1. Introduction

The competitiveness of any country’s economy is closely linked to the sustainable development of its transport infrastructure, including the network of transport communications (roads, bridges, tunnels, rail networks, ports and airports) [1]. It creates prerequisites for the rational organization of logistics and ensuring efficient operation of transport, increasing the level of utilization of the potential of the industry, agriculture, trade activity in both individual regions and the country as a whole [2].

An important prerequisite for the development and stable functioning of the national economy, strengthening of international trade links and ensuring the national security of the country is the availability of a developed road network.

Today, the development and operation of the road network in many countries of the world are based on various forms of partnership between the country and private business. They are used for the implementation of projects of road construction, operation, maintenance, introduction of traffic management systems, development of urban transport and more. The practice of implementing projects...
Many scientists study the issue of determining the expected traffic intensity for a toll road and the impact of various factors on it. Such attention is explained by the fact that the traffic intensity of the toll road is a critical risk factor of disruption of the concession agreement [9]. Its sufficiency is also taken into account in determining the period of concession, dividing responsibilities between the concessionaire and the state partner, assigning compensation payments to the concessionaire by the state [10].

The basis of modeling the traffic flow for a certain section of the transport network is the evaluation of the correspondence matrix. Arrivals and departures of cars and passengers for a particular area, as well as inter-district correspondence (total number of departures and arrivals in the areas) are taken into account. The totality of all correspondences is a correspondence matrix. Its size depends on the number of areas included.

Today, there are many models for evaluating the correspondence matrix. The most common are gravity, entropy and rival models [11]. Statistical models are increasingly being used involving the application of modern information technologies, including the reading of correspondence information by electronic scanners [12, 13]. The resulting information is necessary to further predict the expected traffic flow on a section of the road network.

The methodology for determining the expected traffic flow for a toll road differs in the following cases:

a) an existing road that has been built and operated under concession;

b) substantiation of the project of new construction and further operation of the road under concession.

Thus, in the first case, the authors [14] use a universal time series model to predict traffic volume for toll roads. In [15], the authors propose to estimate the traffic flow using linear and logistic regression models. The latter models provide fairly reliable results with small traffic flows.

The feature of forecasting traffic flows for existing toll roads is the need for statistical information on the traffic of vehicles over a long period of time.

In the second case, the problem becomes more complicated. First, because of the need to forecast traffic flow on the road network section as a whole [16]. Secondly, the problem is to distribute the expected traffic flow between toll and alternative roads. In this case, it is advisable to take into account the behavior principles of road users.

Today there are many methods of forecasting the development of the transport network and expected traffic flows. Advantages and disadvantages of the main ones are presented in [17]. The authors note that the most appropriate method of forecasting traffic intensity is to use the ForFITS (For Future Inland Transport Systems) model. Its practical application requires the use of a large amount of information, which can be divided into socio-economic, technical and corrective (indexes, coefficients) data. Obtaining many of the indicators included in the given array of input data is a time-consuming task. All this complicates the practical use of this method.

Traffic modeling is based on information on the expected average annual daily traffic intensity of vehicles.

Methods for determining the expected average annual daily traffic intensity for projects of new construction and reconstruction (or major repairs) of existing roads are based through joint efforts of the state and private business has become known as public-private partnership (PPP). As a result, projects are being implemented that would be impossible to finance in the near future solely by the state. This ensures not only the development of the road network, but also creates additional jobs, activates the entrepreneurial initiative. All this contributes to social stabilization, increase of well-being and quality of life of the country’s population.

The prevalence of PPP-based road construction in both developed and developing countries is reviewed [3–6]. In the European countries alone in 2018, the total cost of PPP-based projects amounted to 14.6 billion euros. Of these, 7 billion euros is the cost of transport projects. The largest are Ankara-Nigde motorway (1.2 billion euros, Turkey), Gironde broadband network (1.2 billion euros, France), A16 Rotterdam highway (930 million euros, the Netherlands).

At the same time, the capital intensity and quality of the road construction project are not a guarantee of its success. Many projects still fail. From 3.93 % of investments in PPP projects in the Middle East and North Africa to 12.23 % in Latin America and the Caribbean have been discontinued. [7] The reasons for this are:

1) inconsistency of the current institutional environment in the country with conditions of road infrastructure development on the basis of public-private partnership;

2) exceeding the actual cost of the project over estimated, which occurs during its implementation;

3) errors in project preparation (for example, when estimating expected traffic flows, technical errors, neglect or underestimation of risks, etc.).

The most developed form of PPP in road construction is concession. It is an agreement whereby the concessionaire (private partner) undertakes to develop a project, finance, restore an existing or construct a new facility and operate it within a specified period. Upon expiration of the concession period, such facility is transferred to the concessee (state).

As a rule, the source of reimbursement of the concessionaire’s expenses for reconstruction or new construction and maintenance of the road is the fee charged to road users. There is another option for reimbursing the concessionaire’s expenses. This is a “shadow” concession, which provides for payments to the concessionaire by the state, based on the number and quality of services provided to society. It has become widespread in the United Kingdom and Japan [8].

Substantiation of the feasibility of implementing a PPP-based road construction project takes into account the sufficiency of traffic intensity to generate acceptable income to the concessionaire. Income is considered acceptable if it provides not only reimbursement of investments and servicing project financing sources, but also the minimum necessary profit for the concessionaire.

In turn, traffic intensity on a toll road depends on a number of factors. In particular, current and expected traffic flow, availability of alternative roads, their capacity, user costs for fuel and lubricants, tolls and the like.

The foregoing necessitates the study of scientific and practical approaches to determining the expected traffic intensity and development of analytical tools to justify the expected traffic distribution between toll and alternative roads.

2. Literature review and problem statement

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Traffic modeling is based on information on the expected average annual daily traffic intensity of vehicles.

Methods for determining the expected average annual daily traffic intensity for projects of new construction and reconstruction (or major repairs) of existing roads are based
on different approaches. Thus, the future traffic intensity for new road construction projects is determined by analytical methods, taking into account the prospect of socio-economic development of the respective region as a whole. In the case of a project of reconstruction of an existing road, the traffic intensity is assumed as historically developed, taking into account its actual average annual growth [18].

According to [19], traffic intensity forecasting takes into account the probable number of vehicles that will travel between the corresponding settlements. Traffic intensity depends on the number of population, administrative importance and subordination of these settlements, i.e. the level of their connectivity.

In the presence of long-term traffic data, it is recommended to perform traffic intensity forecasting on the basis of mathematical extrapolation using equations of various kinds of functions (linear, logarithmic, polynomial, power, exponential).

When modeling traffic flows for new road construction projects, the potential implications of regional road network development programs, corresponding changes in traffic intensity and redistribution of traffic flows over the road network are taken into account. The expected traffic intensity on a toll road depends on current and expected traffic flow in the region, availability of alternative routes, toll. Such a number of factors influencing the users’ decision-making makes it appropriate to use stochastic models in studies of expected traffic flows [20].

When building a model of traffic flow distribution over a transport network, scientists are often guided by behavior principles of user route selection. They were formulated in [21]:
1) network users independently choose routes seeking to minimize their transport costs;
2) network users choose routes that together minimize total transport costs.

Consideration of traffic flow distribution according to the first Wardrop principle is based on competitive non-cooperative equilibrium. Each traffic participant is interested in reaching the final destination in the fastest and most cost-effective way possible. That is, the network user will choose the route that will provide him with a minimum total cost (time, money, etc.) of travel. This principle is also called user optimization.

The implementation of the first Wardrop principle has some assumptions. First, it is accepted that transport network users are fully aware of the costs of travel by different routes. Secondly, it is assumed that the impact of the individual participant on the costs of other participants of the transport network is negligible.

Traffic participants are distributed over the transport network in the most cost-effective way, seeking to minimize their costs. This gives grounds for considering the second Wardrop principle, called system optimization, in the traffic flow distribution model.

Various methods of modeling transport processes are used for the organization of transport operations. These include discrete-choice models based on utility theory. According to it, the network user chooses the route and mode of transport from a finite number of alternatives, which provides him with maximum usefulness [22–24]. By setting priorities when ranking a set of possible routes and transport modes, the user also determines their usefulness.

The tasks of justifying the traffic flow distribution over the road network involve multivariate cost functions. Known works on problem formulation, proving the existence of a solution, developing solution algorithms are [25–27] and a number of others.

When studying user behavior, scientists take into account various cost components and decision-making factors. Thus, [28] considers only the time spent by users on travel. The “indiscriminate choice” model [29] proposes to take into account factors such as the willingness of users to pay a toll, travel time and comfort level. However, parameters such as user willingness to pay is an indicator that cannot be measured, and comfort level is a relative indicator, the method of measuring which is ambiguous.

In [30], researchers of traffic demand elasticity show that the volume of traffic of a toll road depends on the income of users. Due to the size and stability of their income, they can, to some extent, respond painlessly to rising fuel prices and ensure traffic flow of toll roads. If a certain amount of fuel costs is exceeded, users refuse to take the toll route. The results are useful in simulating the distribution of expected traffic flows between toll and alternative roads under different scenarios of changing economic environment.

Network equilibrium of traffic flows is also investigated on the basis of the nested logit model [31]. This paper proves its higher sensitivity compared to the polynomial logit model. Several alternatives are considered for choosing the types of vehicles to travel from the point of departure to the point of arrival. The information obtained allows determining the composition of the traffic flow for travel, but does not give an answer as to its distribution over the road network.

In [32], scientists study the effect of different types of constraints on rational user behavior on:
1) the emergence of obstacles to the network equilibrium;
2) network performance as a whole.

The study is based on Monte Carlo modeling.

Other researchers [33] based on the regret (disappointment) theory made an attempt to show real user behavior in travel. An equilibrium stochastic multistage model was developed that took into account user groups regarding the extent of their frustration due to traffic delays. Numerical results indicate that more demanding users tend to change the route in the event of traffic jams.

The features of user psychology are also taken into account when modeling traffic flow distribution in [34]. The proposed model is based on the instance-based learning theory and provides social learning for network users. The numerical results of the study show that social communication has a positive effect on route selection behavior as most participants choose recommended routes.

These research findings have practical implications for existing roads. When considering new construction projects, the above approaches cannot be taken as the basis, since their application, on the contrary, increases the level of uncertainty of the expected performance indicators for the concessionaire.

From this point of view, the approach proposed in [35] is interesting. It follows the first Wardrop J. G. principle. To determine the expected traffic distribution in the road network, the traffic density and the time loss of users are taken into account. However, the structure of the user costs included in the model is limited. Costs of fuel and lubricants of users of toll and alternative roads are not taken into account.

The study [36] is also based on the Wardrop principles. A model of traffic flow distribution between toll and alternative roads on the basis of identity of user costs is proposed. Costs
include the toll and time of toll and alternative routes. However, the real costs of users include more components, so the approach proposed in this work is, to some extent, simplified.

The proposed study attempts to develop analytical tools to determine the expected traffic flow distribution between toll and alternative roads. The model takes into account indicators that can be measured unambiguously and do not contain subjective judgments or relative estimates. The presented analytical model considers traffic density on the road and utility of users, expressed through their minimum costs. These costs include the cost of fuel and lubricants consumed, the toll, and travel time, taking into account traffic density and possible congestion.

The presented analytical model of the distribution of expected traffic flows between toll and alternative roads reduces the uncertainty of the input parameters required for the economic justification of road construction projects under concession. In addition, it increases the reliability of the obtained performance indicators of such projects.

3. The aim and objectives of the study

The aim of the study is to obtain reasonable values for traffic flow distribution between toll and alternative roads. This information is necessary both to determine the effectiveness and feasibility of a road construction project under concession and to further substantiate the terms of concession agreements.

The above needed to address the following objectives:
– to develop an analytical model of traffic flow distribution between toll and alternative routes;
– to test the model on the materials of the real project of new construction and further operation of the road under concession.

4. Materials and methods of the study of the expected traffic flow distribution

4.1. Method of determining input parameters for modeling traffic flow distribution

The proposed model of traffic flow distribution between toll and alternative roads is based on the provisions of the traffic flow theory and Wardrop J. G. principles. Based on the first principle, drivers seek to minimize their costs. It is assumed that drivers will not care which route to choose if the following condition is met:

\[ t_s \cdot c_s + C_{\beta,s} + TOLL_{\text{toll},s} \cdot L_s = t_a \cdot c_a + C_{\beta,a} - C_{\beta,s} \times \frac{L_s}{L_a} \quad (3) \]

The value of \( TOLL_{\text{toll},s} \), at which drivers will not care which route to choose is calculated as:

\[ TOLL_{\text{toll},s} = c_s \left( \frac{t_a - t_s}{L_a} \right) + C_{\beta,a} - C_{\beta,s} \times \frac{L_s}{L_a} \quad (4) \]

Another input parameter that is taken into account when modeling the traffic flow distribution between alternative and toll roads is the delay time of the route. Traffic flows in the transport network are distributed taking into account the expected traffic delays due to possible congestion. To account for user time, it is suggested to use a delay function based on a simple deterministic model of queuing theory.

Let \( \lambda \) be traffic intensity (volume of traffic flow per unit of time), veh/day, \( \mu \) – road capacity per unit of time, veh/day. Then, \( p \) – the congestion factor will be defined as

\[ p = \frac{\lambda}{\mu} \quad (5) \]

The case for \( \lambda < \mu \) is considered in the paper.

Calculation of vehicle speed in the model involves adjusting the speed of free movement taking into account the congestion factor [37]:

\[ V_s = V_0 \left( 1 - p \right) = V_0 \left( 1 - \frac{\lambda}{\mu} \right) \quad (6) \]

where \( V_s \) is the estimated vehicle speed taking into account congestion, km/h; \( V_0 \) is the speed of free movement, km/h; \( p \) is the congestion factor; \( \lambda \) is traffic intensity, veh/day; \( \mu \) is the road capacity per unit time, veh/day.

Hence, the estimated travel time on the route, taking into account the traffic intensity is:

\[ t_s = \frac{L}{V_s \left( 1 - \frac{\lambda}{\mu} \right)} = \frac{L}{V_0 \left( 1 - \frac{\lambda}{\mu} \right)} = t_0 \quad (7) \]

where \( L \) is the route length, km; \( t_0 \) is free movement time, h.

The obtained values of indicators are necessary for modeling traffic flow distribution between toll and alternative roads.

4.2. Construction of an economic and mathematical model of traffic flow distribution between toll and alternative roads

Based on the idea that traffic participants are distributed over the transport network in the most favorable way, that is, minimizing their costs, one can expect the implementation of the second Wardrop J. G. principle. This principle is called system optimization.
Let the total traffic flow per day be \( Q \) vehicles. The traffic flow on an alternative road is denoted by \( \lambda_a \). The traffic flow of a toll road is calculated as

\[
\lambda_x = Q - \lambda_a. \tag{8}
\]

Traffic flow distribution between toll and alternative roads is affected by a number of factors. The expected volume of traffic on a toll road can be represented as a functional dependency:

\[
\lambda_x = f\left(Q, \mu_a, \mu_c, t_c, t_a, L_a, V_\alpha, V_\beta, C_{\beta_a}, C_{\beta_t}, C, TOLL_{km}\right)
\]

where \( \mu_a, \mu_c \) is the capacity of the alternative and toll road per unit time, respectively, veh/day;

\( t_c, t_a \) is the actual car travel time by the alternative and estimated travel time by the toll road, respectively, hours;

\( L_a, L_t \) is the length of the alternative and toll road, respectively, km;

\( V_\alpha, V_\beta \) is the speed on the alternative and toll road, respectively, km/h;

\( C_{\beta_a}, C_{\beta_t} \) is the sum of the user’s costs for fuel and lubricants while driving on alternative and toll roads, respectively, in monetary units;

\( C \) is the cost of the unit of time for the road network user, in monetary units per hour;

\( TOLL_{km} \) is the toll for 1 km of the toll road, in monetary units.

According to the system optimization condition, the traffic flow will be distributed over the network in such a way that user costs will be minimized

\[
f(\lambda_x) = \lambda_a - C_e + (Q - \lambda_a) C, \rightarrow \min, \tag{10}
\]

provided that:

\[
\lambda_x \geq 0, \ C_e \geq 0, \ C \geq 0, \ Q \geq \lambda_a. \tag{11}
\]

Such optimization is possible if the unit toll of 1 km per vehicle is the same for both alternative and toll routes. Let us formalize (10) as an optimization search problem. To do this, we introduce the notation:

\[
x_1 = \lambda_a, \ x_2 = C_e, \ x_3 = Q, \ x_4 = C, \tag{12}
\]

Then equation (10) will look like:

\[
f(x_1, x_2, x_3, x_4) = x_1 - x_2 + (x_3 - x_1) x_4 \rightarrow \min, \tag{13}
\]

for restrictions:

\[
x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0, \ x_4 \geq 0. \tag{14}
\]

Since the objective function \( f(x_1, x_2, x_3, x_4) \) is determined by the nonlinear dependency (13) and the constraints (14) are linear, the problem (13), (14) belongs to nonlinear programming problems, for which special methods must be applied. However, it can be simplified and reduced to a linear programming problem if you successfully choose the initial approximation \( (x_{10}, x_{20}, x_{30}, x_{40}) \) and assume that it is near the minimum of the system (13), (14), whose coordinates \( x_{10}, x_{20}, x_{30}, x_{40} \) differ from \( x_{10}, x_{20}, x_{30}, x_{40} \) by small values \( \Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4 \).

Such a hypothesis makes it possible to reformulate the systems (13), (14) in increments \( \Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4 \) by the linearization operation

\[
\Delta f = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \frac{\partial f}{\partial x_3} \Delta x_3 + \frac{\partial f}{\partial x_4} \Delta x_4. \tag{15}
\]

The system (13), (14) can be reduced to

\[
\Delta x_1 \geq 0, \ \Delta x_2 \geq 0, \ \Delta x_3 \geq 0, \ \Delta x_4 \geq 0. \tag{17}
\]

It can now be presented as a linear programming problem. In this case, the linear inequality system will geometrically be a convex polyhedron and, due to the linearity of the objective function (11), its minimum will lie on the polyhedron surface (12).

To find this minimum, standard computer programs can be used. As a result, we obtain the optimal states of the variables:

\[
x_1 = x_{10} + \Delta x_1, \ x_2 = x_{20} + \Delta x_2, \ x_3 = x_{30} + \Delta x_3, \ x_4 = x_{40} + \Delta x_4. \tag{18}
\]

Note that in this particular case, none of the parameters \( x_1, x_2, x_3, x_4 \) can be negative. The constraint system (14) can then be excluded from consideration. Using the input objective function, the problem (13) is reduced to the problem of unconditional extremum and is solved further by analytical transformations.

The expected costs throughout the transport network will be minimal if the user costs on both free and toll roads are the same:

\[
(Q - \lambda_a) \left(t_c \cdot c_e + C_{\beta_a} + TOLL_{km} \cdot L_t \right) = \lambda_a \left(t_a \cdot c_e + C_{\beta_t} \right). \tag{19}
\]

Taking into account the delay time, this equation can be written as follows:

\[
(Q - \lambda_a) \left(1 - \frac{\lambda_a}{C_{\beta_t}} + \frac{\lambda_a}{C_{\beta_a}} \right) = \lambda_a \left(1 - \frac{\lambda_a}{C_{\beta_t}} + \frac{\lambda_a}{C_{\beta_a}} \right). \tag{20}
\]

Using the “Parameter selection” function of the Excel spreadsheet, the values \( \lambda_a \) of the traffic flow on the alternative road under different tolls for 1 km of the toll road are calculated.

The traffic flow of the toll road will be calculated as (8).
The experimental section of the road network covered:

- the segment of the public road adjacent to the design section of the Great Kyiv Ring Road (along the villages of Bobrytsya, Ivankovychi, Zeleny Bor, Roslavichi, Gvozdiv, Markhalivka to Glevakha), which connects the Kyiv-Odesa and Kyiv-Zhytomyr highways. In further studies, this road was considered as an alternative;

- the segment of the existing public road – the entrance (exit) to Kyiv through a stationary police post from the Zhytomyr highway, the interchange at Peremolny avenue and Ring road, Ring road, the interchange at Odesa square, exit (entrance) from Kyiv on Academic Glushkov avenue in the direction of Odesa highway. In further studies, this road was considered as an alternative;

Today, the only road that vehicles can get from the Odesa-Kyiv to Kyiv-Zhytomyr highway and vice versa is the Kyiv ring road. The movement of transport is difficult to manage, and the traffic flow of vehicles, along the Ring road complicates local traffic, has a devastating effect on the road surface, increases the level of pollutant emissions and often leads to a traffic collapse on this section of the road. The construction of the toll Great Ring Road could significantly relieve the city ring road, making it convenient for drivers of both transit and local transport.

For the experiment, selective vehicle traffic recording was used by collecting information within individual hours of the day. Information was collected through video recording, followed by computer processing of the video data. This made it possible to obtain objective data on the traffic intensity in the study area and to determine its composition by vehicle types.

Forecasting of the expected traffic intensity was based on the requirements [19]. The current and expected values of traffic intensity on the studied section of the road network are shown in Fig. 1.

Taking into account the current and expected traffic flows, length of toll and alternative routes, fuel and lubricant consumption rates, unit cost of time, the costs associated with the movement of vehicles were calculated.

According to (4), the toll for 1 km of the toll road was calculated, for which drivers will not care which route to choose $TOLL_{\text{km}}$.

Comparison of tolls for 1 km of the toll road by user groups, calculated according to the author’s method and methodology [32], is shown in Fig. 2.

It can be seen that the tolls calculated by these methods differ significantly. Including the cost of fuel saving in the toll for 1 km of the toll road will help to avoid oversaturation of the toll road due to users’ too low cost when driving it. This will not only avoid congestion and overload on the road surface, but also prevent the concession road operator from unreasonable costs for maintenance. In addition, this will contribute to the formation of higher concessionaire income, which will reduce the risk of project termination due to the inability to reimburse investments, provide loan servicing.

By setting the toll $TOLL_{\text{km}}$ in such a way that the total cost of driving the alternative and toll routes will be the same, the cost of delivering goods by toll and alternative roads will be the same.

The Cheung K. A. and Polak J. W. tolls do not take into account user costs for fuel and lubricants and the corresponding cost savings on the toll road [36]. Because of this, the toll road will be congested. The corresponding concessionaire’s costs for its maintenance will increase. At the same time, the expected income will be extremely low, which may lead to project termination and the need to reimburse the concessionaire for the investments made by the state.

![Fig. 1. Current and expected traffic intensity on the section of the project of construction phase of the Great Kyiv Ring Road](image1)

![Fig. 2. Tolls or different vehicle groups calculated by the author’s model and Cheung K. A. and Polak J. W model](image2)
The algorithm of modeling the distribution of the expected traffic flow between alternative and toll routes by the author model is shown in Fig. 3–5.

**Fig. 3. Generalized algorithm for the first step of modeling the distribution of expected traffic flow between alternative and toll routes**

**Step 1.** Calculation of the toll, for which drivers will not care which route to choose ($TOLL_{m}^{*}$).

Start

Input data for toll $TOLL_{m}^{*}$ calculation by user groups: $t_{w}, t_{a}, c_{w}, L_{w}, C_{m}, c_{2}$

Calculation of $TOLL_{m}^{*}$ by user groups

Array of $TOLL_{m}^{*}$ values by user groups

Exit

**Step 2.** Calculation of the expected traffic flow distribution by $TOLL_{m}^{*}$ ($\lambda_{a}, \lambda_{t}$).

Start

Input data for expected traffic flow calculation for alternative and toll routes: $Q, t_{a}, t_{t}, c_{a}, L_{a}, L_{t}, C_{a}, C_{t}$

Input data for expected traffic flow calculation for alternative and toll routes $TOLL_{m}^{*}$

Array of $\lambda_{a}, \lambda_{t}$ values by user groups

$\lambda_{a} + \lambda_{t} = Q$

Exit

**Step 3.** Simulation of toll change scenarios and expected traffic flow distribution.

Start

Input data for simulation of expected traffic flow distribution: $Q, t_{a}, t_{t}, c_{a}, L_{a}, L_{t}, C_{a}, C_{t}$, $TOLL_{m}^{*}, \lambda_{a}, \lambda_{t}$

Calculation of total user costs for toll and alternative routes (by user groups) $(C_{t}, C_{a})$

Yes

Calculation of $\lambda_{a}$ and $\Delta \lambda_{a} = \lambda_{a}^{*} - \lambda_{a}$ (by user groups)

Calculation of $\lambda_{t}$ and $\Delta \lambda_{t} = \lambda_{t}^{*} - \lambda_{t}$ (by user groups)

Calculation of $\lambda_{t}^{*} + \Delta \lambda_{t}$ (by user groups)

Array of $\lambda_{a}, \lambda_{t}$ values by toll change scenarios by user groups

Exit

**Fig. 4. Generalized algorithm for the second step of modeling the distribution of expected traffic flow between alternative and toll routes**

**Fig. 5. Generalized algorithm for the third step of modeling the distribution of expected traffic flow between alternative and toll routes**

6. Discussion of the results of analytical modeling of traffic flow distribution between toll and alternative roads

According to the results of the study, the developed analytical model provides an opportunity to obtain economical-
ly reasonable expectations regarding traffic flow distribution between toll and alternative routes. This model is based on the Wardrop J. G. road users' behavior principles. The purpose of the model is to minimize the costs of road users (fuel and lubricants consumed, travel and traffic congestion time, road tolls).

As shown in Fig. 3–5, the algorithm has three steps. Each step solves an independent problem, the results of which are included in the input information for the next step.

The first step is to determine the toll at which network users do not care which route to go – toll or alternative.

Accounting for operating costs in the model of toll determination allows a more objective determination of users' expected costs and benefits when driving on toll and alternative routes. This information is taken into account by them when making their subjective decisions about the route.

Considering the cost of user time in the toll calculation, it is necessary to take into account that there is no uniform estimation approach. The results of calculating the toll and expected traffic flow distribution based on the cost of time taken at the hourly rate of the minimum wage in the country are presented. This is because one of the principles of public-private partnership (concession is a form of public-private partnership) is to make services available to all user groups.

At the same time, another, differentiated approach to determining the cost of time for each user group can be used. In this case, for motorcycles and cars, the authors propose to determine the cost of time at the average hourly rate based on the average monthly wage in the country. For trucks, it is suggested that this cost be determined within the average hourly rental rate of a typical truck for a user group.

The second step was calculating the expected traffic flow distribution between the toll and alternative routes by setting the toll in the amount specified in the previous step. As traffic is slower, traffic density affects the speed and, ultimately, travel time. Traffic density was taken into account by introducing the traffic intensity and road capacity ratio into the formula (19). This made it possible to bring it into (20).

In the third step, by means of simulation, scenarios of toll deviations from their values, data sets on expected traffic flows for toll and alternative roads were obtained.

According to the project of the construction phase of the Great Kyiv Ring Road, experimental testing of the proposed analytical model was carried out. In the context of user groups, tolls at which they will not care which route to take were calculated. This will ensure a uniform distribution of vehicles between toll and alternative roads. An increase in the toll will decrease the traffic flow on the toll route. When examining the behavior of user groups under toll change, it was determined that the flow of the user group with a typical “large buses” vehicle responds slower. User groups with a typical “motorcycles” and “small buses” vehicle respond to toll change most quickly.

It should be noted that the proposed analytical model is a continuation of the study [39]. The authors attempt to resolve the issue of traffic flow distribution in a complex manner, taking into account all factors of influence.

Note that the analytical model developed by the authors has an applied value. Its implementation in the Excel spreadsheet makes it easy to use in:

- economic justification of the feasibility of a road construction project under concession,

- determination of relevant indicators regarding the expected traffic flow for concession contracts for new construction and further operation of the road.

The information obtained from the simulation is also necessary to calculate the concessionaire's expected income. Such income is a source of reimbursement for investments in road construction and maintenance. Its sufficiency is a prerequisite for the viability of the project.

For further research, it is interesting to estimate traffic flow distribution between toll and alternative routes using discrete choice and logit models.

7. Conclusions

1. The developed and proposed analytical models provide information that is necessary to justify the feasibility of a PPP-based road construction project. In particular, it is the model of toll determination and the model of traffic flow distribution between toll and alternative roads.

The model of toll determination assumes that the user's total costs will be the same as those on an alternative route. By influencing the toll, it is possible to increase or decrease the user's total cost while traveling the toll route. Thus, it is possible to control the distribution of the current traffic flow between the toll road and the alternative road and provide the necessary traffic intensity for the toll road. This is necessary to generate a reasonable income for the concessionaire, maintain the acceptable level of costs for maintaining the toll road, servicing loans and reimbursing investments.

The model of traffic flow distribution between toll and alternative roads is based on the current traffic flow on the road network section and road capacity. The model takes into account traffic density and such components of network users' costs as the cost of consumed fuel and lubricants, travel time and toll.

The information obtained from the use of models is necessary for:

1) economic justification of the project attractiveness for private investors and project expediency under concession;

2) determination of traffic intensity for the toll road below which it is impractical for the authorities to anticipate concession payments under the concession agreement.

2. Testing of the model was carried out on the materials of the real project for the new road construction – Great Kyiv Ring Road. Cost-effective toll values by user groups were obtained. On the basis of them, scenario modeling of traffic flow distribution between toll and alternative routes was carried out, and the sensitivity of transport flows by user groups to toll changes was determined.

The proposed model has an applied value and can be used in practice, in particular in road construction under concession.

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