Retailer-Oriented Closed-Loop Supply Chain (ROCLSC) is an integration of forward and reverse supply chains with retailer taking charge of the remanufacturing, distribution, and collecting activities. This type of mechanism is quite effective, since the majority of product returns management is performed by the retailer. However, in practical industries, the implementation of ROCLSC is still limited. In this study, we investigate a ROCLSC system that involves an Original Equipment Manufacturer (OEM) and a retailer. OEM plays a role as a producer of new products, while the retailer is in charge of remanufacturing, collecting, as well as selling and distributing both newly manufactured and remanufactured products. We develop a mathematical model to maximize the profit of each party. Although several studies have developed models for cores acquisition, here we apply a different cores switching mechanism. We introduced the fixed rate and flat rate mechanisms used in the business-to-business (B2B) system, where product functions are very important to consumers. In addition, this research focuses on ROCLSC where most of the existing cores acquisition models are Manufacturer-Oriented Closed-Loop Supply Chain (MOCLSC). The result of this study shows that the retailer will get higher profits when the product returns are acquired through the fixed rate mechanism, rather than the flat rate mechanism. Therefore, determining the optimal amount of cores collected through the fixed rate mechanism will increase the retailer’s profit, as well as joint profit of both parties. From the results, we also point out an interesting note that the retailer should increase efforts to sell new products along with the increasing proportion of consumer Willingness to Pay (WTP) for remanufactured products. Hence, both OEM and retailer profits can be increased consecutively.

Keywords: acquisition strategy, retailer-oriented closed-loop supply chain, switching cores

1. Introduction

The concept of Closed-Loop Supply Chain (CLSC) or reverse supply chain emerged as an effort to overcome issues related to sustainability. Recently, issues related to sustainability have become a topic of discussion for many groups, both practitioners and academics. Environmental, economic and social aspects are the most dominant reasons. CLSC is an integration of forward and reverse supply chain, where material moves from OEM to end customers, and will eventually return back to the OEM. The concept of reverse supply chain began to thrive after there were findings that impacts on these aspects can no longer be resolved by the forward supply chain [1], especially the management of product returns. As stated by [2], a product will inevitably experience returns due to various reasons, such as end-of-life, end-of-use, commercial returns, and re-usable components. [3] stated that reverse logistics can handle problems related to end-of-life products more efficiently and effectively.

It is widely recognized that market activities involving remanufacturing processes are more complex than typical manufacturing processes. Several challenges arise in such industries, such as the uncertainty of product returns, the complexity of network channels, as well as the uncertainty of the remanufacturing market. In the field of construction machinery industry, Caterpillar Inc. was the first to implement remanufacturing process [4]. The RL practices they carried out have resulted in a significant increase in its market share, since several consumers were unable to afford new products. The challenge facing this industry is not only how to manage complex reverse flows, but also how to get cores from consumers [5]. According to [2, 6], issues related to the availability of cores mainly occur due to uncertainty of volume/quantity, quality and timing. These uncertainties can end up in return-demand imbalance [7, 8]. Moreover, [9] stated that, with the Interpretive Structural Modeling (ISM) model they developed in the construction machinery remanufacturing industry, it is understood that there are two key variables to successfully
collect return quantity/volume, namely relationship type and logistics convenience.

In reality, the construction machinery industry requires a considerable amount of investment to purchase new machines. Construction machineries are generally used for mining, plantation, construction, and forestry work, which are often located in remote areas. Construction machine units are large, heavy, and bulky so it becomes quite difficult to move them around. With the B2B model, where consumers consider the function of the product very important [2], whenever consumers find their product units are faulty, they have to be prepared to replace it immediately. This encourages the construction machinery remanufacturing industry to develop a strategy for cores acquisition, in order to maintain the sustainable performance of the existing RL system.

In Indonesia, remanufacturing machinery industries have implemented two switching cores mechanisms, namely fixed and flat rate mechanisms. However, companies often encounter decision-making problems in this area. Companies often struggle to determine which mechanism is more profitable for the company. This often results in decisions that are not optimal in terms of costs. Therefore, studies are devoted to developing a core acquisition model that can capture this phenomenon and can be used as a decision-making model to formulate their business policies. Thus, the remanufacturing company, which is the retailer in this case, can determine the mechanism of switching cores that provides better financial benefits. Exploration in this area can provide additional insight for CLSC/Reverse logistics players regarding the different leaders in reverse logistics channels.

2. Literature review and problem statement

Several researchers developed quantitative models for remanufacturing process to investigate the profitability level of various scenarios, including strategies for collecting consumer product returns. Research conducted by [10] discussed the pricing strategy applied by companies that produce durable remanufactured products. There are three pricing schemes offered by the company, namely uniform prices for all consumers, price discounts for new and old customers at a constant rate, and price discounts between new and old customers at a variable rate, which depend on the length of time they have become consumers.

Furthermore, [11] developed a buyback strategy in two scenarios, which are competition and coordination between OEM and remanufacturer. [12] developed a mathematical model that combines advertisement costs, financial incentives and transportation costs. Three types of financial incentives were used in this study, including cash, fixed values and discount rate. Later, [13] developed a mathematical model that aims to maximize profits by considering two strategies for obtaining a used product, namely negotiation and posted pricing. In most practices, some OEMs focus only on their core business and shift their reverse activities to the retailer or third parties [14–16]. This type of system is also known as the Manufacturer-Oriented Closed-loop Supply Chains (MOCLSC) system. Most of the research on quantitative model development is carried out in this area.

In terms of proximity and convenience to customers regarding the collection and remanufacturing processes, the retailer is said to be better than OEM. Research around CLSC, which focuses on the retailer as a leader of the supply chain, which according to [17] is called the Retailer-Oriented Closed-loop Supply Chain (ROCLSC), is still very limited. [17] developed a ROCLSC mechanism to investigate the profitability of reverse channel decisions with the retailer and third parties as the collector used item. However, not many studies have paid attention to the ROCLSC mechanism where the retailer takes charge of both remanufacturing, distribution, and collection activities consecutively. This mechanism is suitable for the B2B model in which consumers consider product functions to be very important.

As discussed earlier, the CLSC concept is basically integrating forward and reverse logistics, which involves the movement of material from upstream to downstream, as well as backward flow of items from customer to producer. In the final stages of life cycle, products can be recovered in various ways with various levels of efficiency in utilizing natural resources. According to [18], there are various alternatives for product recovery, including reusing, repairing, refurbishing, remanufacturing, cannibalization, and recycling.

In practical industries, different market structures may cause differences in dominant position holders in CLSC, such as OEM, retailer or third party [19]. As quoted from [20], difference reverse channel will lead to different collection cost structures. Therefore, an investigation needs to be done to find out which reverse channel can provide maximum profit for the system.

Most of the research on CLSC discusses problems related to MOCLSC. Only a few have paid attention to reverse channel systems with retailers as the leader. In terms of proximity to consumers, according to [17, 20], retailers in CLSC should be given more responsibility by playing a role in collection and remanufacturing activities. Many suggest that it is advisable to conduct a study on a CLSC system that more focuses on the retailer as the leader [17, 20].

The financing structure of the acquisition of used products is the subject of further discussion, since it affects the consumer’s desire to return the used product. As pointed out by [21], there are consumers, called “the reward-driven”, who return the used product only if they get a reward out of it. The availability of cores is very dependent on the number of product returns from consumers. Therefore, the company needs to develop a financing structure, including the cost of acquisition of cores, in order to increase consumer interest in returning the used product. One study on the core acquisition mechanism was conducted by [22], in which they discussed the application of dynamic pricing for used products in car engine remanufacturing companies. The study discusses a company that implements a make-to-stock remanufacturing system where remanufacturers make products using cores obtained from consumers. In the first period, cores are assumed to have single-class, and then in the second period, cores are assumed to have uncertain quality. Dynamic pricing can effectively solve problems related to pricing for items that have random characteristics in both return and demand. Later, [23] discussed a mathematical model that aims to minimize the total cost, which includes the cost of acquiring used products and costs of the remanufacturing process. The model was developed in two conditions, namely discrete and uncertain. [24] developed a quantity discount strategy in which the CLSC system consisted of one manufacturer/remanufacturer and one retailer with the manufacturer/remanufacturer taking the lead. The model was developed in two scenarios, namely
centralization and decentralization. The results show that the centralization scenario provides greater benefits for both manufacturer/remanufacturer and retailer. Most of these studies use buy-back as a mechanism for switching cores, which is only appropriate if the customer is an end user. This kind of mechanism cannot be applied in this study, since product users are companies that use construction machinery for production activities.

Many studies on CLSC have been carried out on various topics, such as reverse channel structure, returned product quality, cost structure (inventory, transportation, acquisition of cores, etc.), and so on. Unfortunately, most of the research has focused solely on MOCLSC. The reason for this may be most of the previous research consider that the reverse logistics management activities are carried out by the manufacturer, not the retailer, which makes related research irrelevant. However, retailers as actors who are closer to consumers should be able to further investigate their role in the CLSC system. Therefore, this research discusses the problem of the ROCLSC system in which the retailer is taking charge of remanufacturing, distribution, and collecting activities. The system studied is the ROCLSC system in the construction machinery and equipment remanufacturing industry. The acquisition strategy is carried out using two switching cores mechanisms, namely fixed rate and flat rate. This study was conducted to determine the characteristics of the ROCLSC system that can increase profits for the system.

3. The aim and objectives of the study

The aim of this study is to obtain an optimal solution related to the cores acquisition strategy model with two switching cores mechanisms, namely fixed rate and flat rate. These mechanisms allow retailers to maintain a return-demand balance while maximizing their profits.

To achieve the aim, the following objectives must be achieved:

- to describe the characteristics of the ROCLSC system in the construction machinery and equipment remanufacturing company. The focus of this research is a ROCLSC system consisting of OEM, who act as producers of new products, and retailers, who act as remanufacturers, distributors, and collectors;
- to formulate a mathematical model, with the objective function of maximizing the retailer profits. Model development is structured to maximize the revenue and minimize the related costs;
- to verify the model with sensitivity analysis to understand and study how parameter changes affect behavior and the optimal solution of the model.

4. Materials and methods

In order to achieve the research objectives, the mathematical model developed in this study includes: the OEM profit functions in the first and second periods, the retailer profit functions in the first and second periods, the total profit function of the OEM and the total profit function of the retailer. Furthermore, research completion consists of three steps, these are the explanation for each step. The first step is describing the system characteristics, this explains about cores acquisition strategy model within a ROCLSC system in which retailer acts as remanufacturer, distributor, and collector. System’s behavior with two switching cores mechanisms, fixed rate and flat rate (price), involving one OEM and one retailer will be investigated. The second step is the mathematical model development, its steps are: defining assumption and notation, mathematical model development on two periods, both on OEM profit model and retailer profit model, model solution procedure by deriving the equation on $\omega_1$, $m_1$, $\omega_2$, $m_2$, and numerical examination. The model developed in this study is a non-linear programming problem, where the optimal solution is obtained with the help of Wolfram Mathematica software. Model verification is performed to find out whether the model can provide optimal performance. Experiments with numerical data were beneficial to ensure that the results are in accordance with the estimation of the model developer. Also, it can represent real conditions in the practical field. The final step is sensitivity analysis, to measure the sensitivity of the model to changes in several key parameters.

5. Result of ROCLSC profit maximization model

5.1. System under discussion

As discussed earlier, the ROCLSC system investigated in this study consisted of an OEM and a retailer as the leader of the supply chain. Here, the retailer’s role is dominant since it plays a role in distributing new products, selling remanufactured products, performing after-sales service and collecting used products. This model considers two switching cores mechanisms, namely flat rate and fixed rate. Switching cores is a method used by retailers to acquire cores that are under customer ownership. The flat rate is a switching cores mechanism where the customer will pay the cost of repairing failure components according to the scope in the flat rate mechanism. If the failure component is not included in the scope, the customer will be charged for the additional fee, which is called a surcharge. Unlike the fixed rate mechanism, where customers only pay a fixed amount of money without any surcharges, since the fixed rate cost structure includes all possible damages. At this point, the retailer will determine the proportion of the cores switching mechanism that can provide maximum profit.

The two-period model developed by [17] was adopted in this study. In the first period, OEMs produced excavators, then distributed these products to retailers. In this first period, retailers acted as distributors of new products. Then in the second period, retailers initially plan a remanufacturing production schedule then collect used products from consumers. Furthermore, the retailer carries out the remanufacturing process, and in the end distributes remanufactured and new products consecutively. Fig. 1 shows the system we investigated, consisting of the players involved, material flows, and supporting variables.

The switching cores mechanism is carried out when the customer encounters damage to the engine or components of the excavator. Based on the request obtained from the customer, the retailer, which in this case also acts as a remanufacturer, will send the remanufactured engine according to the specifications requested by the customer. Subsequently, the customer sends the faulty engine to the nearest remanufacturer site. The logistics factor, especially transportation, is the biggest challenge for the heavy equip-
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To the customer in the second period; period, where one with the fixed rate mechanism compared with the brand new with the fixed rate mechanism in the second period; following decision variables:

Both players, including the OEM and the retailer, have the payment based on the selected cores switching mechanism. The excavator engine is heavy and difficult to move. Then, the customer makes a remanufacturing industry, since the excavator engine from the flat rate mechanism; coming from the fixed rate mechanism;

We use the following notations to develop the model. We develop the model belonging to [17], by including and analyzing different strategies of product acquisition, which involves two switching cores mechanisms: fixed rate and flat rate. This section will explain the demand and profit functions developed in the model.

The following are some of the assumptions and limitations used to develop the model:

1. An excavator can only be remanufactured once during its lifetime.
2. The unit remanufacturing cost is lower than the unit manufacturing cost.
3. Remanufactured products have the same function as new products.
4. The retailer adopts a remanufacture-to-order production system.
5. The acquired returns are products that have been used by customers.
6. In the first period, the retailer only distributes new products. Later in the second period, the retailer sells and distributes both new and remanufactured products.
7. The retailer plans production for remanufactured products.
8. The customer will send faulty products, considered as product returns, to the nearest site owned by the retailer.

5.2.1. Notations and Assumption

We use the following notations to develop the model. Both players, including the OEM and the retailer, have the following decision variables: \( m_1 \) and \( m_2 \) for the OEM and \( m_1, m_2 \) and \( q_{r1} \) and \( q_{r2} \) for the retailer:

- \( Q(P) \): price-sensitive demand;
- \( a \): maximum demand;
- \( k \): demand sensitivity to price;
- \( P \): retailer product price;
- \( q_{r1} \): demand for new products in the first period;
- \( q_{r2} \): demand for new products in the second period;
- \( q_{r2} \): amount of returned products in the second period coming from the flat rate mechanism;
- \( q_{r2} \): amount of returned products in the second period coming from the flat rate mechanism;
- \( \delta_1 \): public perception of the remanufactured products with the fixed rate mechanism compared with the brand new one;
- \( \delta_2 \): public perception of the remanufactured products with the flat rate mechanism compared with the brand new one;
- \( m_1 \): unit profit of the retailer by selling the new product to the customer in the first period;
- \( m_2 \): unit profit of the retailer by selling the new product to the customer in the second period;
- \( P_{r1} \): unit retail price of the new product in the first period, where \( P_{r1} = m_1 + \omega_1 \);
- \( P_{r2} \): unit retail price of the new product in the second period, where \( P_{r2} = m_2 + \omega_2 \);
- \( P_{r2} \): unit retail price of the remanufactured product with the fixed rate mechanism in the second period;
- \( \omega_1 \): unit wholesale price in the first period;
- \( \omega_2 \): unit wholesale price in the second period;

- \( S \): unit surcharge price for the remanufactured product with the flat rate mechanism in the second period;
- \( a \): customer product failure rate;
- \( f \): product failure rate covered by the retailer through the flat rate mechanism;
- \( c_{mr} \): unit cost of producing a new product with the raw materials;
- \( c_{mr} \): unit cost of repairing a remanufactured product from the returns;
- \( c_{ds} \): unit cost of disassembly of a remanufactured product from the returns;
- \( c_{ds} \): unit acquisition costs for product returns;
- \( p \): reverse logistics cost.

The following are some of the assumptions and limitations used to develop the model:

1. An excavator can only be remanufactured once during its lifetime.
2. The unit remanufacturing cost is lower than the unit manufacturing cost.
3. Remanufactured products have the same function as new products.
4. The retailer adopts a remanufacture-to-order production system.
5. The acquired returns are products that have been used by customers.
6. In the first period, the retailer only distributes new products. Later in the second period, the retailer sells and distributes both new and remanufactured products.
7. The retailer plans production for remanufactured products.
8. The customer will send faulty products, considered as product returns, to the nearest site owned by the retailer.

5.2.2. Mathematical Model Development

We develop the model belonging to [17], by including and analyzing different strategies of product acquisition, which involves two switching cores mechanisms: fixed rate and flat rate. This section will explain the demand and profit functions developed in the model.

The coordination scheme between OEM and retailer in the ROCLSC system follows the Stackelberg game with the retailer as a market leader. In the first period, the retailer determines the unit profit of new products, denoted by \( m_1 \), then the OEM decides the wholesale price of \( \omega_1 \). In the second period, the retailer determines the unit profit of new products denoted by \( m_2 \), as well as the number of product returns obtained from the fixed rate and flat rate mechanisms, denoted by \( q_{r2} \) and \( q_{r2} \), and finally OEM determines the new product wholesale price of \( \omega_2 \).

In this study, we assume that the consumer demand function in the first period is deterministic and price-sensitive with a linear relationship, in which demand will decrease linearly along with increasing price. The price-sensitive demand function is given in the following equation

\[
Q(P) = a - kP.
\] (1)

We use a similar assumption to [17, 24] where consumer willingness-to-pay (WTP) is heterogeneous and uniformly distributed in the interval \([0, 1]\). Given each consumer’s WTP for a remanufactured product is a fraction of their WTP for a new one, and the market size is normalized to 1, we get the price function as given by
Furthermore, the demand model for manufacturing (OEM) and retailer was formulated, for both period one and period two. Given \( m_1 \) is the retailer profit from selling new products in the first period and \( m_2 \) is the retailer profit from selling new products in the second period, the demand function for the first period is known as

\[
q_{1N} = 1 - m_1 - \omega_1. \tag{3}
\]

The OEM and retailer profit functions in the first period are given as follows:

1. OEM’s profit in the first period.
2. Retailer’s profit in the first period.

In the first period, the retailer only distributes and sells new products, so that retailer profit is derived from the total profit from selling new products. The retailer’s profit function in the first period can be calculated as follows

\[ \pi_{1r} = TR_{1R} - TC_{1R} = (1 - m_1 - \omega_1)m_1. \tag{4} \]

The profit functions of OEM and retailer in equation (4) and (5) will be decentrally maximized along with the optimization of \( \omega_1 \) and \( m_1 \) variables.

In this research, we propose a model with two different cores acquisition strategies, which are referred to as the flat rate and fixed rate mechanisms. The representation of this occurs in the second period, where in addition to new products, the retailer also offers remanufactured products. The OEM profit function in the second period is similar to the first period, except that the demand function of new products is influenced by the value of the fraction of consumer WTP for remanufactured products of both fixed and flat rates, to the new products. Given \( \delta_1 \) and \( \delta_2 \) denote the fraction of consumer WTP for fixed rate and flat rate, respectively, we can calculate demand function for the new products in the second period as given by

\[
q_{2N} = 1 - P_{2N} - \delta q_{2R1} - \delta_2 q_{2R2} =
= 1 - m_2 - \omega_2 - \delta q_{2R1} - \delta_2 q_{2R2}. \tag{6}
\]

The OEM and retailer profit functions in the second period are given as follows:

1. OEM’s profit in the second period.
2. Retailer’s profit in the second period.

In the second period, OEM profit is the total profit from selling an amount of \( q_{2N} \) units. The OEM profit in the second period can be calculated as

\[ \pi_{2M} = TR_{2M} - TC_{2M} =
= (1 - m_2 - \delta q_{2R1} - \delta_2 q_{2R2} - \omega_2)(\omega_2 - c_m). \tag{7} \]

3. Retailer’s profit in the second period.

Implementation of fixed and flat rate mechanisms affects the retailer annual revenue. If it is known that the demand is sensitive to price, then we get the following relation

\[
P_{2R1} = \delta_1(1 - q_{2N} - q_{2R1} - \delta q_{2R2}) \tag{8}
\]

and

\[
P_{2R2} = \delta_2(1 - q_{2N} - \delta q_{2R1} - q_{2R2}) \tag{9}
\]

with \( P_{2R1} \) and \( P_{2R2} \) denote the fixed rate price and flat rate price, respectively. Based on the assumptions outlined in the previous section, it is known that if the proportion of the consumer failure rate is greater than the product failure rate covered within the flat rate mechanism \((\alpha > \gamma)\), the consumer will be incurred a surcharge, which value is equal to

\[
S = \frac{(\alpha - f)}{f} P_{2R2}. \tag{10}
\]

In the second period, the revenue of the retailer is obtained from the total sales of both new and remanufactured products. In this period, demand for new products follows the function given in equation (6). Whereas, the demand for remanufactured products consists of returned products coming from the fixed rate mechanism, denoted by \( q_{2R1} \), and returned products coming from the flat rate mechanism, denoted by \( q_{2R2} \). In the flat rate mechanism, \( f \) and \( \alpha \) parameters appear showing the proportion of the covered failure component and the actual customer failure rate, respectively. Then, the revenue of retailer in the second period is given in the following equation:

\[
TR_{2R} = q_{2N} \times P_{2N} + q_{2R1} \times P_{2R1} + q_{2R2} \times P_{2R2} + q_{2R3} \times S =
= (1 - m_2 - \delta q_{2R1} - \delta_2 q_{2R2} - \omega_2)(m_2 + \omega_2) +
\]

\[
+ \delta_1 q_{2R1}(m_2 + q_{2R1}(\delta_1 - 1) + \omega_2) +
+ \delta_2 q_{2R2}(m_2 + q_{2R2}(\delta_2 - 1) + \omega_2) +
+ \delta_1 q_{2R1}(\alpha - f)(m_2 + q_{2R1}(\delta_1 - 1) + \omega_2). \tag{11}
\]

During this period, the retailer incurs costs related to purchasing new products, remanufacturing costs and collection costs for used products. Remanufacturing costs consist of disassembly costs \((c_d)\) and repair costs \((c_r)\). Meanwhile, the cost of collecting used products consists of acquisition costs \((c_{ac})\) and reverse logistics costs \((c_{rl})\). Acquisition costs are costs incurred by the retailer to get consumers to return their products, such as incentives and promotional costs. Reverse logistics costs is the cost of collecting physical components of the used product, which includes expenses for handling the RL network and transportation. Hence, the retailer cost function is given as

\[
TC_{2R} = q_{2N} \omega_2 + (q_{2R1} + q_{2R2})c_d + \alpha(\delta q_{2R1} + q_{2R2})c_m +
+ (\delta q_{2R1} + q_{2R2})c_i + (q_{2R1} + q_{2R2})c_p =
= (1 - m_2 - \delta q_{2R1} - \delta_2 q_{2R2} - \omega_2)(\omega_2 +
+ (\delta q_{2R1} + q_{2R2})(c_d + \alpha c_m + c_i + c_p). \tag{12}
\]

Finally, we get the function of retailer profit as given by
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\[ \pi_x = \pi_{1x} + \pi_{2x} = (1 - m_x - \omega_x)m_x + \]
\[ + (1 - m_x - \delta q_{r1} - \delta q_{r2} - \omega_x)m_x + \]
\[ + \delta q_{r1}(1 - m_x - q_{r1} - \omega_x) + \]
\[ + \delta q_{r2}(m_x + q_{r2} - \omega_x) + \]
\[ + \delta q_{r2}((\alpha - f)m_x + q_{r2}(\delta_1 - 1) + \omega_x) - \]
\[ f - (q_{r1} + q_{r2}) (c_a + \alpha c_m + c_s + \rho). \]  

The profit functions of OEM and retailer in equation (7) and (13) will be decentralized maximized along with the optimization of \( \omega_x \) and \( m_1 \) in equation (21). Afterwards, the total profit function is the sum of the profit of OEM and retailer in both first and second periods. The following is the formulation of OEM and retailer total profits:

1. OEM’s total profit function.

\[ \pi_x = \pi_{1x} + \pi_{2x} = (1 - m_x - \omega_x)m_x + \]
\[ \times (\omega_x - c_a) + (1 - m_x - \delta q_{r1} - \omega_x) \]
\[ \times (\omega_x - c_a) (1 - m_x - \omega_x) (\omega_x - c_a). \]  

2. Retailer’s total profit function.

The optimization is carried out using a decentralized policy where OEM and retailer maximized their profits separately. Profit functions in equation (14) and (15) will be maximized using the proposed solution procedure explained in the next section.

5.2.3. Solution Procedure

The proposed model aims to maximize profits for both OEM and retailer in the ROCLSC system and determines the optimal values of \( \omega_x \), \( m_1 \), \( \omega_x \), \( m_2 \), \( q_{r1} \), and \( q_{r2} \). The solution procedure to get the optimal solutions is given below.

1. By assuming the value of \( m_1 \) to be constant, find the value of \( \omega_x \) that maximizes \( \pi_x \) in equation (4)

\[ \frac{d\pi_{1x}}{d\omega_x} = 1 + c_a - m_x - 2\omega_x = 0, \]
\[ \omega_x (m_x) - \frac{1}{2} (1 + c_a - m_x). \]  

2. By substituting the value of \( \omega_x (m_x) \) in equation (16) to equation (7), find the value of \( m_1 \) that maximizes \( \pi_x \) in equation (5)

\[ \frac{d\pi_{1x}}{dm_1} = \frac{1}{2} (1 - c_a - 2m_x) = 0, \]
\[ m_1 = \frac{1}{2} (1 - c_a). \]  

3. By substituting the value of \( m_1 \) in equation (17) into equation (16), we obtain the value of \( \omega_x \) as

\[ \omega_x = \frac{1}{4} (1 + 3c_a). \]  

4. By assuming the value of \( m_2 \) to be constant, substituting the value of \( m_1 \) and \( \omega_x \) in equation (17) and equation (18) into equation (7), find the value of \( \omega_x \) that maximizes \( \pi_{2x} \)

\[ \frac{d\pi_{2x}}{d\omega_x} = 1 + c_a - m_x - \delta q_{r1} - \delta q_{r2} = 0, \]
\[ \omega_x (m_x) = \frac{1}{2} (1 + c_a - m_x - \delta q_{r1} - \delta q_{r2}). \]  

5. By substituting the value of \( \omega_x (m_x) \) in equation (19) into equation (13), we obtain the value of \( m_2 \) that maximizes \( \pi_{2x} \)

\[ \frac{d\pi_{2x}}{dm_2} = \frac{1}{2} \left( 1 + c_a - m_x - \delta q_{r1} - \delta q_{r2} \right), \]
\[ m_2 = \frac{1}{2} \left( 1 + c_a - \delta q_{r1} - \delta q_{r2} \right) \alpha = \frac{1}{4} \left( 1 + 3c_a - \delta q_{r1} - \delta q_{r2} \right). \]  

6. By substituting the value of \( m_1 \) in equation (20) into equation (19), we obtain the value of \( \omega_x \) as

\[ \omega_x = \frac{1}{4} \left( 1 + 3c_a - \delta q_{r1} - \delta q_{r2} \right) \alpha = \frac{1}{4} \left( 1 + 3c_a \right). \]  

7. To obtain the optimal value of \( q_{r1} \) and \( q_{r2} \), we use constrained optimization with the help of Wolfram Mathematica software that meets the following condition:

Objective function:

Maximize \( \pi_{2x} (q_{r1}, q_{r2}) \)

s.t.

\[ 0 \leq q_{r1} \leq 1, \]
\[ 0 \leq q_{r2} \leq 1, \]
\[ 0 \leq q_{r2} \leq 1, \]
\[ P_{21} > P_{21} > P_{22}, \]

Here, we consider several constraints as given by equation (23)–(26). Constraints in equation (23)–(25) denote that the size of the market is normalized into 0 to 1 [2]. Whereas, the constraint in equation (26) denotes that the price of new products is always greater than the price of used products, and the price of used products coming from the fixed rate mechanism is always greater than the price of used products coming from the flat rate mechanism.

5.2.4. Numerical Example

The numerical example illustrates the implementation of the proposed model to solve problems related to the ROCLSC system. The input values are adopted from [17] with several adjustments. In this model, the objective is to maximize total profits of the system offering two switching core mechanisms: fixed rate and flat rate. The input parameters are: \( c_a = 0.2; c_f = 0.05; c_v = 0.07; c_d = 0.03; p = 0.05; f = 0.3; \alpha = 0.5; \delta = 0.8; \) and \( \delta = 0.45. \)
By applying the proposed solution procedure to the model, we obtain the optimal values of $\omega_1, m_1, \omega_2, m_2, q_{2,1}$, and $q_{2,2}$ as summarized in Table 2.

The maximum total profit is obtained when the retailer margin is set at $m_1=0.400$ and $m_2=0.432$ for both periods. The surface plot of $\pi_R, q_{2,1}$ and $q_{2,2}$ is shown in Fig. 2.

The three-dimensional depiction of the optimal solution in Fig. 2 shows the relationship between the amount of return products coming from the fixed rate ($q_{2,1}$) and flat rate ($q_{2,2}$) mechanisms with the retailer total profit ($\pi_R$). From Fig. 2, it is clear that both variables $q_{2,1}$ and $q_{2,2}$ have concave effects on $\pi_R$. The peak point of the surface plot in Fig. 2 shows that the maximum profit for the retailer is 0.2378, which is obtained at $q_{2,1}^* = 0.2354$ and $q_{2,2}^* = 0.2167$.

### Table 2

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Notation</th>
<th>Optimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td></td>
<td>0.4000</td>
</tr>
<tr>
<td>$m_1$</td>
<td></td>
<td>0.4000</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td></td>
<td>0.2408</td>
</tr>
<tr>
<td>$m_2$</td>
<td></td>
<td>0.4325</td>
</tr>
<tr>
<td>$q_{2,1}$</td>
<td>$q_{2N}$</td>
<td>0.0408</td>
</tr>
<tr>
<td>$q_{2,2}$</td>
<td>$q_2r_1$</td>
<td>0.2354</td>
</tr>
<tr>
<td>$q_{2,2}$</td>
<td>$q_2r_2$</td>
<td>0.2167</td>
</tr>
<tr>
<td>Model’s Performance Criteria</td>
<td>$\pi_M$</td>
<td>0.0417</td>
</tr>
<tr>
<td>$\pi_R$</td>
<td>0.2378</td>
<td></td>
</tr>
<tr>
<td>$\pi_{Total}$</td>
<td>0.2795</td>
<td></td>
</tr>
</tbody>
</table>

### 5.3. Sensitivity Analysis

#### 5.3.1. Sensitivity Analysis on $\delta_1$

In this section, we perform sensitivity analysis to study the effects of several key parameters on the model optimal solution and performance criteria. Table 3 shows the sensitivity analysis results of parameter $\delta_1$ to the model optimal solution.

### Table 3

<table>
<thead>
<tr>
<th>$\delta_1$</th>
<th>Period 1</th>
<th>Period 2</th>
<th>$\pi_M$</th>
<th>$\pi_R$</th>
<th>$\pi_{Total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-50%$</td>
<td>0.40 0.40 0.40</td>
<td>0.302 0.446 0.102 0.024 0.311 0.050 0.217 0.267</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-25%$</td>
<td>0.48 0.40 0.40</td>
<td>0.294 0.445 0.094 0.072 0.298 0.049 0.219 0.267</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.80 0.40 0.40</td>
<td>0.241 0.432 0.041 0.235 0.217 0.042 0.238 0.280</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+25%$</td>
<td>1.00 0.40 0.40</td>
<td>0.219 0.428 0.019 0.285 0.184 0.040 0.247 0.288</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 5.3.2. Sensitivity Analysis on $\delta_2$

Further, we will study how the variation in $\delta_2$ will affect the model optimal solution. The value of $\delta_2$ was changed from $-80\%$ to $+80\%$ while the values for other parameters remain unchanged. Table 4 shows the sensitivity analysis results of parameter $\delta_2$ to the model optimal solution.

### Table 4

<table>
<thead>
<tr>
<th>$\delta_2$</th>
<th>Period 1</th>
<th>Period 2</th>
<th>$\pi_M$</th>
<th>$\pi_R$</th>
<th>$\pi_{Total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-80%$</td>
<td>0.40 0.40 0.40</td>
<td>0.263 0.400 0.063 0.344 0.000 0.044 0.217 0.261</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-40%$</td>
<td>0.27 0.40 0.40</td>
<td>0.258 0.408 0.058 0.219 0.083 0.043 0.220 0.262</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>0.45 0.40 0.40</td>
<td>0.241 0.433 0.041 0.235 0.217 0.042 0.238 0.280</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+40%$</td>
<td>0.63 0.40 0.40</td>
<td>0.222 0.476 0.012 0.092 0.360 0.040 0.277 0.317</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+80%$</td>
<td>0.81 0.40 0.40</td>
<td>0.200 0.500 0.000 0.000 0.370 0.040 0.339 0.379</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.3.3. Sensitivity Analysis on \( f \)

Parameter \( f \) denotes the proportion of product failure covered by the retailer through the flat rate mechanism without any surcharges. The value of \( f \) was changed by \(-80\% \) to \(+80\% \), while the values for other parameters remain unchanged. Table 5 shows the sensitivity analysis results of parameter \( f \) to the model optimal solution.

<table>
<thead>
<tr>
<th>( f )</th>
<th>Period 1</th>
<th>Period 2</th>
<th>( \pi_M )</th>
<th>( \pi_R )</th>
<th>( \pi_{Total} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-80% )</td>
<td>0.060</td>
<td>0.200</td>
<td>0.274</td>
<td>0.000</td>
<td>0.190</td>
</tr>
<tr>
<td>(-40% )</td>
<td>0.180</td>
<td>0.265</td>
<td>0.296</td>
<td>0.005</td>
<td>0.406</td>
</tr>
<tr>
<td>0</td>
<td>0.800</td>
<td>0.241</td>
<td>0.433</td>
<td>0.041</td>
<td>0.235</td>
</tr>
<tr>
<td>(+40% )</td>
<td>0.420</td>
<td>0.256</td>
<td>0.403</td>
<td>0.056</td>
<td>0.311</td>
</tr>
<tr>
<td>(+80% )</td>
<td>0.540</td>
<td>0.263</td>
<td>0.400</td>
<td>0.062</td>
<td>0.344</td>
</tr>
</tbody>
</table>

5.3.4. Sensitivity Analysis on \( \alpha \)

Here, we also performed a sensitivity analysis to measure the sensitivity of the model to changes in \( \alpha \), which denotes the proportion of consumer's actual failure proportion to components. Table 6 shows the sensitivity analysis results of parameter \( \alpha \) to the model optimal solution.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Period 1</th>
<th>Period 2</th>
<th>( \pi_M )</th>
<th>( \pi_R )</th>
<th>( \pi_{Total} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-50% )</td>
<td>0.250</td>
<td>0.251</td>
<td>0.500</td>
<td>0.051</td>
<td>0.362</td>
</tr>
<tr>
<td>(-25% )</td>
<td>0.375</td>
<td>0.248</td>
<td>0.406</td>
<td>0.048</td>
<td>0.310</td>
</tr>
<tr>
<td>0</td>
<td>0.500</td>
<td>0.241</td>
<td>0.433</td>
<td>0.041</td>
<td>0.235</td>
</tr>
<tr>
<td>(+25% )</td>
<td>0.625</td>
<td>0.234</td>
<td>0.471</td>
<td>0.034</td>
<td>0.162</td>
</tr>
<tr>
<td>(+50% )</td>
<td>0.750</td>
<td>0.226</td>
<td>0.521</td>
<td>0.026</td>
<td>0.083</td>
</tr>
</tbody>
</table>

6. Discussion of numerical experimental results

The ROCLSC system model developed in this study aims to maximize the total profit at the retailer, with two switching cores mechanisms, namely flat rate and fixed rate. The acquisition of cores is a critical point in the reverse logistics system, considering that material suppliers from this industry are customers who have various perceptions of used products. Retailer, as a leader in the system, has to maximize efforts to obtain product returns from customers through the mechanisms offered. The characteristics of the ROCLSC system where the retailer plays a role in distributing new products, selling remanufactured products, carrying out after-sales services, as well as collecting used products, cause an increase in retailer profits followed by an increase in total profits of the supply chain system.

The optimal solution for the retailer is obtained when the retailer sets its margin as \( m_1=0.40 \) per unit in the first period, and \( m_2=0.433 \) per unit in the second period. At this level, the optimal amount of returned products for the fixed and flat rate mechanisms is 0.235 and 0.217, respectively (Table 2). From the numerical example result, we understand that the optimal solution for the retailer is to increase the acquisition of product returns through the fixed rate mechanism. Through the switching cores mechanism, remanufacturing companies can control the number of cores in order to reduce supply and demand imbalances.

The results of the sensitivity analysis (Tables 3–6) show that the consumer’s WTP for remanufactured products from both fixed rate and flat rate, has an impact on the retailer profit, since the number of product returns also increased along with an increase in consumer’s WTP. Therefore, in order to increase profit, the retailer has to properly manage customer perceptions of remanufactured products. This is important because as the proportion of consumer’s WTP for remanufactured products increases, the demand for new products decreases. Consequently, the retailer relies on the switching cores mechanism to increase revenue. In addition, a study of the value of the proportion of faulty components covered by the flat rate mechanism without any surcharges shows that a decrease in the value of \( f \) causes retailer profits to be higher. A high \( f \) value means that the retailer will cover more faulty components, which is certainly not profitable for the retailer. Thus, the retailer must increase the number of return products through the flat rate mechanism so that the profit remains maximum. In addition, to be able to increase profits, the retailer must increase the number of product returns through the flat rate mechanism, because retailers will receive additional costs due to the proportion of failure to the actual components that exceeds the proportion covered by the retailer.

The RCLSC core acquisition model for the machinery remanufacturing industry was developed and optimized in this study. This study introduced the fixed rate and flat rate mechanisms for core acquisition. These mechanisms are in accordance with the characteristics of the heavy construction machinery products, and the characteristics of the customers that consider product function as their priority. Within the switching mechanism, acquiring cores only takes a short time for customers, so there is no significant loss in order fulfillment time. For the retailer, as a remanufacturing company, the switching core mechanism provides maximum profit, where the optimization result explains that 48% of the returned product is obtained from the fixed rate mechanism and 44% is obtained from the flat rate mechanism.

The limitation in this study is the limited number of actors involved in the supply chain system. The role of the retailer as collector, remanufacturer and distributor can be added or replaced by a third party, so that the system has dual channels for a part or all of these roles. Other limitations of this study we only consider deterministic variables.
and short time horizon. Future development can be done by taking these limitations into account. Another possible development is to consider other cores acquisition mechanisms within the scope of transactional switching cores mechanism. Since the function of products is a very important part in construction machinery type of products. Here, we only analyze four parameters in the sensitivity analysis section, which are $\delta_1$, $\delta_2$, $f$ and $a$. Incorporating more parameters in the sensitivity analysis will be possible for future development. Thus, hopefully the decision maker can obtain a more comprehensive solution, which is in accordance with the real conditions experienced by the industry.

### 7. Conclusions

1. The ROCLSC system model with a dominant role of retailer, along with the acquisition strategy of two switching cores mechanisms, can generate maximum profits for the supply chain system when product returns are obtained from these two mechanisms.

2. Based on the numerical example, the optimal solution is obtained when the amount of returned products coming from the fixed rate mechanism is set as 0.238, where flat rate amount is 0.217. It means that the fixed rate amount should be set 9% higher than the flat rate amount, in order to maximize the profit of the system.

3. The sensitivity analysis result indicates that the parameters of consumer WTP for the fixed rate product and flat rate product, and the component failure rate of the consumer have a positive correlation with the retailer profit. On the contrary, the parameter of component failure rate covered by the retailer in the flat rate mechanism has a negative correlation with the retailer profit.

### References


### Acknowledgments

This study was supported by a Doctoral Research Grant from the Ministry of Research, Technology and Higher Education of Indonesia, with contract No. 3594/UN10.14/KU/2015.