This paper reports a method for improving the firing efficiency of an artillery unit that results in enhanced effectiveness. Given the modern use of artillery for counter-battery warfare, the effectiveness of shooting is not enough assessed by accuracy only. It is also necessary to take into consideration and minimize the time spent by the unit in the firing position and the consumption of shells to hit the target.

It has been shown that in order to assess the effectiveness of an artillery shot due to the initial velocity of the projectile, the most rapid and simple means is to classify the quality of the shot by the acoustic field. A procedure for categorizing the shot has been improved by applying an automatic classifier with training based on a machine of support vectors with the least squares. It is established that the error in the classification of the effectiveness of the second shot does not exceed 0.05. The concept of the effectiveness of a single artillery shot was introduced. Under the conditions of intense shooting, there may be accidental disturbances in each shot due to the wear of the charging chamber of the gun, its barrel, and incomplete information about the powder charge. When firing involves disturbances, the firing of an artillery unit can be described by a model of a discrete Markov chain. Based on the Markov model, a method for improving the efficiency of artillery fire has been devised. The method is based on the identification of guns that produce ineffective shots. The fire control phase of the unit has been introduced. In the process of controlling the fire of the unit, such guns are excluded from further firing. A generalized criterion for the effectiveness of artillery firing of a unit, based on the consecution of criteria, has been introduced. It is shown that the devised method significantly improves the effectiveness of shooting according to the generalized criterion

**Keywords:** artillery unit, firing efficiency, acoustic field of shot, Markov model, generalized criterion of effectiveness

1. Introduction

The experience of using artillery in military conflicts of the XXI century shows that barrel artillery continues to play one of the main roles during hostilities. At the same time, the bulk of the tasks for the fire defeat of the enemy is assigned to the artillery of large calibers (more than 120 mm) [1]. In this aspect, much attention is paid to improving the efficiency of artillery fire. Since efficiency is a complex concept, there are different approaches to determining the effectiveness of the use of artillery weapons. Thus, in [2], this efficiency is considered as a weighted convolution of tactical and technical indicators of an artillery gun. Efficiency as the maximum probability of hitting a target is considered in [3]; that concept was then developed in [4]. At the same time, the need for large-caliber artillery to use the tactics of “shoot and scoot” (short-term fire with rapid destruction of the target, literal translation – shoot and ran away) is now being intensively discussed [5]. The tactics of “shoot and scoot” are associated with the intensive development in the armed forces of high-tech states of accurate means of technical reconnaissance of artillery. In accordance with the concept of counter-battery warfare, in the event of detection and evaluation of the coordinates of the position of the firing artillery unit (AU), the
enemy in the shortest possible time opens return fire to the
defeat of this position [6]. Thus, according to [7], the safe
time of stay of an artillery battery in position after the first
shot is (5...12) minutes. Since the first shot usually begins
the target shooting, during the specified time the adjustment
of fire must be completed, and shooting to kill must be per-
formed. Therefore, as the first criterion for the effectiveness
of the use of artillery, it is necessary to take a minimization
of the time of stay of guns in a combat position, which de-
termines the preservation of the viability and combat capa-
ability of AU. On the other hand, in the minimum time, the
unit must perform the task of accurately hitting the target.
Therefore, the second criterion for the effectiveness of ar-
tillery is the accuracy of hitting the target. The third criterion
should be the consumption of shells spent to hit the target. It
should be minimal. The formation of a similar criterion was
proposed in [8] in which the dependence of the performance
on random factors was analyzed.

Devising methods to improve the effectiveness of shoot-
ing, taking into consideration the three formulated criteria,
is a task whose solution makes it possible to bring fire con-
trol to a fundamentally new level. Such methods minimize
the lag time of obtaining information about the quality of
the shot necessary to form the subsequent control action.
In essence, the existing information feedback based on the
adjustment of fire based on the results of previous shots, on
which the current concept of artillery fire control is based,
is eliminated. In practical terms, solving this task would
improve the mobility and survivability of artillery units. On
the other hand, the solution to this scientific task of increas-
ing efficiency could make it possible to perform a fire task
with a significant saving of ammunition.

2. Literature review and problem statement

The classic concept of ensuring the effectiveness of ar-
tillery fire by improving accuracy is based on information
feedback between successive shots from the gun. It is given
in work [9]. According to this concept, after each shot (or a
series of shots), the initial firing settings of the next shot are
corrected according to the results of the previous one, taking
into consideration the shot error made during the shot. The
error is estimated as the difference in the coordinates of the
target and the explosion of the projectile. Typically, the error
is estimated by metric $L_2$ [10]. Information about the shot er-
eror is used in the firing position to adjust the initial settings
of the next ($i+1$)-th shot. At the same time, the procedures
of information feedback, according to [9], are divided into two
categories – "shot-look-shoot" (for targets observed from a
firing position) (SLS) or "shoot-adjust-shoot" (SAS). The
latter procedure applies to hidden firing positions. In both
cases, an assessment of the coordinates of the projectile ex-
losion when fired is required for adjustment. This requires
the use of artillery reconnaissance under the mode of servic-
ing its firing, which significantly complicates and lengthens
the procedure. These means determine the coordinates of the
projectile explosion at the point of its landing by observing
and recording the physical fields of the explosion [10].

The traditional and simplest means of this group is to
observe explosions using optical instruments [11]. Opto-
electronic observation tools [12] make it possible to record
explosions at any time of the day and under difficult me-
teorological conditions with higher accuracy than optical
means [12]. When firing at visually unobservable targets,
aerial reconnaissance means [13], or unmanned aerial vehi-
cles (UAVs), are used [14]. The means of recording the ex-
plosions on the acoustic fields created by them also
include sound reconnaissance of artillery [15]. In this case,
sound-metric systems are used under the mode "Mainte-
nance of firing of one's artillery" [16]. The obvious disadvan-
tages of the means of adjusting the shot specified in [10–16]
include the need to attract additional expensive combat and
technical resources to defeat the target. The main drawback
of those means is the ability to assess the coordinates of the
shell explosion and the formation of a correction in the gun
installation only after the projectile lands. In this case, the
flight time of the projectile when firing at maximum ranges
can be 60 s or more. Artillery radar stations (ARS) require
less time to adjust the fire [17]. ARS register only a part – a
few points of the flight trajectory of the projectile [18]. Fur-
ther, the entire trajectory is interpolated at several points
with the calculation of the coordinates of the point of the
shot (reconnaissance of enemy artillery) and/or the landing
point of the projectile (maintenance of its firing) [19]. How-
ever, modern ARSs are complex technical systems equipped
with phased array antennas. Their use in the conditions of a
battery moving through firing positions to carry out fire
correction is impractical. In addition, ARSs are active (radi-
ing) systems; from this point of view, they are unmasking
targets for enemy fire.

More operational are the devices for measuring the main
ballistic parameter of the shot – the initial velocity of the
projectile when it leaves the muzzle of the barrel – artillery
ballistic stations (ABS) [20]. ABS makes it possible to evalu-
ate the trajectory and calculate the coordinates of the land-
ing point in a time of 30–40 s [21]. The disadvantages
of ABS are a fairly high cost and unmasking property since
ABS is radiating active systems. An option for overcom-
ing the described problems can be means of assessing the
initial velocity of the projectile from the acoustic fields of
the shot [22]. These means are based on the registration of the
ballistic wave (BW) and muzzle wave (MW) that
occur when shot [23]. According to the analysis of the fine
structure of BW and MW signals recorded by microphones
located at a distance of (30–100) m, placed directly at the
firing position [24], it is possible to conduct a threshold
classification of the effectiveness of the shot by the initial ve-
locity of the projectile. Acoustic methods are simple, do not
require expensive equipment. That makes them very promis-
king for assessing the landing coordinates of projectiles. Until
now these methods have been used to assess the level of the
wear of barrels, their application to fire correction requires
additional research. The use of acoustic methods eliminates
the process of information feedback for the correction of
the shot, eliminates the time to wait for the projectile to fly
along the trajectory, and the SLS or SAS procedures. With
this approach, the process of shooting is almost completely
excluded. Thus, the time of AU stay in the firing position
is significantly reduced while the probability of hitting the
target increases.

When firing each individual gun, random errors may oc-
cur that are not repeated for all shots of the gun. These errors
(individual random perturbations of the shot) have three
causes [25]. The first reason is the extension of the gun's
charging chamber [26]. In the conditions of intensive fire ac-
tivity, instrumental control of both the wear of the charging
chamber and the wear of the barrel can be difficult for tac-
tical and technical reasons. Due to the wear of the barrel, a second random disturbance of the shot occurs – a decrease in the initial velocity of the projectile, $\Delta v_0$, due to wear of the gun barrel. The third disturbing factor is incomplete information about the state and parameters of the charge. In the practice of artillery shooting, with a shortage of charges or an excess of their storage period, it is possible to fire charges with unreliable data on their quality and energy. Incomplete information about the state and parameters of the charge causes a third random perturbation – a change in the initial velocity of the projectile due to the uncertainty of the charge energy [27]. Random perturbations of the three types affect the shot in the same way – accidentally changing the initial velocity of the charge. It is not possible to separate them, so it is advisable to consider the total individual deviation of the initial velocity of the projectile from the tabular for this shot for a given gun. Techniques of accounting for these random disturbances and their correction to improve the accuracy of the shooting have not been found in the literature. Nevertheless, acoustic methods for assessing firing errors seem promising for accounting for random perturbations of shots [22–24].

Our review of the literature data [5–7, 9, 10, 25] has shown that the main directions of research to improve the effectiveness of shooting are based on the principle of information feedback. The principle is based on the assessment of the coordinates of the projectile explosion with the subsequent correction of firing installations (shooting). Known tools for assessing shooting errors [11–16] are time-consuming and costly. Issues related to the accounting and compensation for accidental disturbances of the shot remained unresolved. The issues of shooting without feedback with a reduction in shooting time have not been investigated at all, as well as the issues related to reducing the consumption of shells to hit the target. This suggests the following: It is advisable to conduct a study on improving the effectiveness of shooting by eliminating information feedback by assessing the initial speed of the projectile by prompt and simple means. Ways to eliminate random perturbations that may be present in the shot should be explored. To quantify the effectiveness of shooting, it is advisable to form a generalized criterion of effectiveness. Such a criterion should take into consideration not only the accuracy of shooting but also the time of stay of the artillery unit in position, as well as the consumption of shells to hit the target.

3. The aim and objectives of the study

The aim of this work is to study the possibilities of improving the effectiveness of firing by an artillery unit with firing control in the presence of random perturbations of shots. The study results could make it possible to increase the accuracy of shooting, reduce the minimum time spent by AU in the firing position, and minimize the consumption of shells.

To accomplish the aim, the following tasks have been set:

– to devise a method for controlling the firing by an artillery unit of increased efficiency, which is based on the Markov model;

– to form a generalized quantitative criterion for assessing the effectiveness of firing by an artillery unit in the presence of random perturbations of shots.

4. The study materials and methods

The object of the study is the process of fire control by an artillery unit, taking into consideration random disturbances present in the shots of individual guns.

The study is based on the following working hypothesis. The main quantitative parameter that determines the effectiveness of an artillery shot is the initial velocity of the projectile when it leaves the muzzle of the barrel. If the initial velocity of the projectile is not lower than the tabular velocity, the shot is effective.

In accomplishing this task, the following simplifications were adopted. It is believed that all systematic errors of AU firing associated with the topological features of the firing position and target location and meteorological conditions along the trajectory of the projectiles are taken into consideration and corrected at the stage of firing preparation. The study only considers the control over random perturbations, which may be present in each shot of an individual gun.

To determine the effectiveness of an artillery shot, it is necessary to estimate the initial velocity of the projectile at the muzzle of the gun. The most suitable method for this is the analysis of the acoustic field created by the shot. This method is passive, requires minimal equipment (two measuring microphones). The method has proven effective in assessing the initial velocity of projectiles to determine the level of the wear of the barrels [24]. A binary classification algorithm based on a support vector machine (SVM) was employed. However, qualitative classification requires prior training of the classifier on sufficiently large samples. Additionally, in the task of assessing the quality of the shot directly at the firing position in the shortest possible time, it is necessary to maximize the classification time. In addition, in studies conducted earlier, the only indicator of the quality of the classification was the reliability of the classification. It is desirable to evaluate other classification indicators, in particular, the proportion of erroneous decisions on the quality of the shot based on the parameters of the acoustic field based on the classifier by the Least Squares Support Vector Machine (LSSVM) [28, 29]. This method requires a smaller training sample and has a higher performance. To assess the quality of classification by the methods of a simulation experiment, a whole system of recognition quality indicators based on the concept of binary classification logic was evaluated. At the same time, the material for simulation modeling was a set of real records of acoustic fields recorded during the firing from the M109 howitzer [30].

When assessing the effectiveness of the firing by an artillery unit, methods for assessing probabilistic firing errors are used. Making corrections to the gun settings is done with feedback, that is, to correct the next shot, one needs to know the result of the previous one. This feedback principle increases the shooting time. In addition, the existing shooting procedure does not eliminate accidental disturbances present in the shots of individual guns. To eliminate these shortcomings, the apparatus of Markov chains was
used [31, 32]. The description of the firing by AU by successive states of the Markov chain made it possible to devise a method for controlling the firing by AU in the presence of random disturbances in the shots of individual guns to achieve maximum firing efficiency.

The task of this study is to select and maximize the objective function of shooting efficiency, built on the basis of specific criteria for shooting accuracy, minimum shooting time, and minimum ammunition consumption. To form a generalized quality criterion, the methods of forming a multi-criteria convolution used in modern decision theory were applied [33, 34].

5. Results of studying the method of shooting control of increased efficiency

5.1. Procedure for categorizing the effectiveness of a single shot from a gun

To determine the concept of the effectiveness of a single shot, the following hypothesis was put forward. If the initial velocity of the projectile is not less than 0.95 of the tabular value \( v_0 \geq 0.95 v_{\text{table}} \), then, subject to compensation for all other errors at the stage of preparation of firing, the projectile falls into the circular vicinity of the target. The radius of the median circular error at firing range \( D \) is \( \text{CEP}_p \geq 1 \% D \) with a probability of \( p \geq 0.5 \). To test the working hypothesis, a simulation experiment was set up. During the experiment, successive shots from the 155-mm howitzer M109 (M107 shell, maximum charge – 5 DMT2 modules) were simulated in 5 series of 100 shots each. The initial speed of each shot was selected randomly in the range \( v_0 \in \text{rand}(0.95...1) \), \( \text{rand} \) is the standard pseudo-random number generation operator with uniform distribution. 5 episodes of 100 shots each were simulated. Each series corresponded to a firing range of \( D=1000 \text{ m}; 3000 \text{ m}; 5000 \text{ m}; 9000 \text{ m}; 12000 \text{ m} \), respectively. For each shot, the trajectory of the projectile was calculated when firing at the target with the coordinates \( (x_0, y_0) \) of the landing point \( (x, y) \), \( i=1,100 \). The calculation was performed according to a flight model with five degrees of freedom, built on the basis of the NATO standard STANAG 4106 [34]. For modeling, the program code for the MATLAB® R2017a system, developed at North-West University (South Africa) and given in [35], was used. For each shot, the fact of hitting/not hitting the landing point of the projectile in a circle with a radius of \( R=1 \% D \) and the number of hits were evaluated. For each series, the root mean square deviations (RMS) \( \sigma_x, \sigma_y \) in the direction of firing and perpendicular direction were calculated. The transition from RMS to median circular error was carried out according to the ratios given in [36]

\[
\text{CEP} = \begin{cases} 
0.615\sigma_x + 0.562\sigma_y, & \sigma_x < \sigma_y \\
1.17\sigma_y, & \sigma_x \leq \sigma_y \\
0.615\sigma_x + 0.562\sigma_y, & \sigma_x > \sigma_y 
\end{cases} 
\]

The results of the simulation experiment are given in Table 1. The second line of Table 1 shows an estimate of the probability of projectiles hitting the circle of radius \( R \) for each series of shots \( \hat{p} \).

Data in Table 1 confirm the working hypothesis. Based on the results of the simulation experiment, two definitions were introduced. An effective shot is a shot in which the initial velocity of the projectile \( v_0 \) is at least 0.95 of the tabular velocity: \( 0.95 v_{\text{table}} < v_0 < v_{\text{fire}} \). The ineffective shot is a shot whose initial speed is less than 0.95 of the tabular one: \( v_0 < 0.95 v_{\text{table}} \). That is, hereafter, the effective shot is a shot in which the projectile falls into a circle of radius \( R=1 \% D \) with a probability of at least 0.5. This holds if the initial velocity of the projectile is not more than 5 % less than the tabular velocity. The rest of the shots are considered ineffective.

<table>
<thead>
<tr>
<th>( D, \text{m} )</th>
<th>1000</th>
<th>3000</th>
<th>5000</th>
<th>9000</th>
<th>12000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{p} )</td>
<td>0.78</td>
<td>0.73</td>
<td>0.69</td>
<td>0.71</td>
<td>0.68</td>
</tr>
<tr>
<td>CEP, m</td>
<td>9.23</td>
<td>24.8</td>
<td>43.6</td>
<td>85.4</td>
<td>112.7</td>
</tr>
</tbody>
</table>

To devise a procedure for assessing the effectiveness of a shot, we apply an analysis of its acoustic field. The acoustic field formed by an artillery shot is formed by two fundamentally different types of waves. When a projectile flies out of the barrel at supersonic speed, a ballistic (shock) wave (BW) is formed. The center of this wave coincides with the nose of the projectile flying along the trajectory. The ballistic wave propagates along with the projectile flying along the trajectory. It can be recorded by a microphone inside the Mach cone formed by the projectile during flight [37]. The ballistic wave has an N-shape. Its duration is 3–6 ms, the amplitude of sound pressure is 90–120 Pa. From the point of view of spectral analysis, BW is a wideband audio signal with a spectrum width of 10–700 Hz. The muzzle wave (MW) propagates from the cut of the gun barrel at the speed of sound. MW is a fading sinusoidal oscillation lasting 1.5–3 periods. The central frequency of the MW spectrum lies in the range of 10–30 Hz; the width of the spectrum is about 40 Hz. The amplitude of the sound pressure of the muzzle wave lies in the range of 150–350 Pa at a distance of 50–100 m from the muzzle section of the gun. In this case, the amplitude of MW fluctuates greatly depending on the weather conditions at the point of its registration by the microphone. It was established that a shot from a barrel with an initial velocity less than a tabular velocity, on the generated acoustic field is equivalent to a shot from a smaller caliber gun [22, 23]. Practically, this means that the duration of pulsed acoustic BW and MW signals with an ineffective shot is less than with an effective one, therefore, their spectra are wider. This effect makes it possible to build an automated classifier of the effectiveness of the shot, based on a set of temporal and spectral features of the BW and MW signals. The construction of the classifier is based on the positive results reported in [22–24]. To build the classifier, 2 classes of objects were formed – class 1 – “ineffective shot”, and class 2 – “effective shot”. Class 1 includes shots from barrels for which the initial velocity of the projectile is \( v_0 < 0.95 v_{\text{table}} \), where \( v_{\text{table}} \) is a tabular value of the initial velocity of the projectile when fired with a complete absence of random disturbances. Class 2 includes barrels with a projectile initial velocity of \( v_0 \geq 0.95 v_{\text{table}} \). To build the classifier, computer simulations were carried out. The purpose of the simulation was to form a representative sample of recordings of acoustic signals from effective and ineffective shots. To form a sample, similarly to [22], real recordings of acoustic signals recorded during the firing of the M109A3GN howitzer were used [30]. The signals were recorded when firing by broadband measuring microphones at distances of 20 m and 250 m from the firing position. The initial tabular velocity of the projectile is \( v_0 = 684 \text{ m/s} \). To simulate ineffective shots from barrels with wear, the sections on the recordings containing BW signals (duration, 10 ms) and
MW signals (duration, 150 ms) were converted to the spectral domain using the discrete Fourier transform (DFT). In the spectral region, the BW and MW spectra were shifted towards high frequencies by the value \( n_{\text{SHIFT}} = \text{rand}[0.95:1] \), where \( \text{rand} \) is a software-implemented operator for uniform random selection of a number from a given interval. Next, the BW and MW signals “deformed” in this way were converted back to the time domain using the inverse discrete Fourier transform. In total, 150 recordings of BW and MW signals simulating class 1 and 2 shots were formed in this way. The sample of simulation signals contains approximately equally the signals of class 1 and 2. The next stage of the simulation experiment was the formation of vectors of information features. For each record containing BW and MW signals, the amplitude, temporal, spectral, and cumulative characteristics of BW and MW were calculated. The features are described in detail in [22]. Since all features have different physical nature and a different range of values, they were normalized by the minimax method of normalization over an interval of \((0; 1)\) [38]. Next, the normalized values of the features were collected sequentially into the AS vector of dimension \( \text{dim}(\text{AS}) = 14 \). Each of the components of the AS vector is a numerical characteristic of the acoustic field created by an effective or ineffective shot. It should be noted that the vectors of features of shots are comparable only when firing from guns of the same type, with the same charge, and the same shells. The next step in building an automatic classifier was to select the type of classifier. In works [22, 24], as a result of the analysis of various types of classifiers of barrel wear, a binary SVM classifier was used [39]. To assess the effectiveness of the shot, a more effective method than the method of support vector machines (SVM) used in [22, 24] was chosen — the least squares (Least Squares Support Vector Machines (LSSVM) method [40]. LSSVM is a reformulated classical SVM method that leads to the solution of linear systems and boils down to solving the linear programming problem. The classic SVM method implements the solution to the quadratic programming problem. As a result, the LSSVM method reduces the estimated time by almost an order of magnitude with the same classification quality indicators. In addition, the use of the LSSVM method makes it possible to reduce the volume of the training sample by 2–4 times [41]. In addition, there is a well-established and verified toolbox from the MATLAB system for this method [42]. The classifier was trained on a sample size of 50 \( \text{AS}_{\text{LEARN}} \) feature vectors randomly selected from the generated sample. Next, a test sample \( \text{AS}_{\text{TEST}} \) with a size of 50 vectors was fed to the input of the LSSVM classifier trained and hyper-parameter-tuned classifier. Its elements were also randomly selected from the overall AS sample. In this case, a condition was set — the elements of the training and test samples should not intersect. The randomly formed test sample included 28 feature vectors of ineffective shots and 22 characteristic vectors of effective shots.

Until now, the quantitative quality of the classification of shots by the initial velocity of the projectile was assessed by one quantitative indicator — reliability, which is the ratio of correctly classified objects to the total number of objects. This assessment is not complete enough. Therefore, for the classification of shots by effectiveness, various quality indicators of the classification given in [43] were constructed and calculated. The classification logic is presented in Table 3 in columns 2, 3. Table 3 shows the true classes to which the shot belongs. The rows of Table 3 demonstrate classifier solutions showing which of the two classes the classifier assigns the shot to. In the cells of the table, at the intersection of the corresponding rows and columns, the number of true or false decisions of the classifier is given. Since the main task of classification is to detect shots with an initial velocity below the tabular one (ineffective shots), the state “Shot is really ineffective” is taken as a truly positive state. Then the opposite state — “The shot is really effective” is truly negative. The decisions of the classifier can be as follows: “A shot is ineffective” is a truly positive decision, and the result of classification: “A shot is ineffective” is a truly negative decision. The meaning of erroneously positive and erroneously negative decisions is clear from Table 2. The corresponding cells in Table 2 show the quantitative results of the classification of the sample under study.

Table 3 gives quantitative quality metrics for classification by a test sample of \( \text{dim}(\text{AS}_{\text{TEST}}) = 50 \) shots.

### Table 2

<table>
<thead>
<tr>
<th>Shot state</th>
<th>A truly positive state</th>
<th>A truly negative state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classifier solution</td>
<td>“The shot is really ineffective”</td>
<td>“The shot is really effective”</td>
</tr>
<tr>
<td>Classified Class: “Shot ineffective”</td>
<td>True Positives (TP) 26</td>
<td>False Negatives (FN) 1</td>
</tr>
<tr>
<td>Classified class: “Shot effective”</td>
<td>False Positives (FP) 2</td>
<td>True Negatives (TN) 21</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Metric</th>
<th>Formula</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accuracy</strong></td>
<td>( \text{ACC} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} \approx 0.94 )</td>
<td>Accuracy. It evaluates the total ratio of correctly classified objects to the total number of objects</td>
</tr>
<tr>
<td><strong>Error Rate</strong></td>
<td>( \text{Err} = 1 - \text{ACC} = 0.06 )</td>
<td>The ratio of incorrectly classified objects to the total number of objects</td>
</tr>
<tr>
<td><strong>Specificity, True Negative Rate</strong></td>
<td>( \text{TNR} = \frac{\text{TN}}{\text{TN} + \text{FP}} = 21 / (21 + 2) = 0.93 )</td>
<td>Specificity estimates the proportion of effective shots classified correctly</td>
</tr>
<tr>
<td><strong>Precision</strong></td>
<td>( \text{Pre} = \frac{\text{TP}}{\text{TP} + \text{FP}} = 26 / (26 + 2) = 0.92 )</td>
<td>Precision estimates the proportion of ineffective shots classified correctly</td>
</tr>
<tr>
<td><strong>False Positive Rate</strong> (fall-out, false alarm rate)</td>
<td>( \text{FPR} = \frac{\text{FP}}{\text{TN} + \text{FP}} = 1 - \text{TPR} = 0.04 )</td>
<td>Proportion of ineffective shots that are erroneously classified as effective</td>
</tr>
<tr>
<td><strong>Sensitivity, True Positive Rate, Recall</strong></td>
<td>( \text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}} = 26 / (26 + 1) = 0.96 )</td>
<td>Sensitivity shows how much of the inefficient barrels are classified as inefficient</td>
</tr>
<tr>
<td><strong>F-metric</strong></td>
<td>( F = \frac{2 \text{TP}}{2 \text{TP} + \text{FP} + \text{FN}} = 52 / (52 + 3) = 0.95 )</td>
<td>Harmonic mean between accuracy and completeness (overall classification quality)</td>
</tr>
</tbody>
</table>
The proximity of the ACC, TNR, Pre, TPR, and F integral assessment measures to unity indicates a high quality of classification. One should note the low value of the frequency of false-positive FP estimates or errors of the second kind. Their value shows how many ineffective shots are classified as effective. The value of this value of 0.04 indicates that during operation the developed binary classifier mistakenly recognizes only 4% of ineffective shots as effective.

5.2. The Markov model of an artillery unit firing with random disturbances of individual gun shots

Let an AU of n guns fire a series of single shots from each gun. The interval between shots from guns, \( \Delta t \), is considered constant and determined by the rate of fire. Represent successive AU states with a simple homogeneous Markov chain with \((n+1)\) state whose graph is depicted in Fig. 1. The assumption about the Markovian nature of the process is quite reasonable. The effectiveness of the subsequent shot from the gun is determined only by the state of the elements of the gun (the charging chamber and barrel), which the gun entered as a result of the previous shot. In addition, the efficiency is determined by the state of the prepared charge. The states of the Markov chain are as follows. The initial state, \( S_0 \), an AU with \( n \) guns is ready to be fired in successive shots from each gun. State \( S_1 \) - one of the AU \( n \) guns fires a shot. In each shot, one or more of the random perturbations considered above may be present - a change in the initial velocity of the projectile \( \Delta v_0 \) due to the wear of the charging chamber \( \Delta v_{chamb} \), due to the wear of the gun barrel charge \( \Delta v_{barrel} \), and due to the uncertainty of the charge energy \( \Delta v^{urb} \). The listed disturbances in the shot can manifest themselves both separately and in various combinations. The worst option for an ineffective shot is the presence of all three types of perturbations in \( \Delta v_0 = \Delta v_{chamb} + \Delta v_{barrel} + \Delta v^{urb} \). The probabilities of the absence of disturbances (effective shot) are set and equal to: \( p_i = p(\Delta v_{chamb}) \), \( p_2 = p(\Delta v_{barrel}) \) and \( p_3 = p(\Delta v^{urb}) \). The probabilities of occurrence in a single shot from a gun of perturbations from the three listed types are the probabilities of opposite events: \( q_i = p(\Delta v_{chamb}) \), \( q_2 = p(\Delta v_{barrel}) \), and \( q_3 = p(\Delta v^{urb}) \). The next AU state \( S_2 \) is a shot fired from the 2nd gun of the AU, which may also contain or not contain perturbations. Thus, the subsequent AU states \( S_i \) (1 = \( \ell n \)) are sequentially set for the entire series of shots.

The Markov transition probability matrix is as follows:

\[
P = \begin{bmatrix}
p_{00} & p_{01} & \cdots & p_{0i} & \cdots & p_{0n} \\
p_{10} & p_{11} & \cdots & p_{1i} & \cdots & p_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
p_{i0} & p_{i1} & \cdots & p_{ii} & \cdots & p_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
p_{n0} & p_{n1} & \cdots & p_{ni} & \cdots & p_{nn}
\end{bmatrix},
\]

where \( p_i \) (1, j = \( \ell n \)) is the probability of the transition of the Markov chain from state \( S_i \) to state \( S_j \). The Markov graph corresponding to matrix (2) is shown in Fig. 1.

Fig. 1. Markov graph of states and transient probabilities
The corresponding probabilities of ineffective shots with the presence of appropriate perturbations are equal to $q_i = 1 - p_i$, $i = 1, 2, 3$. In the worst-case event when each gun is fired, all three types of perturbations are present. Then the probability of an effective shot is $p_{123} = p_1 p_2 p_3 = 0.81225$. The probability of an ineffective shot with three types of random perturbations is $q_{123} = 1 - p_{123} = 0.18775$.

The matrix of transient probabilities, built according to rule (3), is given in Table 4.

Fig. 2 shows the dependence of the probability of an effective shot when firing by AU from $n$ guns on the number of consecutive firing cycles.

### Transient probability matrix

<table>
<thead>
<tr>
<th>State</th>
<th>$S_0$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_1$</td>
<td>0.18775</td>
<td>0.81225</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.93252</td>
<td>0.305</td>
<td>0.6975</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_3$</td>
<td>0.00661</td>
<td>0.08589</td>
<td>0.9716</td>
<td>0.5358</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0.00124</td>
<td>0.02150</td>
<td>0.9785</td>
<td>0.4024</td>
<td>0.4327</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S_5$</td>
<td>0.00023</td>
<td>0.0054</td>
<td>0.04366</td>
<td>0.18889</td>
<td>0.40861</td>
<td>0</td>
<td>0.35254</td>
</tr>
<tr>
<td>$S_6$</td>
<td>4.38E−05</td>
<td>0.00113</td>
<td>0.01229</td>
<td>0.07095</td>
<td>0.23014</td>
<td>0.39827</td>
<td>0.28717</td>
</tr>
</tbody>
</table>

Markov’s process of changing the states of the effectiveness of shots is discrete. The plots shown in Fig. 2 are only a qualitative visualization demonstrating that if there are three types of random disturbances in the shots, the probability of an effective shot drops sharply with an increase in the number of consecutive shots.

**5.3. Method to control firing by an artillery unit of increased efficiency**

Based on the formulated model, the following method of controlling the firing by AU is proposed, taking into consideration random disturbances. Random disturbances due to the reasons of the wear of the charging chamber and barrel and the uncertainty of information about the state of charge may be present in each shot of each AU gun. The task of hitting the target with AU fire is formulated as follows. The coordinates of a single fixed target are specified. Firing is carried out in a series of simultaneous shots of $n$ shots ($n \geq m$, $m$ is the number of AU guns). A target is considered hit if $n_{eff}$ effective shots are fired at the target. In accordance with the definition introduced earlier, at least $n_{eff}$ shots must have a probability of at least 0.5 having a hit in a circle of a given radius with a center at the point of location of the target. The following additional specific performance criteria must be met. The time of fire by AU from the moment of the first shot to the destruction of the target $time_{fire}$ should not exceed the maximum possible time for the safe stay of AU in the firing position. The total consumption of AU shells for hitting the target should not exceed $n_{shell}$.

### Table 4

**Stage 1. Preliminary preparation.**

The preliminary preparation is based on the above-formulated procedure for categorizing the effectiveness of a shot in accordance with the initial velocity of the projectile based on the acoustic field of the shot. The procedure is based on the use of a binary LSSVM classifier that requires prior training. Therefore, at this stage, according to previous firings from each gun with the number $i$, $L_m$ training samples are constructed for each $j$-th of the $L$ type of charge used when firing from a given gun $AS_{i,j}$. $i = 1, m; j = 1, L$. At the stage of forming training samples, it is desirable to use an artillery ballistic station to confirm the reliability of the classification. However, this requirement is not necessary, since it is shown above that the probability of error of classification of the 2nd kind FPR (the probability of categorizing an ineffective shot as effective) does not exceed 0.04. $L_m$ training samples are entered into the classifier’s field computer together with the LSSVM classification program.

**Stage 2. Preliminary preparation of firing in the firing position.**

After the unit has a firing position, measuring microphones are installed at each gun at a distance of 30 m and 100 m from the muzzle of the gun to classify the shots; they are connected to the field computer of the classifier. The initial firing settings of each gun are introduced. Next, all possible corrections to the initial installations of the gun are introduced: ballistic corrections, meteorological corrections, topographical corrections. Thus, only possible random perturbations remain unaccounted for. Then the Markov model of shooting by AU is built, in particular the matrix of transient probabilities $P$ (2). A matrix of states $P_M$ after $M$ firing cycles is calculated (3). We set the minimum probability of the number of effective shots after $M$ firing cycles as $p_{M}^{eff}$.

**Stage 3. Initial firing without fire control.**

Firing is opened to hit the target in accordance with the installations introduced by the full composition of AU from $m$ guns in successive cycles. For each gun, each shot is rated by the classifier as effective or ineffective. After each firing cycle, the total number of shots fired $N_0$, as well as the number of effective $N_0^{eff}$ and ineffective $N_0^{ineff}$ shots fired according to the classifier, is counted. Obviously, $N_0 = N_0^{eff} + N_0^{ineff}$. After each firing cycle, the mathematical expectation of the number of effective shots according to the Markov model is estimated as $N_0^{eff} = N_0^{eff}$. Meeting the condition $|N_0^{eff} - N_0^{eff}| / N_0^{eff} < 0.05$ is a simultaneous confirmation of the correctness of the classifier and the fairness of the Markov model (4).

The end of the initial firing without fire control is determined by meeting the following condition:
Control processes

\[ N_{\text{eff}} = N_0 (1 - p_{\text{eff}}^* ) > 0.2n_{\text{shot}}. \] (6)

Condition (6) means that more than 20% of the resource of AU shells is spent on ineffective shots. Under this condition, the transition to shooting with fire control is carried out.

**Stage 4.** Shooting with fire control.

At this stage, shooting to hit the target is carried out only with AU guns whose effective shot is most likely. In particular, guns in which an ineffective shot was observed more than 3 times in a row have an irreparable defect in the field – the wear of the charging chamber or the wear of the barrel. This leads to a decrease in the initial velocity of the projectile when fired to a value below 0.85 of the fire table. The probability of an ineffective shot more than 3 times in a row from a serviceable gun is negligible. Stage 4 continues until the condition \( N_{\text{eff}} > n_{\text{shot}} \) is met. Meeting this condition means that the task of hitting the target is completed.

To verify the proposed method, a simulation experiment was set up. The following conditional combat mission was simulated. The coordinates of the target are set located at a distance of \( D = 12,000 \) m. AU consists of \( m = 6 \) guns. The radius of the effective shot when calculating CEP \( R = 0.01 D = 120 \) m. The number of effective shots to hit the target \( n_{\text{eff}} = 40 \). The resource of shots is \( n_{\text{shot}} = 75 \).

Two methods of the shooting were simulated. The first method is the sighting shooting method (SSM) involving three shots from the main AU gun. Corrections of shooting errors are transferred to all AU guns. The assessment of the coordinates of explosions in the process of shooting is carried out by an artillery reconnaissance unit. The second method is the controlled shooting method (CSM), proposed in this paper. It should be noted that the authors of [44, 45] successfully applied the method of controlling inefficient equipment as a result of its regrouping. They also proposed a method for comparing the effectiveness of control for different cases. Therefore, to simulate the methods considered in this paper, 2 guns with defects were identified as part of the artillery unit: one with excess wear of the charging chamber, the second gun with barrel wear. All shots from these guns in both experiments were simulated as ineffective, with an initial velocity of 0.85 of the fire table. The simulation of ineffective shots due to incomplete information about the state of charge was carried out by simulating every tenth shot with a value of the initial velocity of 0.85 of the fire table, while the number of the gun with such a shot was chosen randomly with the number of the gun \( i = \text{rand} [1, 2, 4, 6] \). At the same time, guns with defects (with numbers \( i = 3, 5 \)) were excluded from random selection. To assess CEP, as earlier, for both methods, the shots were simulated as firing from a 155 mm M109 howitzer using a flight path calculation program [35].

The results of the calculation are given in Table 5.

<table>
<thead>
<tr>
<th>Simulated method</th>
<th>CEP, m</th>
<th>Number of ineffective shots fired</th>
<th>Number of effective shots fired</th>
<th>Total number of shots fired</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSM</td>
<td>116</td>
<td>29</td>
<td>40</td>
<td>69</td>
</tr>
<tr>
<td>CSM</td>
<td>103</td>
<td>11</td>
<td>40</td>
<td>51</td>
</tr>
</tbody>
</table>

To estimate the time budget for the fire task, the following calculations were carried out. For firing by howitzers of a caliber more than 122 mm, the rate of fire is 5 rounds per minute [46]. It takes \( \Delta t^* = 2.3 \) minutes to fire 69 rounds of six guns in a row to fire based on MFS. Since the firing of the guns is not synchronized, we shall increase this time by 50%. The shooting time is \( \Delta t^* = 3.45 \) minutes. The time for each shooting shot consists of flight time (according to the firing tables, for the M109 howitzer – 32 s) and the time of propagation of the break sound signal from the break point to the sound receivers of the sound reconnaissance station, equal to \( \Delta t_{\text{SOUND, RANGE}} = 12,000 \text{ m/340 m/s} = 35 \text{ s} \). For three shots, this time would be about \( \Delta t_{\text{SOUND}} = 2 \text{ minutes} \). The time for the introduction of initial settings and corrections of shooting would be equal to \( \Delta t_{\text{CORR}} = 5 \text{ minutes} \). The total time budget for shooting method 1 is about \( T_1 = \Delta t^* + \Delta t_{\text{SOUND, RANGE}} + \Delta t_{\text{CORR}} = 11 \text{ minutes} \).

For 51 shots based on MSU method, taking into consideration the lack of synchrony of the firing of the guns, about \( \Delta t^*_0 = 3 \) minutes would be spent. The additional time spent would be \( \Delta t_{\text{MSU, RANGE}} = 2 \text{ minutes} \) for the transition from Stage 3 to Stage 4 (excluding defective guns from further firing). The total time budget for shooting based on method 2 is \( T_2 = \Delta t^*_0 + \Delta t_{\text{ADH}} = 5 \text{ minutes} \).

**5.4. The generalized criterion of firing effectiveness by an artillery unit**

As a generalized criterion for the effectiveness of AU shooting, a convolution of the following particular criteria is proposed. All particular criteria are standardized accordingly.

The particular criterion of shooting accuracy:

\[ \text{Crit}_{\text{accuracy}} = \frac{\text{CEP}_{\text{fire}}}{\text{CEP}_0}, \] (7)

where \( \text{CEP}_{\text{fire}} \) is the median circular firing error, \( \text{CEP}_0 \) is the median circular firing error for range \( D \).

According to [46], for artillery with a caliber greater than 122 mm, \( \text{CEP}_0 = 1 \%D \). From the practice of shooting, it is known that the value of this criterion cannot be less than 0.75 [36].

The particular criterion for the time of task execution:

\[ \text{Crit}_{\text{time}} = \frac{\text{time}_{\text{fire}}}{\text{time}_{\text{fire}}}, \] (8)

where \( \text{time}_{\text{fire}} \) is the AU fire time to hit the target, \( \text{time}_{\text{lim}} \) the limit time of fire in a given position.

Under real conditions, it is believed that the time of fire activity should be 0.6–0.7 of the total time spent in the firing position [5, 9].

The particular criterion of the efficiency of the consumption of shells

\[ \text{Crit}_{\text{project}} = \frac{n_{\text{shot}}}{n_{\text{shot}}}, \] (9)

where \( n_{\text{shot}} \) is the number of ineffective shots, \( n_{\text{shot}} \) is the total number of shells fired.

In practice, the value of this criterion often exceeds 0.3, that is, almost a third of the shells are ineffective [9].

Fig. 3 shows the above particular criteria calculated from the data of the simulation experiment (Table 5).

Let us form a general criterion for the effectiveness of AU shooting \( \text{Crit}^{\text{eff}}_{\text{project}} \), as a convolution of the above particular criteria [32, 33]. The most popular methods for assessing the
effectiveness of several criteria \( n \) are additive and multiplier convolution. Additive convolution is the following operation:

\[
Crit^\text{add}_i = \sum_{i=1}^{n} c_i Crit_i, \quad i = 1, \ldots, n,
\]  

(10)

where \( c_i \) are the weights of the particular criteria, \( Crit_i \) are the particular criteria. Multiplier convolution:

\[
Crit^\text{mult}_i = \prod_{i=1}^{n} c_i Crit_i, \quad i = 1, \ldots, n.
\]  

(11)

where \( Crit^\text{ideal-point}_i \) are the values of the particular criteria at the so-called ideal point – a dummy point in the \( n \)-dimensional space in which the values of the particular criteria reach an extremum (in this case, the minimum). In expression (12), the distance values of the particular criteria from the ideal point can be weighted by the corresponding weighting factors. However, the choice of weighting coefficients that determine the significance of the corresponding particular criteria is largely subjective. In addition, it can be determined by a specific tactical situation (for example, a limited resource of shells, in which case a particular criterion (9) is the most significant). Therefore, in the considered conditional combat mission, particular criteria are taken to be equally significant. The particular criteria chosen to assess the effectiveness of shooting, in this case, were minimized:

\[
Crit^\text{ideal-point}_i = \sum_{i=1}^{n} \left( \frac{Crit_i - Crit^\text{ideal-point}_i}{\text{Median circular firing error}} \right)^2, \quad i = 1, \ldots, n.
\]  

(12)

6. Discussion of results of studying ways to improve the effectiveness of firing by an artillery unit

To tackle the issue related to developing ways to improve the effectiveness of AU shooting, the concept of an effective shot was introduced. With the help of simulation of the flight...
trajectory of the M109 howitzer projectile with a ballistic model with five degrees of freedom on 5 series of 100 shots in each, the following was confirmed. When fired with a projectile initial velocity deviating from the tabular one downwards by no more than 5% of the tabular value, provided that all other firing errors are compensated for, the projectile enters the circular vicinity of the target. The radius of the circular error \( R = 0.01 D \) with a probability greater than 0.5 (Table 1). Such a shot is termed effective. An improved procedure was devised to categorize shots into effective and ineffective. The procedure is based on the binary classification of the shot according to the parameters of the ballistic and muzzle waves formed during the shot. An advanced LSSVM classifier is used for categorization. The performance of the classifier on a field computer of average performance (MacOS Catalina v10.15.5, CPU: Intel Core i7 2.2 GHz, RAM: 16Gb) does not exceed 15 s. Based on the logic of binary solutions, the quality characteristics of the classification of the shot into classes effective/ineffective have been determined. It is established that the overall quality of the classification according to the F-measure is 0.95 (Table 2). It should be especially noted that the FPR indicator (classification error of the 2nd kind or the proportion of ineffective shots that are erroneously classified as effective) is 0.04 (Table 2).

A limitation of the devised procedure is the need for preliminary training of the classifier for each AU gun when firing on all types of charges used.

When firing by AU, there may be random disturbances in each shot of an individual gun. Disturbances arise due to the wear of the charging chamber \( \Delta e_{\text{charge}} \), of the barrel wear \( \Delta e_{\text{barrel}} \), and the uncertainty of information about the applied charge \( \Delta e_{\text{charge}} \). The built Markov model of the AU firing state makes it possible to estimate the probability of effective shots with random disturbances after \( n \) series of shooting. Fig. 3 shows that this probability decreases dramatically if there are random disturbances in each shot of individual guns. However, it should be borne in mind that the probability of all three types of perturbations in each shot is very small.

Based on the Markov model of AU firing, a method to control shooting of increased efficiency has been developed. The method makes it possible at the second stage of shooting to identify guns that have the wear of the charging chamber or the wear of the barrel. Such guns as ineffective are excluded from further firing. The method devised makes it possible to actually eliminate the feedback procedure “error of the previous shot—settings for the next shot“. This significantly reduces the time of the fire task. In addition, the method can significantly reduce the influence of random disturbances on the accuracy of shooting (Table 5). The limitation of the developed method is the need for a sufficiently correct assessment of the probabilities of the absence/presence of random perturbations. For a conditional combat mission, specific performance criteria have been calculated for sighting shooting and for shooting with fire control according to the proposed method (Fig. 3). A significant effect obtained when shooting with control has been demonstrated. A generalized efficiency criterion (14) has been formed, which is a multi-criteria convolution according to the ideal point method. Note that convolution by the ideal point method is quite subjective due to the choice of coordinates of the ideal point. However, the use of additive and multiplicative convolutions in a given problem is impossible since, for them, the “shortage” of the indicator for one particular criterion can be compensated for by exceeding the other particular criterion.

In the process of further research, it is planned to separate the random perturbations present in the shot with deep learning methods. It is also possible to eliminate subjectivity in the formation of a generalized criterion for the effectiveness of shooting by introducing standardized weights of particular performance criteria. Such studies could eliminate the limitations of the proposed method.

7. Conclusions

1. We have devised an improved procedure for categorizing the effectiveness of a single shot from a gun based on a binary classification of the effectiveness of a shot according to the parameters of the acoustic field created by it. Quantitative indicators of the quality of the classification of the effectiveness of the shot have been calculated. It is shown that the overall quality index of the classification according to the F-measure is 0.95, while the probability of a categorization error of the 2nd kind is 0.04.

2. A Markov model of the effectiveness of sequential firing by an artillery unit has been constructed. The model makes it possible to take into consideration random perturbations, the presence of which is possible in each shot of individual guns. These are disturbances caused by previously undetected wear of the gun's charging chamber, wear of the gun barrel, and incomplete information about the state of the charge used in the shot. It is shown that the presence of random disturbances in the shots reduces the probability of an effective shot to 0.25 when firing by AU three guns after three series of shots.

3. A method for controlling the firing by an artillery unit of increased efficiency based on the constructed Markov model has been developed. The method makes it possible to control the fire by AU by identifying guns with the wear of the charging chamber or the wear of the barrel and excluding them from further shooting. The method reduces the time of the fire task by eliminating feedback between consecutive shots from 11 minutes to 5 minutes for AU with six guns. It is shown that excluding two identified defective guns from the firing by AU makes it possible to perform the task of hitting the target with a saving of 18 shots.

4. A generalized quantitative criterion for assessing the effectiveness of firing by an artillery unit in the presence of random perturbations of shots has been formed. The criterion is built on the basis of a multi-criteria convolution of particular criteria according to the ideal point method. It is shown that when shooting with control, the value of the generalized criterion of effectiveness is almost 5 times higher than its value at sighting shooting.

References


42. LS–SVMlab toolbox. Available at: https://www.esat.kuleuven.be/sista/lssvmlab/