Supercapacitors are commonly used for a guaranteed launch of diesel generators. However, the processes caused by the starting current until the starter shaft rotates are disregarded. The duration of this moment is short but its effect on the rechargeable battery, taking into consideration its service life, is significant. The shape of this pulse, its duration significantly depend on the ratio of system parameters: supercapacitor (rechargeable battery) – starter – diesel generator.

A system of differential equations has been proposed to describe the compatible electromagnetic and electromechanical processes that occur when the starter of the diesel generator is powered from the supercapacitor. A charge is used as a variable quantity. The transitional processes occurring in the stationary starter rotor and the subsequent processes caused by the growth of the electromagnetic starter moment have been taken into consideration.

This paper reports establishing those patterns that are related to the beginning of the starter movement, its entering the mode at the falling voltage of the supercapacitor, the exchange of electrical and magnetic energy accumulated in the inductive elements of the starter.

Using the charge as a variable quantity has made it possible to combine the final values of the preceding process (stationary rotor) with the initial ones of the next one (output to starting speed). Thus, a mathematical notation has been derived that considers most of the parameters of the charge circle of the supercapacitor. The possibility of using an inflated voltage of the supercapacitor to increase the accumulated energy has been clarified.

The processes have been theoretically substantiated, which makes it possible to use a small internal resistance of the starter circuit, the presence of inductive components, an abnormal capacity of the supercapacitor to form the desired shape of the electromagnetic moment. That would make it possible to take into consideration the specific requirements of various systems of guaranteed power supply.

Keywords: electromagnetic transition processes, electromechanical transition processes, electromagnetic moment, moment of resistance.

1. Introduction

Given their unique properties such as an abnormally large capacity, low internal resistance, performance at low temperatures, supercapacitors are increasingly used in various industries. One of the weakest links in backup power supply systems is the launch of diesel generators at low temperatures. The situation is similar in automotive technology [1]. Rechargeable battery service life depends significantly on how frequent the launches were at low temperatures and how deep the charge pulses were. To ensure guaranteed power supply, improve the operating conditions of batteries, supercapacitors that are reliably functioning under difficult conditions have been used recently.

The studies that address this issue do not always take into consideration a series of factors. In particular, when...
using a load plug as a diagnostic agent, the change in its internal resistance due to temperature is disregarded. When applying the compatible operation of the supercapacitor and rechargeable battery, the redistribution of currents between them is ignored. The launch technology involving the supercapacitor only [2] is not perfect due to the absence of a mathematical apparatus that could describe such transition processes.

In general, the results obtained are not highly reliable. During the full-scale studies involving a starter, electro-mechanical transition processes are usually taken into consideration while electromagnetic ones remain unaddressed. However, the peak current is caused precisely by the electromagnetic transition process when the starter shaft has not yet rotated. That is, the starter's counter EMF has not yet appeared; only its inductance in a stationary state “operates”.

The desire of manufacturers of guaranteed power supply systems to improve the reliability of the launch of the diesel generator by increasing the capacity of rechargeable batteries does not resolve the issue. Since critical to the launch is not the amount of energy accumulated but the ability of the source to provide for appropriate power for a certain period. In this respect, supercapacitors demonstrate a significant advantage over rechargeable batteries. Traditionally, when studying the starter’s operation, the energy accumulated in its inductive elements is not used. The introduction of the supercapacitor fluctuation charge mode in the starter makes it possible to apply it, which could reduce the overall indicators of the launch system.

Deriving a mathematical notation of the processes that occur when the starter operates from the supercapacitor would make it possible to elucidate the influence of the following factors: the active and inductive resistance of the starter, the capacity and internal resistance of the supercapacitor, the moment of resistance on the shaft from the diesel side, the level of the charging voltage of the supercapacitor. The next step would be to solve the optimization problem depending on the customer's conditions. By varying these parameters, at the constant energy of the supercapacitor, one can form different shapes of the torque over time. Approaches may vary depending on the mode of operation of the diesel generator: cold reserve, hot reserve, running time to start, etc.

2. Literature review and problem statement

Paper [1] investigated the possibilities of starting the engine from supercapacitors. The results of research using a semiconductor converter are given. It is shown that for a specific situation, this option of using a supercapacitor is quite suitable. However, the details remained unresolved, regarding the characteristics of the dynamic moment of resistance of the engine shaft, the mechanical characteristics of the starter, during different phases of the transition process. The reason for this may be the issues related to the separation of various transition processes in full-scale research, at currents of 200–1,000 A. In general, this direction may not be appropriate.

A likely option to overcome the complications of a reliable start-up from the supercapacitor may be to use a synchronous bidirectional converter [2], which ensures the operation of the auxiliary power plant. Despite the practical significance of this approach, questions remained about how this would affect weight-dimensional indicators. The issue of converter operation on peak currents remains unresolved. One of the areas to overcome such difficulties is the combination of the rechargeable battery and supercapacitor. This approach is used in work [3] where the supercapacitor is used as a damper for peak current and thereby improves the operating conditions of the rechargeable battery. However, the circuitry solutions to control load distribution during the start-up process have not been sufficiently worked out. That is likely due to the complexity of selecting the parameters for existing supercapacitors, starters, and rechargeable batteries. The results of studying the system of guaranteed power supply with various power drives are reported in [4]. In particular, the authors investigated an option to start a diesel generator only from the supercapacitor. The results obtained indicate the prospects for the joint use of rechargeable batteries and supercapacitors. However, the issues related to the ratios of the parameters of the start-up schemes and their impact on the duration of transition processes remain unresolved. The reason for this is likely the lack of obvious existence, at the first stage, only of the electromagnetic transition process and the complexity of its delimitation in full-time research. Work [5] reports the results of studying supercapacitors as compensators of peak current and analyzes their ability to force charge and discharge. However, there are no data on the number of cycles for a period; the characteristics of sources that could provide such operation and methods of implementation of forced modes are not given. Paper [6] investigates the ability to accumulate energy by the supercapacitor, followed by its powering a vehicle engine. To analyze the results, numerical modeling methods are used in the MATLAB Simulink environment (University of New Mexico (UNM), USA). This approach is obvious but the specified mathematical support is not designed for non-standard circuit solutions, which include powering the engine by the supercapacitor under transition modes. A mathematical model of the operation of urban electric transit involving supercapacitors is proposed in work [7]. The authors generalized the processes of charging and discharging the supercapacitors but gave no detail of the processes that would take into consideration compatible transition processes. The reason for this is likely the objective difficulties associated with describing the properties of DC motors in transition processes during the falling voltage of power supply. An attempt to provide a reliable mathematical notation of the operational processes of a supercapacitor along with a rechargeable battery was made in work [8]. However, the research addressed mainly thermal processes and their impact on the duration of the warranty resource. An option to overcome these difficulties may be to build a power source control system (fuel cell, rechargeable battery, and supercapacitor). This approach is used in paper [9]; controllers for managing power sources were developed in the MATLAB/Simulink environment. The result of achieving control over energy flows between the sources and drive engine is the data that made it possible to analyze the performance of processes. At the same time, the peculiarities caused by the joint action of transitional electro-machine and electromagnetic processes remained unaddressed. Paper [10] reports the results of studying a guaranteed power supply system, which includes a supercapacitor, a rechargeable battery, and a diesel generator. A multi-power management strategy was implemented, which provides obvious advantages for the guaranteed launch of
standalone objects. However, questions remained about the nature of change in the voltage, current, moment, and speed during the start-up.

All this allows us to assert that it is advisable to continue the study that could clarify the nature of transient electromagnetic and electromechanical processes, their impact on the speed and moment developed by the starter when it is powered by the supercapacitor.

3. The aim and objectives of the study

The aim of this study is to establish patterns in the starter operation when it is powered by a supercapacitor. Given the fact that the supercapacitor has an order of magnitude better performance indicators in terms of specific power, compared to a rechargeable battery, there is reason to hope for obtaining the predefined law of change in the electromagnetic moment. That, in turn, would ensure the adaptability of the launch mode depending on the starting characteristics of the diesel engine and operating conditions.

To accomplish the aim, the following tasks have been set:
– to describe by a differential equation those electromagnetic processes that occur in a stationary starter, using the value of charge coming from the supercapacitor as a variable;
– to describe by differential equations those compatible electromagnetic and electromechanical processes that take place in the system supercapacitor–serial excitation motor under a load.

4. Materials and methods to study compatible electromagnetic and electromechanical processes in the starter

This study is aimed at establishing those patterns that occur during starter operation from a source with a falling nature of voltage (a supercapacitor). The sequential transition of the electromagnetic transition process with the subsequent joint electromagnetic and electromechanical transition processes is taken into consideration. Using the obtained results and entering the parameters for a particular system starter–supercapacitor, we obtain a characteristic of the process. Varying the parameters, one can find out how far the system is from the optimal one and by what parameter. The next step is to select the appropriate equipment. When starting from the rechargeable battery, such questions were not acute. Since its internal resistance is several orders of order greater than the internal resistance of the supercapacitor, the voltage accepts a relatively constant value; no electromagnetic fluctuation processes are observed. Therefore, it was not relevant to take into consideration the magnetic energy that accumulated in the starter at the first stage of launch. Over time, the energy was lost, turning into heat. The use of the supercapacitor, taking into consideration the parameters of the charge circuit, makes it possible to form a “shock electromagnetic moment” not at the initial stage but move it in time. That is, combine currents caused by the active resistance of the circuit and utilize the energy accumulated in the inductive elements of the starter. It is important, to rotate the cold diesel engine, to move the crankshaft, then the mechanical moment of resistance begins to fall. Accordingly, it is possible to reduce the amount of energy flow from the supercapacitor, which is executed due to its properties.

The process of starting a diesel generator from a starter, powered by a supercapacitor, was divided into two interrelated stages. The first lasts from hundredths to a few seconds, when the voltage is supplied to the starter but its shaft has not yet begun to rotate. Consequently, there is no high-speed counter EMF (counter-electromotive force), which depends on the rotational speed of the rotor in the magnetic field created by the current passing through the rotor and the excitation winding. The current is due only to the parameters of the LRC circuit. The second stage comes when the starter’s shaft comes into motion and a high-speed counter EMF appears.

Since the starter shaft does not rotate, we have a consistent connection of the supercapacitor, the active resistance (the sum of the active resistance of the starter’s windings, wires, supercapacitor), and inductance of the starter; Fig. 1.

Fig. 1. Charge circuit of the supercapacitor

The differential equation of the transition process of such a circuit is generally known. A current is a variable value. However, in the case where the power voltage of the supercapacitor is a variable quantity, it is advisable to proceed to the differential equation in which a variable value is the charge of the supercapacitor, coulombs:

\[ Lq' + rq + \frac{q}{C} = 0, \]  

where \( q \) is the charge that changes over time as the supercapacitor is discharged; \( C \) is the capacity of the supercapacitor; \( L \) is the starters’ inductance; \( r \) is the active resistance of the circuit (a supercapacitor, connecting wires of starter’s windings).

Since we have a second-order differential equation, it is necessary to find the initial value of the charge of the supercapacitor and the initial values of the first derivative from the charge for time, that is, the initial current:

\[
\begin{align*}
q(0) &= C \cdot U_0, \\
q' (0) &= 0,
\end{align*}
\]

where \( U_0 \) is the voltage value on the supercapacitor.

The next step is to determine the moment when the starter shaft is set into motion. This time would likely correspond to the value of the current when the electromagnetic moment developed by the starter is equal to the moment of resistance of the starter's shaft. We shall build a system of differential equations, which combines the electromagnetic and electromechanical transition processes:

\[
\begin{align*}
Lq' + rq + \frac{q}{C} &= c_s \omega, \\
\omega(q)^2 - M_0 &= j\omega,
\end{align*}
\]

where \( c_s \) is the structural parameter characterizing the electromagnetic properties of the starter; \( \omega \) is the instanta-
neous speed of the engine shaft; \( j \) is the moment of inertia of the system starter’s shaft-engine crankshaft; \( c_n \) is the structural parameter characterizing the electromechanical properties of the starter; \( M_0 \) is the moment of resistance on the starter’s shaft.

The first equation of system (3) corresponds to the sum of voltages on inductance, active resistance, the voltage on capacity, and the \( c_n \) term (counter EMF), which is proportional to the speed of rotor rotation. The second equation characterizes the relationship between the electromagnetic moment developed by the engine \( c_n(q) \), the moment of resistance on the shaft \( M_0 \), and the dynamic moment \( j \omega \). Since starters, due to technological considerations, are made as serial excitation motors, this means that the current of the armature circuit passes the circuit of excitation as well. The electromagnetic moment, in turn, depends on the product of magnetic flux and current, so we obtain a quadratic dependence.

Fig. 2 shows the scheme for enabling the starter. It includes the mechanical part: the diesel engine crankshaft DG and the starter rotor St. Electric part includes C – a supercapacitor, L – the general inductance of the rotor and the starter excitation coil, \( r \) – active resistance, combining the internal resistance of the supercapacitor, starter, and connecting wires.

![Fig. 2. Scheme for enabling the starter](image)

The onset of the movement of the starter rotor would correspond to the moment when the electromagnetic moment and the moment of resistance on the shaft are equalized:

\[
e_n(q)^2 = M_0, \tag{4}
\]

hence:

\[
q = i = \frac{M_0}{c_n} \tag{5}
\]

Comparing a current \( i_1 \) with the current found when solving differential equation (1), we find the time \( t_1 \) when the shaft begins to move, and the amount of charge remaining on the supercapacitor \( q_{00}(t_1) \).

To solve the differential system of equations (3) numerically, it should be represented in the explicit Cauchy form relative to the higher derivative. Following a series of standard mathematical transformations, we obtain:

\[
q'' = \frac{1}{L} r q' - \frac{q}{C} \frac{c_n}{j} (q)^2 - \frac{c_n M_0}{j}, \tag{6}
\]

the third-order differential equation. To solve it, it is necessary to find a value of the second derivative for a charge. To this end, we use the first equation from system (3). Since the shaft does not rotate \( \omega = 0 \), we can write:

\[
L q''(t) + r_i + \frac{q_{00}}{C} = 0,
\]

hence:

\[
q(t) = \frac{1}{L} \left( r_i + \frac{q_{00}}{C} \right). \tag{7}
\]

Fig. 2.

Thus, we derive the initial conditions to solve equation (6) numerically

\[
\begin{align*}
&\{ t_i; \\
&q_{01}(t_i); \\
&q_{01}(t_i);
\end{align*}
\]

In general, we obtain the following structure: the supercapacitor has a charge corresponding to voltage \( U_0 \); at moment \( t_0 = 0 \), the voltage is supplied to the starter, which remains stationary; the charge decreases, the current increases. These processes are described by equation (1); the initial conditions are (2). At time \( t_1 \), the current values became such that the electromagnetic moment of the starter was equal to the moment of resistance \( M_0 \); the rotational movement of the rotor begins, the counter EMF appears. The processes are described by equation (3) with the initial conditions of (7). The final conditions of the preceding process are the initial ones of the next one.

### 5. Results of studying the supercapacitor launch of the starter

#### 5.1. Electromagnetic transition processes at a stationary rotor

The resulting numerical solution to differential equation (1) showed how the charge of the supercapacitor and the current in the circuit of the starter change over time during the stage when the rotor is still stationary. The results are shown in Fig. 3.

![Fig. 3. The supercapacitor charge curves: curve \( q \) – the dependence of the amount of charge on the supercapacitor on time (the left axis), \( i \) – a change in current over time (the right axis)](image)
The ratio of the negative current values and positive voltage (charge) values in Fig. 3 is explained by a decrease in the charge of the supercapacitor, that is, the derivative from the charge accepts a negative value. The characteristics correspond to the mode of the slowed rotor. That is, within 10 s, the rotor (artificially) remains stationary. From this experiment, we find at which current, according to (4,5), the moments are equalized and we obtain the start time of the starter rotor movement. For greater obviousness, the time axis is given by an uneven scale since, starting from 1.5 to 10 s, the process proceeds relatively smoothly. It is established that the nature of transient electromagnetic processes depends on the ratios L, r, C and the number of charges on the supercapacitor; in this case, 700 K, which, at the supercapacitor capacity of 20 F, corresponds to the voltage of 35 V. We observe the aperiodic nature of the discharge of the supercapacitor.

5.2. Compatible electromagnetic and electromechanical transition processes

Fig. 4 shows the characteristics of voltage and current obtained from solving equation (3) numerically. The section with an interval of t=0–2.167 s corresponds to the chart in Fig. 3 when the rotor is stationary; that has made it possible to find the start time of rotor rotation. Points 1 and 2 in Fig. 4 correspond to this moment: current, 200 A; voltage, 24–20 V.

Here comes a high-speed counter EMF, which leads to a slowdown in current growth and a drop in voltage. Point 3 in Fig. 4 – voltage transition through zero (an element of the fluctuation discharge) due to the energy accumulated on the inductance of the starter and the maximum current through the supercapacitor: in this case, 700 K, which, at the supercapacitor capacity of 20 F, corresponds to the voltage of 35 V. We observe the aperiodic nature of the discharge of the supercapacitor.

Fig. 4. Change in the voltage and current of the starter powered by the supercapacitor: U – starter voltage (the left axis), i – starter current (the right axis)

Our results indicate that the mathematical notation of the start-up of the diesel engine from the supercapacitor, based on equations (1), (3), corresponds to the real processes that take place during the operation of a serial excitation motor. The falling nature of the voltage source, the small active resistance of the charge circuit, the inductive component of the starter could initiate the oscillatory processes.

6. Discussion of results of studying the operation of a serial excitation motor powered by the source of the falling voltage

The use of supercapacitor power for the starter makes it possible to reduce the active resistance of the charge circuit. The internal resistance of the supercapacitor is several orders less than the internal resistance of the rechargeable battery, especially at low temperatures. The inductance of the starter acts as a current-balancing element, before its movement. The energy accumulated in the electromagnetic elements of the starter during its operation contributes to an increase in the rotational electromagnetic moment. Thus, the active energy of the supercapacitor is transmitted to the starter with minimal losses.

When describing the processes associated with the starter, the equations are used where the supply voltage is a constant quantity (a source of infinite power). That allows us to take advantage of known equations where the variable quantity is current and continue to derive the required ratios. Although in reality the rechargeable battery voltage at the time of start-up is reduced by 10–20 %, it is difficult to take this into consideration in a differential equation, due to the lack of analytical dependence of the voltage value on
current. That issue could be solved if one knew the number of coulombs in the rechargeable battery at the time of launch; currently, this issue is also not resolved. In this sense, the properties of the supercapacitor make it possible to obtain an unambiguous dependence of the power voltage on the amount of charge and relate various transition processes in a single system of differential equations (3). The simplifications that are assumed are as follows. The starter’s magnetic line operates under an unsaturated mode. This gives reason to assert that when the magnetic system of the motor is not moving its inductance is a constant quantity. With the beginning of the rotor movement, an armature reaction appears that distorts the magnetic flux but, due to the non-saturation, its value remains constant. Structurally, the rotor is executed symmetrically; if one neglects the pulsations caused by the switching of the collector layers, it is permissible to assume that the inductance of the rotor is also constant. It should be noted that the serial starter excitation coil is stationary, so the inductance is steady, and the flux coupling changes only by changing the current values. With the onset of movement, the counter EMF appears, which is due to the movement of the rotor. Since the magnetic system is not saturated, it is proportional to the velocity taken into consideration in the first equation in (3). Based on the stated simplifications, the second electromechanical equation of system (3) was built.

In the system of differential equations (3), the moment of resistance on the starter shaft is considered as a constant value. In fact, the moment of resistance to the diesel engine shaft has an inversely proportional dependence on the revolutions. However, solving this equation requires a different mathematical apparatus; it is postponed for the future. On the other hand, the proposed option describes a heavier mechanical start-up processes of the starter makes it possible to influence the beginning of the start-up process. Depending on the specific situation, we can vary the energy supplied to the starter. For a “tough” start-up, it is necessary to provide for a delay in time, accumulate energy in inductive elements, and organize a shock electromagnetic moment. Alternatively, when using the same energy, it is necessary to distribute it “smoothly” over a time interval, we obtain an electromagnetic moment that changes relatively “gently” at the start interval. Such aspects should be taken into consideration depending on the properties of the diesel engine and the ambient temperature.

2. The combination of the electromagnetic and electromechanical start-up processes of the starter makes it possible to evaluate the effect of a particular parameter on the nature of the development of the process in general. An important indicator when starting a cold diesel engine is the minimal revolutions at which the mixture is ignited. The minimal permissible revolutions increase significantly as the ambient temperature decreases. Using the results obtained, one can combine the nature of change in maximum speed – a steeper peak and a shorter time interval, or vice versa, at the same amount of energy.

The limitations also include assumptions regarding the operation of the magnetic system under an unsaturated state. The cost and mass-dimensional indicators of engines operating on the linear characterization of the hysteresis loop are higher compared to those where the cell operates in a saturated state. It is not yet possible to describe the operation of the starter when using a nonlinear region of the hysteresis loop.

Further research into the supercapacitor launch should focus on devising an optimal law of change in the electromagnetic start-up moment. A rechargeable battery does not make it possible to vary this setting. The supercapacitor start-up can form the torque from pulsed, for example, at low temperatures, to smoothly rising, at high temperatures, at unchanged accumulated energy. It is advisable to choose parameters in this way so that in the case of the first unsuccessful start, the energy reserve is enough for several follow-up launches, with a change in the shape of a dynamic electromagnetic moment, taking into consideration the preceding history of the process.

7. Conclusions

1. Knowing the law of current change over time, with a stationary rotor, and taking into consideration the time when the electromagnetic moment is equal to the moment of resistance to the crankshaft of the diesel engine, we obtain a possibility to influence the beginning of the start-up process. Depending on the specific situation, we can vary the energy supplied to the starter. For a “tough” start-up, it is necessary to provide for a delay in time, accumulate energy in inductive elements, and organize a shock electromagnetic moment. Alternatively, when using the same energy, it is necessary to distribute it “smoothly” over a time interval, we obtain an electromagnetic moment that changes relatively “gently” at the start interval. Such aspects should be taken into consideration depending on the properties of the diesel engine and the ambient temperature.

References


1. Introduction

Electrical impedance tomography (EIT) is one of the methods of medical imaging, the principle of which is based on evaluation of a change in the field of conductivity in the assigned cross-section of an object at the injection of high-frequency current of a small amplitude through it [1].