \( \kappa = 0.25 \).

4. It was established that with a decrease in the chamber filling degree, \( \kappa \), specific power intensity and productivity of the self-oscillating grinding process carried out in the tumbling mill significantly decrease. This is due to manifestation of the technological effect of a significant increase in the impact action of the grinding bodies on particles of the crushed material and some decrease in the drive power spent for rotation of the filled drum with a decrease in \( \kappa \).

### References


