This paper reports a study into the errors of process forecasting under the conditions of uncertain dynamics and observation noise using a self-adjusting Brown’s zero-order model. The dynamics test models have been built for predicted processes and observation noises, which make it possible to investigate forecasting errors for the self-adjusting and adaptive models. The test process dynamics were determined in the form of a rectangular video pulse with a fixed unit amplitude, a radio pulse of the harmonic process with an amplitude attenuated exponentially, as well as a video pulse with amplitude increasing exponentially. As a model of observation noise, an additive discrete Gaussian process with zero mean and variable value of the mean square deviation was considered. It was established that for small values of the mean square deviation of observation noise, a self-adjusting model under the conditions of dynamics uncertainty produces a smaller error in the process forecast. For the test jump-like dynamics of the process, the variance of the forecast error was less than 1%. At the same time, for the adaptive model, with an adaptation parameter from the classical and beyond-the-limit sets, the variance of the error was about 20% and 5%, respectively. With significant observation noises, the variance of the error in the forecast of the test process dynamics for the self-adjusting and adaptive models with a parameter from the classical set was in the range from 1% to 20%. However, for the adaptive model, with a parameter from the beyond-the-limit set, the variance of the prediction error was close to 100% for all test models. It was established that with an increase in the mean square deviation of observation noise, there is greater masking of the predicted test process dynamics, leading to an increase in the variance of the forecast error when using a self-adjusting model. This is the price for predicting processes with uncertain dynamics and observation noises.

Keywords: forecasting errors, self-adjusting Brown’s zero-order model, process dynamics uncertainty

1. Introduction

Forecasting the processes in the socio-economic area is considered one of the most important tasks related to resolving the issues of ensuring the sustainable development of society. However, actual processes are quite complex to predict since they are usually characterized by uncertain dynamics and unknown statistical parameters of observation noise. Therefore, the prediction of these processes based on the use of the conventional hypothesis about the
stability of their certain probabilistic characteristics in time is hardly possible. For such processes, the stability of probabilistic characteristics evolves over time according to a law unknown in advance or may be absent altogether. That significantly complicates the prediction of such processes based on known models and methods. Due to the high uncertainty in the dynamics of the specified processes and the conditions for their observation, it is possible to resolve this issue based on self-adjusting models and forecasting methods. The need to address this task is exacerbated by the fact that most actual processes [1] that disrupt the sustainable development of mankind belong to this very type of process. An important class of such processes is those processes that are associated with the occurrence of various accidents and emergencies [2]. The most representative of these processes are fires in various types of ecosystems [3] and at man-made facilities [4, 5]. The most dangerous are those fires that occur on the premises (FaP) [6]. This is because FaP is a massive phenomenon for all countries that cause significant damage to human life [7], objects [8], and the environment [9]. In this regard, studying the self-adjusting models for forecasting processes with uncertain dynamics and unknown observation noises becomes a particularly relevant task.

2. Literature review and problem statement

To predict processes under the conditions of dynamics uncertainty, adaptive and self-adjusting models give the best results. The most widely used model for predicting such processes is Brown’s zero-order model (BZOM) [10]. According to BZOM, the forecast is carried out on the basis of an iterative procedure, which is based on the use of the previous forecast and its corresponding correction by the deviation of the current observation from the previous forecast. This procedure allows the BZOM to be adapted to current observations. The BZOM adaptive properties depend on a single parameter \( h \), which acts as a corrective weight for current deviations. BZOM is universal and makes it possible to predict both stationary and evolutionary processes [11]. In accordance with BZOM, process forecasting is carried out at a fixed value of the smoothing parameter. To predict processes, the value for \( h \) is typically selected from the classical set between 0 and 1. There are many different ways to select this parameter. In [12], it is proposed to choose the value for \( h \) from the condition \( 0.1 < h < 0.3 \). However, that range is not justified. In [13], it is noted that with a significant variance in the observation noise, the values for \( h \) should not exceed 0.2. It is indicated that for a particular process, the choice of the value for the parameter \( h \) should be made based on the specified purpose of the forecast. In [14], to predict processes of an evolutionary nature, it is recommended to be guided by the condition \( 0 < h < 1 \), and for chaotic processes \( 1 < h < 2 \). It is noted that for each process there is the best value of the parameter \( h \). There is also an approach [15] based on the choice of \( h = 2/(k+1) \), where \( k \) is the number of steps included in the smoothing interval. In this case, the value of \( k \) is determined empirically while the process dynamics are limited to a random constant. In [16], a different approach is considered, based on the calculation of the Hurst index for processes with complex and uncertain dynamics. The main limitation of that approach is the presence of segments of stationarity in the predicted processes necessary to calculate the Hurst index, as well as the impossibility of its application in real time. Neither self-adjusting BZOMs, nor studying them under the conditions of a priori uncertainty in the dynamics of predicted processes and observation noises is considered in [10–16]. In [17], the variant of a self-learning model for forecasting socio-economic processes based on the application of provisions from the theory of functions of the complex variable is considered. A significant advantage of the model in comparison with known BZOM modifications is that it does not require a priori information about the dynamics of the predicted process. However, the model turns out to be difficult to apply and quite sensitive to errors in the selection of initial values for a complex smoothing coefficient. There are also models for forecasting the processes caused by hazardous factors (HF) associated with FaP [18]. However, such models are deterministic, roughly describing the actual physical processes in the evolution of HF associated with FaP. Therefore, the scope of application of such models is quite narrow and is limited only to the tasks of designing and evaluating the effectiveness of fire prevention measures. At the same time, [19] notes that the actual air environment associated with FaP is a complex dynamic system; neglecting this information leads to significant forecasting errors. More constructive are the models of FaP forecasting, based on the methods of nonlinear dynamics of complex systems [20], in relation to the dynamics of HF [21]. A review of the basic methods of quantitative analysis of the nonlinear dynamics of complex systems is given in [22]. However, papers [20–22] do not consider studying the self-adjusting models for predicting processes with uncertain dynamics at unknown observation noise. A special role belongs to works reporting experimental studies into the processes with uncertain dynamics using an example of considering the processes of the dynamics in the HF associated with FaP. Thus, in [23], the characteristics of HF dynamics in the event of FaP are experimentally investigated. An experimental study into the influence of thermal radiation on the dynamics of the rate of heat generation by various materials is reported in [24]. The results of the experimental investigation of the dynamics of the heat transfer rate at FaP are given in [25]. Papers [23–25] confirm that the dynamics of actual processes caused by HF associated with FaP are complex and non-stationary. The task of improving the efficiency of forecasting FaP based on the HF dynamics is tackled in [26]. In [27], the application of the current correlation dimensionality of the components of the state of the gas medium as a forecast parameter for the detection of FaP is considered. In [28], the autocorrelations and pairwise correlations of HF associated with FaP are investigated. At the same time, it is noted that the current values of the HF dynamics, and not their correlation, are more informative for predicting fires. Methods suitable for predicting processes with uncertain dynamics observed in noise are considered in [29]. However, the cited work is limited to the consideration of stationary models of processes. The method of estimating the non-stationary dynamics of processes is considered in [30]. However, the method is based on the Fourier transform. At the same time, it is not possible to isolate stationary fragments in the non-stationary dynamics in the case of their uncertainty. Methods of frequency-time identification of nonlinear systems are con-
considered in [31, 32]. However, the method reported in [31] is difficult to implement, does not involve forecasting processes with uncertain dynamics. In [32], frequency-time identification is solved by the method of short-term Fourier transforms. This leads to limited actual-time forecasting of processes with uncertain dynamics.

In [33], the frequency-time method for estimating the HF dynamics of HF in fires is considered. It is noted that this method is also quite complex that does not allow it to be used to predict FaP. In this regard, paper [34] proposes BZOM for the current measure of recurrence of increments of HF states. Unlike [33], a given model has high efficiency in forecasting processes with uncertain dynamics. The quality of the forecast is determined, in that case, by the smoothing parameter and the dynamics of the process under study [35]. In [36], a method for the operative detection of dangerous conditions in a dynamic system of atmospheric air based on the current measure of recurrence of states with uncertain dynamics is suggested. The use of the uncertainty function to solve a similar problem is considered in [37]. At the same time, the forecasting of dangerous states of the atmosphere with uncertain dynamics of pollution is not considered in [36, 37], nor the accuracy of forecasting is tackled. In [38], the accuracy of FaP forecasting is investigated based on the use of BZOM for fixed values of the smoothing parameter. It is noted that in the case of a parameter from the beyond-the-limit set, the accuracy of forecasting FaP increases. At the same time, the accuracy of the forecast is limited by the HF dynamics and the selected smoothing parameter. In [39], a variant of the self-adjusting BZOM, according to observations, is proposed for forecasting the dynamics of irreversible processes and phenomena. However, forecasting errors based on the proposed model are limited to the experimental dynamics of the recurrence of the increments of the state of the gaseous medium when igniting materials in the laboratory chamber. Due to the uncertainty in the dynamics of the recurrence of the increments of the state of the gas medium in the chamber, an objective assessment of errors when predicting using the model [39] is not possible. This means that for an objective assessment of the forecasting errors of the self-adjusting model [39], it is necessary to investigate the prediction errors of this model on test processes with different dynamics and observation noises. In this regard, an important and unresolved part of the problem relates to studying errors when forecasting the processes by self-adjusting BZOM under the conditions of uncertainty in the dynamics and observation noise.

3. The study materials and methods

The aim of this work is to study errors when forecasting processes by self-adjusting Brown’s zero-order model under the conditions of uncertainty in the dynamics of processes and observation noise.

To accomplish the aim, the following tasks have been set:
- to determine test models of the dynamics of predicted processes and observation noise for investigating errors of forecasting by the self-adjusting Brown’s zero-order model under the conditions of uncertainty in the dynamics of predicted processes and observation noise;
- to investigate the prediction errors of the self-adjusting Brown’s zero-order model for the test models of process dynamics and observation noise.

4. The study materials and methods

The object of this research is the self-adjusting model [39] of forecasting processes under the conditions of uncertainty in the dynamics and observation noise. The subject of this study is the errors in forecasting by the self-adjusting model [39] for test models of the dynamics of processes and observation noise. Conventionally, prediction errors by known models are evaluated on test processes with characteristic dynamics at different values of the mean square value and zero mean additive Gaussian observation noise. Three pulse models of the predicted process with known time parameters were chosen as the test dynamics. The first test dynamics were determined in the form of a rectangular video pulse of fixed amplitude. The second is in the form of a rectangular radio pulse of the predefined frequency with a fading amplitude. The third is in the form of a rectangular video pulse with an exponentially increasing amplitude. As a model of observation noise, the Gaussian process with zero mean and the predefined value of the mean square deviation was considered. The study examined the case of additive observation noise for the three specified test models of the dynamics of the predicted process. Smoothed variances in a forecasting error (EV) were investigated as prediction errors. Errors of forecasting by the self-adjusting BZOM and adaptive BZOM were investigated at the parameter of adaptation from the classical and beyond-the-limit sets [40]. At the same time, the value of the adaptation parameter from the classical set was 0.2, and from the beyond-the-limit set – 1.8. The models of test processes and observation noise, as well as processing the results from studying forecast errors, were treated in the Mathcad 14 (USA) computing environment.

5. Results of studying forecasting errors under the conditions of uncertainty in the dynamics and observation noise

5.1. Test models of the dynamics of predicted processes and observation noise

The first test model of the dynamics in the form of a rectangular video pulse of fixed amplitude was determined in the following form:

\[ T_{11} = \Phi(i-n) - \Phi(i-k), \]  \( i \) is the current moment of pulse onset; \( n \) is the moment of pulse onset; \( k \) is the moment of pulse termination. The second test model of the dynamics in the form of a rectangular radio pulse with an exponentially attenuating amplitude:

\[ T_{22} = (\Phi(i-n) - \Phi(i-k)) \times \] 
\[ \times e^{-j(\Phi(i-n) - \Phi(i-k))(1-n)\Delta t/10} \sin(2\pi f \Delta t), \]  \( f \) is the frequency of the harmonic process (Hz); \( \Delta t \) is the sampling interval of observations (s). The third test model of the dynamics in the form of a rectangular video pulse with an exponentially increasing amplitude takes the form:

\[ T_{33} = (\Phi(i-n) - \Phi(i-k)) \left( e^{j(\Phi(i-n) - \Phi(i-k))(1-n)\Delta t/10} - 1 \right) \].

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As a test model of additive observation noise at discrete moments, a stationary Gaussian process with a zero mean value and a variable value of the standard deviation (SD) was chosen. Test models (1) to (3) of the dynamics are characteristic of most actual processes observed against the background of additive noise of various levels. These models have been defined as the basis for studying the self-adjusting BZOM under the conditions of a priori uncertainty in the dynamics of predicted processes and observation noise.

5. 2. Investigating forecast errors for test models of the dynamics and observation noise

Exponentially smoothed with parameter 0.2 values of current EV and an absolute forecast error for test processes (1)–(3) for three different values of SD in observation noise equal to 0.005, 0.05, and 0.5 were investigated. During the study, many results were obtained for intermediate values. Therefore, the figures below show only some of them. Our results generally indicate the advantages of the self-adjusting BZOM [40] in comparison with the known adaptive forecasting models at the studied values of the parameter from the classical and beyond-the-limit sets. Fig. 1 shows, as an illustration, the dynamics of the smoothed EV of the forecast (σ^2) for test model (1) at different values of the observation noise SD. Hereinafter, S denotes the corresponding test dynamics of the predicted process without observation noise (normal), as well as taking into consideration the additive observation noise (noise).

Fig. 1. Dynamics in the smoothed variance of a forecast error for model (1) when using the self-adjusting and adaptive BZOMs for various observation noise SD: a – 0.005; b – 0.5

Similar dependences for test models (2), (3) are shown in Fig. 2, 3, respectively.

In Fig. 1–3, the curves of red color (auto) correspond to the self-adjusting BZOM, and the curves of blue and green color – adaptive BZOM, respectively, with the values of the adaptation parameter (h=0.2) from the classical set, and (h=1.8) from the beyond-the-limit set.

Fig. 2. Dynamics in the smoothed variance of a forecast error for model (2) when using the self-adjusting and adaptive BZOMs for various observation noise SD: a – 0.005; b – 0.5

Fig. 3. Dynamics in the smoothed variance of a forecast error for model (3) when using the self-adjusting and adaptive BZOMs for various observation noise SD: a – 0.005; b – 0.5

6. Discussion of results of studying the self-adjusting Brown’s forecast model

Our results show that for small values of the observation noise SD (0.005) under the conditions of uncertainty in the dynamics of processes in comparison with known adaptive models for the adaptation parameter from the classical and beyond-the-limit set, a more accurate forecast is produced by the self-adjusting model. For example, the
EV of forecasting abrupt changes in the process dynamics (Fig. 1, a) is less than 1 %. And the EV of forecasting abrupt changes in the process dynamics in the case of the adaptation parameter from the classical and beyond-the-limit sets is about 20 % and 5 %, respectively. At the same time, the EV of forecasting the fixed process dynamics is less than 0.1 % for all types of BZOMs considered. However, the specific EV value of the forecast of a fixed value in this case, based on the self-adjusting model, is somewhat larger compared to the adaptive model with a parameter from the classical set but less compared to the adaptive model with a parameter from the beyond-the-limit set. Similar advantages of the self-adjusting model are characteristic of the test model of the dynamics in the form of a fading harmonic oscillation (2). In this case, the forecast EV (Fig. 2, a) is on average less than 0.1 %. At the same time, the forecast EV for the adaptive model with an adaptation parameter from the classical and beyond-the-limit sets is less than 25 % and 1 %, respectively. In the case of a test model of the dynamics in the form of a video pulse with an exponentially increasing amplitude (3), the self-adjusting model provides the forecast EV (Fig. 3, a) not exceeding 0.01 %, which increases to 25 % by the time the pulse ends. At the same time, adaptive models with a parameter from the classical and beyond-the-limit sets provide the forecast EV not exceeding 1 % and 0.1 %, respectively. For this case, the forecast EV at the time of the end of the pulse is 45 % and 30 %, respectively. With observation noises at SD=0.5, the forecast EV of the test dynamics of processes (1) to (3) for the self-adjusting and adaptive models with a parameter from the classical set is from 1 % to 20 %. However, for the adaptive model, with a parameter from the beyond-the-limit set, the EV is close to 100 % or more. A similar result (Fig. 3, b) for this type of adaptive model is characteristic of the dynamics of process (3). This means that the known adaptive model for the studied value of the parameter from the beyond-the-limit set is unsuitable for predicting processes with test dynamics (1) to (3). The result obtained is explained by the fact that with an increase in the observation noise SD, the predicted test dynamics of the processes are masked.

The limitations of this study include a small set of test dynamics for the predicted process and consideration of the noise model in the form of a Gaussian additive process. As a direction to further advance this study, it is necessary to consider extending the set of the test dynamics of predicted processes and observation noise that take place within the economic, financial, and social applications of the transition period, as well as dangerous processes in the technical and natural fields.

7. Conclusions

1. We have determined the test models of the dynamics of predicted processes and observation noise, which make it possible to investigate errors in forecasting when using the self-adjusting and adaptive Brown's zero-order model under the conditions of uncertainty in the dynamics of predicted processes and observation noise. The proposed models describe the characteristic dynamics for most actual irreversible processes and phenomena in various fields. The test models are determined by time-discrete equations that reflect the dynamics of predicted processes over a limited observation interval. We have considered the test dynamics of processes in the form of a rectangular video pulse with a fixed unit amplitude, a radio pulse of a harmonic process with an amplitude attenuating exponentially, as well as a video pulse with amplitude increasing exponentially. As observation noise, the model of the additive discrete Gaussian process with zero mean and variable value of the mean square deviation was considered.

2. The errors of forecasting have been investigated when using the self-adjusting and adaptive Brown's zero-order model of the test models of process dynamics at different values of the standard deviation of observation noise. It has been established that for small values of the standard deviation in the observation noise and the uncertainty of process dynamics, the self-adjusting model provides a more accurate forecast compared to known adaptive models for the selected parameter values from the classical and beyond-the-limit sets. For the self-adjusting model, the forecast EV of the jump-like dynamics of the process is less than 1 %. At the same time, for the adaptive model, the forecast EV with a parameter from the classical and beyond-the-limit sets is about 20 % and 5 %, respectively. With significant observation noise, the forecast EV of the test dynamics of processes for the self-adjusting and adaptive models with a parameter from the classical set is in the range from 1 % to 20 %. However, for an adaptive model, with a parameter from the beyond-the-limit set, the forecast EV is close to 100 % for all the tested dynamics types. This means that the known adaptive model for the studied value of the adaptation parameter from the beyond-the-limit set is unsuitable in the case of predicting the test types of process dynamics under consideration. In general, our results indicate that with an increase in the standard deviation of observation noise, the masking of the predicted test dynamics of processes increases. This leads to an increase in the forecast EV when using the self-adjusting model and it is the price for forecasting under the conditions of uncertainty in the dynamics of processes and observation noise.

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