This paper reports the theoretically investigated aerodynamic imbalance of the propeller blade, as well as correcting masses for balancing it.

It has been established that the aerodynamic forces acting on the propeller blade can be balanced by the adjustment of masses. This is also true for the case of compressed air (gas) provided that the blades are streamlined by laminar flow. That makes it possible to use rotor balancing methods to study the aerodynamic forces acting on the propeller blade.

The rotating blade mainly generates torque aerodynamic imbalance due to a lift force. A much smaller static component of the aerodynamic imbalance is formed by the drag force acting on the blade. The correcting mass located in the propeller plane balances both static and torque components of the aerodynamic imbalance in its correction plane. A second correcting mass (for example, on the electric motor shank) balances the torque component of aerodynamic imbalance in its correction plane.

The calculations are simplified under the assumption that the equilibrium of aerodynamic forces is perpendicular to the chord of the blade. For approximate calculations, one can use information about the approximate location of the pressure center.

The aerodynamic forces acting on the blade can be determined on the basis of the correcting masses that balance them. The accuracy in determining the aerodynamic forces could be improved by measuring a lift force.

The computational experiment has confirmed the theoretical results formulated above. The experiment further proves the possibility of applying the devised theory for propellers whose rotation speed changes in the angles of blade installation.

Keywords: propeller, blade, aerodynamic imbalance, mass imbalance, aerodynamic balancing, mass adjustment balancing

1. Introduction

Propellers are widely used in machines for various purposes. These propellers include axial fan impellers, aircraft engine propellers, wind tunnel fans, etc. There is a common issue related to reducing the vibrations arising during the operation of propellers. The main source of vibrations is regular and aerodynamic imbalance. In propellers, there is an analogy between these two kinds of imbalance. Therefore, they could be simultaneously balanced, for example, by adjusting the masses, aerodynamically, or by mixing balanced. However, the aerodynamic imbalance, unlike the regular one, depends on the air density. Therefore, if one balances the propeller only by adjusting the masses, the balancing accuracy would be disturbed with the change in air density.

Due to the analogy between the two kinds of imbalance, there is a particular task to identify the regular and aerodynamic components from the overall imbalance. This is a relevant issue regarding the design of propellers, when devising methods to balance them, or checking the quality of balancing, refusing defects, in operational monitoring, etc.

2. Literature review and problem statement

Aerodynamic (gas-dynamic) imbalance in a broad sense occurs in the propellers of large [1] and small [2] aircraft,
turbo-assemblies [3], turbochargers of ICEs [4], axial fans of low [5] and high [6] pressure, GTEs [7], etc.

The imbalance of any origin increases the vibrations of machines [1–7], reduces the speed of rotation of the impeller, the traction force of the propeller [2], the efficiency of ICE [4] or GTE [7], etc.

The common property of aerodynamic imbalances is that they have a significant rotational component. In turn, the rotational component can be balanced by adjusting the masses. However, such balancing would be disturbed at a change in the operating mode of the machine [3], in the air (gas) temperature and pressure [6], in the impeller rotation speed, etc. Therefore, it is advisable to balance impellers both by adjusting the masses and aerodynamically. It is also expedient to control the regular and aerodynamic imbalance of impellers during manufacturing and operation.

The aerodynamic imbalance of propellers is almost all rotational [1] and, therefore, has more analogies with imbalance due to correcting masses. Adjusting the masses has a greater effect because the balance changes less with a change in the modes of operation of the propeller. However, it should be noted that broadband noise and vibrations excite the aerodynamic forces acting on propeller blades due to the detachment of airflows from the edges of the blades. Therefore, separate tasks are the design of the impeller [8], the optimization of the blades’ geometrical parameters [9], the modernization of blade shapes [10], and so on. However, such studies do not tackle the regular and aerodynamic imbalance of the propeller.

When fabricating axial fans, the blade rotor is consistently balanced aerodynamically and by adjusting the masses before operation. An example is a technique described in [11]. The production and operation of propellers are complicated by the fact that aerodynamic balancing methods are mainly reduced to checking the geometric symmetry of the propeller and, therefore, they are time-consuming and not always effective. In addition, given the analogy in the manifestations of the two types of imbalance, it is difficult to distinguish components caused by the imbalance of masses and the aerodynamic imbalance in the propeller.

A promising direction towards overcoming these difficulties is to continuously balance the imbalance of masses and the aerodynamic imbalance by passive auto-balancers [12–14]. The operation of these devices is based on the fact that, under certain conditions, correcting loads in auto-balancers come to the position in which they balance the rotor. Consider some of the results of those studies.

Paper [11] has analytically proven the possibility of the on-the-go static balancing of the axial fan impeller by a passive auto-balancer.

In [12], a possibility of continuous dynamic balancing of the disk with blades by two passive auto-balancers was studied. The numerical methods, for a particular system, have established that at the above-the-resonance speeds of rotation of the disk the auto-balancers effectively eliminate both the imbalance of masses and the aerodynamic imbalance.

Work [13] established a mathematical analogy between the regular and aerodynamic imbalances of the impellers of axial fans of low pressure. The possibility of their simultaneous balancing by adjusting the masses or by passive auto-balancers has been proven. It was found that the aerodynamic imbalance is directly proportional to air density and, therefore, varies depending on the weather conditions and operating conditions of the fan, but is not disturbed when the impeller rotation frequency changes. It was established that it is impossible to dynamically balance a dynamically flat impeller (short rotor) with two auto-balancers. However, if it is installed in a heavy case with the possibility of rotation, and the case is visco-elastically fixed, then the auto-balancing would be set at above-the resonance speeds.

Study [14] has experimentally investigated the accuracy of dynamic balancing of the axial fan impeller VO 06-300-4 by ball auto-balancers and mass adjustment. It was found that both the imbalance of masses and the aerodynamic imbalance are simultaneously balanced by two techniques. At the same time, the accuracy of balancing the two types of imbalance by adjusting the masses is limited only by the accuracy provided by a balancing instrument.

The use of passive auto-balancers requires an additional space, which is not always available. Therefore, it is advisable to control the aerodynamic and regular imbalance of propellers during production. It is advisable to introduce the passport specifications for propellers, such as the residual imbalance of masses and the residual aerodynamic imbalance. Therefore, there is a separate scientific issue related to studying various types of propellers’ imbalance. To resolve this task, paper [15] determined the main vector and torque of aerodynamic forces acting on the rotating impeller. The aerodynamic imbalance was found in the following cases: when mounting the impeller onto the rotor shaft with eccentricity and distortion; while installing one blade at another angle of attack; when violating the uniform distribution of blades in a circle. It was established that the greatest aerodynamic imbalance is generated when installing blades at different angles. At the same time, the aerodynamic imbalance has a moment component, which is an order of magnitude greater than static.

The next step in addressing this issue is a theoretical study into the aerodynamic imbalance formed by one blade, as well as into the correcting masses for balancing it.

3. The aim and objectives of the study

The purpose of this study is to study theoretically the aerodynamic imbalance of the propeller blade installed at different angles, as well as correcting masses for its balancing. That would make it possible to investigate the aerodynamic forces acting on the blade by rotor balancing methods, to estimate the aerodynamic imbalance of the propeller in its manufacture in terms of errors, etc.

To accomplish the aim, the following tasks have been set:
- to establish a relation between the aerodynamic forces and centrifugal forces of inertia that balance them for the blade installed at different angles;
- to derive correcting masses as functions of aerodynamic forces;
- to investigate by a computational experiment the aerodynamic imbalance, generated by the blade installed at different angles and with error.

4. Methods to study the aerodynamic imbalance generated by a propeller blade

Elements of the theory of the propeller and wings [16] were used to build the blade model. It is assumed that the blades are streamlined by laminar flow while the aerodynamic
forces acting on the blade are reduced to equivalent. It is assumed that the propeller operates under such conditions that the dependence of the aerodynamic forces on the Mach and Reynolds numbers is not yet manifested. To study the aerodynamic imbalance, we applied elements from the theory of rotor balancing, theoretical mechanics. The computational experiment is based on the Mathcad computer algebra system (PTC, Inc).

5. The results of studying the aerodynamic imbalance of a propeller blade

5.1. The relationship between the aerodynamic forces and the centrifugal forces of inertia that balance them

5.1.1. Description of the model of a propeller blade and correcting masses.

We consider symmetrical blades with symmetric profiles (Fig. 1). The center of the blade mass is at point $C_B$. The main central axial moments of blade inertia are $J_\xi$, $J_\eta$, $J_\zeta$, and $J_\xi < J_\eta < J_\zeta$.

Fig. 2, a shows the balanced propeller schematically. In it, the experimental blade is turned at angle $\alpha$, which is not turned. When changing the angle $\alpha$, the center of masses of the experimental blade remains on the longitude axis of the blade; the static balance is not disturbed. The propeller rotates around the stationary $z$ axis at a fixed angular speed $\omega$.

The rotating blade is exposed to the drag force $F_D$ and the lift force $F_L$. They are applied in the center of pressure $P$, which is at a distance $b$, from the longitudinal axis of the blade. The case of the positive angle of rotation of the experimental blade is shown in Fig. 2, b; negative – Fig. 2, c.

Rotating parts in the assembly are balanced by two point masses $m_1$, $m_2$, in two correction planes (Fig. 2, a). In the first plane, there are the centers of masses of the blades. The distance between the two planes of correction is $L$. The masses $m_1$, $m_2$ are respectively, at the distances $r_1$, $r_2$ at angles $\varphi_1$, $\varphi_2$. The mass $m_1$ creates the centrifugal force of inertia $I_1$, and the mass $m_2$ – $I_2$.

The centrifugal forces’ modules of inertia are determined from the following formulas:

$$I_j = m_j r_j \omega^2, \quad / j = 1, 2 /.$$

We shall neglect the forces of gravity, the gyroscopic forces induced by a rotating blade, the aerodynamic forces acting on the non-rotated blades. We believe that the propeller is balanced and, therefore, the reactions of the supports holding the rotating parts are zero (not shown in the diagram).

5.1.2. A system of equations relating the aerodynamic forces and centrifugal forces of inertia to balance them

The aerodynamic force brought to point $O_1$ (the center of a propeller) is determined as follows:

$$F_\alpha(x) = 0,$$

$$F_\alpha(y) = -F_D(\alpha),$$

$$F_\alpha(z) = F_L(\alpha) \sin(\alpha) + F_D(\alpha) b \cos(\alpha),$$

$$M_\alpha(x) = F_L(\alpha) r \sin(\alpha),$$

$$M_\alpha(y) = -F_D(\alpha) b \sin(\alpha).$$

In (2), $F_L(\alpha)$, $F_D(\alpha)$ are the modules, respectively, of the lift force and drag force.
The centrifugal forces of inertia, brought to point O1, are determined as follows:

\[ I_x = I_{x_1} + I_{x_2}, \quad I_y = I_{y_1} + I_{y_2}, \quad I_z = 0, \]
\[ M_x = -I_{2z} L, \quad M_y = I_{2z} L, \quad M_z = 0, \]

where

\[ I_{x,j} = I_j \cos \varphi_j = m_j r_j \omega^2 \cos \varphi_j, \]
\[ I_{y,j} = I_j \sin \varphi_j = m_j r_j \omega^2 \sin \varphi_j, \quad /j = 1, 2/ \]

– projections of the centrifugal forces of inertia onto the Oxyz coordinate axes.

The conditions under which the centrifugal forces balance the aerodynamic forces:

\[ I_{x,j} + I_{x,j} = 0, \]
\[ I_{y,j} + I_{y,j} = 0, \]
\[ I_{y,j} = I_{y,j} + F_D(\alpha), \]
\[ I_{y,j} = b_j \left[ F_c(\alpha) \cos(\alpha) \sin(\alpha) + F_o(\alpha) \sin(\alpha) \right], \]
\[ I_{y,j} = F_c(\alpha) \rho \sin(\alpha). \]

It follows from (5) that the aerodynamic forces acting on the blade streamlined by a laminar flow can always be balanced by the adjustment of masses in two correction planes. It has been proven in [13, 15] that such balancing would persist at a constant air density at any rotational speed for the impellers of low-pressure axial fans.

5.2. Determining correcting masses as the functions of aerodynamic forces

The system of equations (5) has the following solution relative to the centrifugal forces:

\[ I_{x,j} = F_c(\alpha) \rho \sin(\alpha) / L, \]
\[ I_{y,j} = b_j \left[ F_c(\alpha) \cos(\alpha) \sin(\alpha) + F_o(\alpha) \sin(\alpha) \right] / L, \]
\[ I_{y,j} = -I_{x,j}, \]
\[ I_{y,j} = -I_{y,j} + F_D(\alpha). \]

Using (4), one can derive from (6):

\[ m_{x,j} = m_j \cos \varphi_j = F_c(\alpha) \rho \sin(\alpha) / (r_j \omega^2 L), \]
\[ m_{y,j} = m_j \sin \varphi_j = \frac{b_j \left[ F_c(\alpha) \cos(\alpha) \sin(\alpha) + F_o(\alpha) \sin(\alpha) \right]}{r_j \omega^2 L}, \]
\[ m_{x,j} = m_j \sin \varphi_j = \frac{F_D(\alpha)}{r_j \omega^2} - \frac{m_j r_j \omega^2}{r_1}, \]
\[ m_{y,j} = -m_{x,j} \frac{r_j}{r_1}. \]

From (7), we find the following parameters for correcting masses:

\[ m_j = \sqrt{m_{x,j}^2 + m_{y,j}^2}, \]
\[ \varphi_j = \frac{180 \cdot \arccos \left( \frac{m_{x,j}}{m_j} \right)}{\pi}, \quad \text{if } y \geq 0; \]
\[ \varphi_j = \frac{360 - 180 \cdot \arccos \left( \frac{m_{x,j}}{m_j} \right)}{\pi}, \quad /j = 1, 2/. \]

In (8), the angles \( \varphi_1, \varphi_2 \) are defined in degrees and vary from 0 to 360 degrees. The angles are counted from the longitudinal axis of the blade in the direction of propeller rotation. This is convenient when using a balancing device in full-scale experiments [14].

Correcting loads eliminate the following (modulo) imbalance in their correction planes:

\[ S_j = m_j \rho, \quad /j = 1, 2/. \]

If the accuracy of installing the blade in the propeller is \( \pm \Delta \alpha \), then one can use the following assessment of the imbalance, which can be induced by the blade when mounted in a propeller with an error:

\[ \Delta S_j(\alpha) = S_j(\alpha + \Delta \alpha) - S_j(\alpha - \Delta \alpha), \quad /j = 1, 2/. \]

In practice, the easiest way is to measure a lift force while the rest of the parameters for an aerodynamic force are more difficult to determine accurately [1]. Therefore, consider some additional assumptions that could be adopted in the processing of full-time experiments.

If the aerodynamic force is perpendicular to the chord of a characteristic cross-section (a flat blade plane), then:

\[ F_c(\alpha) = F_c(\alpha) \sin \alpha \],
\[ F_o(\alpha) = F_o(\alpha) \cos \alpha, \]
\[ F_D(\alpha) = F_D(\alpha) \tan \alpha. \]

For approximate calculations, one can use information about the (approximate) location of the pressure center [16]:

\[ b_j = 0.25 \cdot b, \quad R = 0.7 \cdot D / 2, \]

where \( b_j \) is the length of the cross-section chord, where the pressure center is located, \( D \) is the diameter of the propeller.

5.3. Expressing aerodynamic forces through the centrifugal forces of inertia

Let the correcting masses be known; it is necessary to find the parameters for the aerodynamic force – \( F_c(\alpha), F_o(\alpha), R, b \).

Before determining the aerodynamic force, it is necessary to check the correctness of determining the correcting masses. They are determined properly if:

\[ I_{x,j} + I_{x,j} = 0 \quad (m_j r_j \cos \varphi_j + m_j r_j \cos \varphi_j = 0). \]

The three remaining equations (5) are not enough to determine the four parameters. Therefore, we consider some cases where it is possible to solve the problem:

1. The aerodynamic force is perpendicular to the chord of the characteristic cross-section (11). Using the system of equations (5), from (11), we find:
\[ F_{pl}(\alpha) = I_{i\alpha} + I_{s\alpha} \]

\[ F_z(\alpha) = F_{pl}(\alpha) \cdot \text{sign} \alpha, \]

\[ r = I_{s\alpha} L \cdot \text{sign} \alpha / F_z(\alpha), \]

\[ b_z = I_{s\alpha} L / \left[ F_z(\alpha) \cdot \text{sign} \alpha + F_z(\alpha) \cdot \sin \alpha \right] \] \hspace{1cm} (14)

Since all the values that are determined are positive, the additional verification of the correctness of determining the correcting masses takes the following form:

\[ I_{s\alpha} \cdot \text{sign} \alpha > 0 \left[ \cos \phi, \text{sign} \alpha > 0 \right], \]

\[ I_{s\alpha} \cdot \text{sign} \alpha > 0 \left[ \sin \phi, \text{sign} \alpha > 0 \right]. \] \hspace{1cm} (15)

Note that formulas (14) may produce significant errors because \( F_z(\alpha) \) is a large value and, therefore, it is experimentally determined with the greatest relative error. Since all other values are determined through \( F_z(\alpha) \), then they are also determined with the highest relative error.

2. If the measured aerodynamic lift force is \( F_z(\alpha) \), then we find from (5):

\[ F_{pl}(\alpha) = I_{i\alpha} + I_{s\alpha} = F_z(\alpha) \cdot \tan \alpha, \]

\[ r = I_{s\alpha} L \cdot \text{sign} \alpha / F_z(\alpha), \]

\[ b_z = I_{s\alpha} L / \left[ F_z(\alpha) \cdot \cos \alpha \cdot \text{sign} \alpha + F_z(\alpha) \cdot \sin \alpha \right] \] \hspace{1cm} (16)

Unlike (14), a solution to (16) gives much smaller errors because \( F_z(\alpha) \) is a large value and, therefore, it is experimentally determined with the least relative error. Since all other values are determined through \( F_z(\alpha) \), then they are also determined with the least relative error.

The conditions (15) must be met for formulas (16) to give the correct result.

5.4. A computational experiment to study the aerodynamic imbalance generated by a propeller blade

The estimation data for the propeller shown in Fig. 1:

\[ D = 0.4 \text{ m}, \quad L = 0.15 \text{ m}, \quad b = 0.07 \text{ m}, \]

\[ r_1 = r_2 = 0.034 \text{ m}, \quad g = 9.81 \text{ m/s}^2. \] \hspace{1cm} (17)

The blades are flat, symmetrical. It is accepted that the aerodynamic force is perpendicular to the plane of the blade.

Additional information:

– the propeller is designed on the basis of a household fan with a power of 50 W with a diameter of the impeller of 400 mm;
– rotating parts in the assembly form a folded rotor weighing 960 g.

For an induction single-phase motor with a power of 50 W, we have experimentally derived the following (almost linear) laws of change in the angular speed of propeller rotation and the lift force depending on the angle of blade installation (a room temperature, 20.5 °C; atmospheric pressure, 1.006 hPa):

\[ \omega(\alpha) = \frac{\pi}{30} \left[ \frac{n_{\text{max}} - |\alpha|}{n_{\text{max}} - n_{\text{min}}} \right] \text{ [rad/s]}, \]

\[ F_z(\alpha) = \frac{F_{\text{max}} g |\alpha|}{16} \text{ [N].} \] \hspace{1cm} (18)

where

\[ n_{\text{max}} = 1438 \text{ rev/s, } n_{\text{min}} = 1257 \text{ rev/s, } F_{\text{max}} = 0.240 \text{ kg}. \] \hspace{1cm} (19)

The calculations were carried out according to formulas (7) to (11) using ratios (12).

Fig. 3, a – c shows the plots of dependence of the correcting masses (g), imbalance (g-mm), the installation angles of correcting loads (degree) in two correction planes on the angle of blade installation (degree).

Fig. 3, a, b, respectively, demonstrate that the correcting masses or imbalance in the two correction planes are almost directly proportional to the angle of the blade installation.

The main component of the aerodynamic imbalance is the torque component. The first correcting mass balances both the static and torque components of the aerodynamic imbalance...
in the propeller plane. The second correcting mass balances the torque component of the aerodynamic imbalance in the second plane of correction. The installation angle of the second correcting mass is stable for the installation angles of the blade of one sign and changes at almost 180° when changing the blade installation angle sign to the opposite. The installation angle of the first correcting mass linearly depends on the installation angle of the blade of one sign and changes at almost 180° when changing the blade installation angle sign to the opposite.

Fig. 3, d shows that the accuracy of the installation of the angle of attack of the blade of ±1° generates changes in the aerodynamic imbalance in the first plane of correction — up to 25 g·mm, in the second — up to 20 g·mm. These are relatively large quantifies for the folded rotor weighing 960 g.

6. Discussion of results of studying the aerodynamic imbalance (force) generated by a propeller blade

The established relationship between the aerodynamic forces and the centrifugal forces of inertia (5) shows that the aerodynamic forces can be balanced by adjusting the masses. The blade model uses the fact that the aerodynamic forces acting on the blade can be reduced to the equilibrium force applied in the center of pressure. This is true for the case of compressed air (gas). Therefore, balancing the aerodynamic imbalance by adjusting the masses is possible for both low- and high-pressure propellers. Since the balancing of rotors by adjusting masses has been most theoretically and practically investigated, it makes it possible to use rotor balancing methods to study aerodynamic forces.

The rotating blade mainly creates torque aerodynamic imbalance due to the lift force. Static imbalance is generated by the drag force acting on the blade. The correcting mass located in the propeller plane balances both the static and torque components of the aerodynamic imbalance in its correction plane. The second correcting mass (on the shank of the electric motor) balances the torque component of the aerodynamic imbalance in its correction plane.

The calculations are simplified by the assumption that the resultant of the aerodynamic forces is perpendicular to the blade’s chord (11). For approximate calculations, one can use information about the approximate location of the pressure center (12).

The aerodynamic forces acting on the blade could be determined based on the adjusting masses that balance them (14). The accuracy in determining the aerodynamic forces could be improved by measuring a lift force.

The computational experiment confirms the above theoretical results. The experiment further proves the possibility of applying the devised theory for propellers whose rotational speed changes with a change in the angles of blade installation.

Our study results are applicable to propellers whose blades are streamlined by laminar flow. The findings make it possible to relate the aerodynamic forces and the correcting masses to balance them, to investigate aerodynamic forces by rotor balancing methods, to estimate the aerodynamic imbalance of the propeller induced by installing the blade with an error.

The disadvantage of the constructed theory is not taking into consideration the surface friction forces acting on the blade from the air (gas), the consideration of only symmetric blades with a symmetric profile, the neglect of gyroscopic forces generated by a blade at rotation.

In the future, it is planned to use the built theory for an experimental study into the aerodynamic forces acting on the blade at different angles of installation, as well as determining the aerodynamic imbalance of the propeller.

7. Conclusions

1. The aerodynamic forces acting on the propeller blade can be fully balanced by adjusting the masses. This is true for the case of compressed air (gas), provided the blades are streamlined by laminar flow. That allows the rotor balancing methods to be used to study the aerodynamic forces acting on a propeller blade.

2. The rotated blade mainly creates the torque aerodynamic imbalance due to a lift force. The static imbalance is induced by the drag force acting on a blade. The correcting mass located in the propeller plane balances both the static and torque components of the aerodynamic imbalance in its correction plane. The second correcting mass (on the shank of the electric motor) balances the torque component of the aerodynamic imbalance in its correction plane.

The calculations are simplified by the assumption that the resultant of the aerodynamic forces is perpendicular to the blade’s chord. For approximate calculations, one can use information about the approximate location of the pressure center.

3. The aerodynamic forces acting on a blade could be determined based on the correcting masses that balance them. The accuracy in determining the aerodynamic forces could be improved by measuring a lift force.

4. The computational experiment shows that the accuracy of setting the angle of attack of the blade of ±1° generates changes in the aerodynamic imbalance in the first correction plane — up to 25 g·mm, in the second — up to 20 g·mm. These are relatively large values for the folded rotor weighing 960 g. The experiment further proves the possibility of applying the devised theory to propellers whose rotation speed changes with a change in the angles of blade installation.

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