ENGINEERING TECHNOLOGICAL SYSTEMS

OPTIMIZING THE PARTIAL GEAR RATIOS OF THE TWO-STAGE WORM GEARBOX FOR MINIMIZING TOTAL GEARBOX COST

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1. Introduction

The size, mass, hence total cost of a gearbox are significantly affected by gear ratios. Accordingly, mechanical designers typically consider the contribution of the partial gear ratios [1–3]. In particular, in the optimal design problems, finding out the optimal ratios of the gearbox is an important task. An example documented by [4] showing the dependence of


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the gearbox mass on the optimal partial ratios is presented in Fig. 1. It is clearly visualized that the total gear mass reaches 233 kg when \(u_2=6\). The corresponding value of gear mass is 233 kg if \(u_2=1\) is utilized. Inversely, the mass of gears minimizes at 175 kg if \(u_2=2\) is adopted. Hence, it can be said that \(u_2=2\) is the optimal value of the partial gearbox ratios in this study.

Optimal determination of the partial gearbox ratios has been conducted by several researchers so far [5, 6]. The methodology used in gearbox optimization is methods based on graph, practice, and model. For the first one, the optimal gear ratios can be found from the graphical relations between the partial and total ratios. For instance, consider the intersection between the \(u_1\) curve and the \(u_2\) curve as exhibited in Fig. 2 in which the first and second stages \(u_1\) and \(u_2\) of a helical gearbox with three-stage are identified in the graphical form. The method based on practice is the fact that the ratios of gears can be optimally determined by the real data given by gearbox companies. The results presented in [7] reveal that the gearbox mass can be minimized if the center distance of the second stage is 1.4–1.6 times that of the first stage.

**2. Literature review and problem statement**

There have been few studies dealing with optimizing worm gearbox, which are utilized in transmission systems due to the high reduction ratio with rigid size. The model method has been applied to optimize the worm gearbox. Optimization of the volume of two-stage gear drive and simple planetary gear by using a genetic algorithm was conducted by [18]. The weight of worm gears was minimized by applying the artificial immune algorithm presented in [19]. The impacts of lubricant viscosity appearing on the worm gear were conducted in [20]. Regarding the energy resulting from worm gear, the power loss in worm gears was considered in [21]. In this case, the genetic algorithm was used. The authors [22] presented a new and useful method for the calculation of the total transmission ratio of two-step worm reducers for the best reasonable gearbox housing structure. In that study, based on the moment equilibrium condition of the mechanic system including the two units and their regular resistance condition, an effective model for splitting the total ratio of two-step worm reducers was found. By giving explicit models, the transmission ratio of two steps can be calculated fast and effectively. However, it was shown that the proposed method cannot be widely utilized in other cases of gearbox optimization. With a similar objective function, it was revealed that the optimized gear ratios have a strong effect on the size of the gearbox [23]. Determination of the optimal values of gear ratios of a two-stage worm gearbox with the objective function of the reasonable gearbox structure was carried out [24]. It can be seen that the above-mentioned studies do not take into account the cost parameters, which is very important in the problem of optimal design of the gearbox. Hence, this issue should be more crucially concerned by research communities.

**3. The aim and objectives of the study**

The aim of the study is to find the optimal set of the main design parameters, which can minimize the total cost of gearbox.

To achieve the aim, the following objectives were set:
- to investigate the effects of the main design parameters on the \(u_2\) response;
- to determine the regression equation to calculate \(u_2\);
- to evaluate the fitness of the proposed model.

**4. Materials and methods**

4.1 Cost analysis

In fact, the total gearbox cost is significantly affected by the cost of components like bearings, gears, shafts and casing. The calculation of bearing cost will be ignored because of its complication herein. For a two-speed helical gearbox, the cost, \(C_{gb}\), can be calculated as follows:

\[
C_{gb} = C_g + C_w + C_{gb} + C_b + C_s + C_p.
\]  

where, \(C_g\), \(C_w\), \(C_{gb}\), \(C_b\), and \(C_s\) are the cost of the gearbox housing, gears, worm, worm wheel, shafts, and bearing pairs of the gearbox. These cost components can be calculated by:
4. 2. Determining the mass of the gearbox housing

In this study, the housing mass, \( m_{gh} \) is identified as follows:

\[
m_{gh} = \rho_{gh} V_{gh},
\]

where, \( \rho_{gh} \) is the density of the material, for cast iron, \( \rho_{gh} = 7.2 \) (kg/dm\(^3\)); \( V_{gh} \) is assigned to the housing volume (m\(^3\)). The housing shape can be schematically determined by different subareas as shown in Fig. 1, 3:

\[
V_{gh} = 2V_{a1} + V_{a1} + V_{s1} + V_{s2} + V_{s3},
\]

(9)

where, the subareas noted by \( V_{a} \), \( V_{s1} \) and \( V_{s2} \) are calculated by:

\[
V_{a1} = (L + 2S_c) \cdot H \cdot B_t \cdot H_l,
\]

(10)

\[
V_{a1} = (B - 2S_c) \cdot (L + 2S_c + L_e).
\]

(11)

\[
V_{s1} = (B - 2S_c) \cdot (H - 2S_c),
\]

(12)

\[
V_{s2} = (B - 2S_c) \cdot L_e,
\]

(13)

\[
V_{s3} = (B - 2S_c) \cdot (H - S_0 - H_t).
\]

(14)

\[
V_{s3} = (B - 2S_c) \cdot (H - 2S_c),
\]

(15)

where, \( L, H, B_t, B_2 \) and \( S_0 \) can be estimated by:

\[
L = d_{x2} + 20,
\]

(17)

\[
H = \frac{d_{x21} + a_{x2}}{2} + \max \left( \frac{d_{x21} + d_{x2} \cdot 2}{2} \right) + 8.5S_c,
\]

(18)

\[
L_t = b_{x1} + 4S_c,
\]

(19)

\[
H_t = \frac{d_{x21} + a_{x2} + d_{x2} \cdot 2}{2} + 8.5S_c,
\]

(20)

\[
B = \max (d_{x21}, d_{x2}) + 20 + 2S_c,
\]

(21)

\[
S_c = 0.005L + 4.5.
\]

(22)

where, \( C_{g,n}, C_{a,n}, C_{s,n}, C_{w,n}, C_{k,n} \) are the cost per kilogram (USD/kg) of gear, worm, worm wheel, housing, and shaft, respectively. \( C_{hi} \) is the cost of a bearing on the \( i^{th} \) shaft (\( i = 1 \) to 3). This cost component will be defined later. \( m_g, m_w, m_h, m_s, \) and \( m_0 \) (kg) are noted for gear, worm, worm wheel, housing, and shaft mass. The mentioned components of mass can be calculated in the following sub-sections.

4. 3. Determining the mass of gears

It is discerned that the gear mass in a gearbox can be identified by totalizing the mass of element gears:

\[
m_g = \rho_c \left( \pi \cdot e_1 \cdot d_{x1}^2 \cdot \frac{b_{w1}}{4} + \pi \cdot e_2 \cdot d_{x21}^2 \cdot \frac{b_{w2}}{4} \right).
\]

(23)

where, \( \rho_c \) is the material density, \( \rho_c = 7.8 \) (kg/dm\(^3\)) for steel [25]; for the first stage \( e_1 \) and \( e_2 \) are the volume coefficients of the drive and driven gear.

In fact, \( e_1 \) and \( e_2 \) are chosen as 1 and 0.6 respectively; \( b_{w1} \) (mm), the gear width, is determined by:

\[
b_{w1} = X_{w1} \cdot a_{x1},
\]

(24)

where, \( d_{w1} \) and \( d_{w2} \) are the pitch diameters determined by [26]:

\[
d_{w1} = 2 \cdot \frac{a_{x1}}{u_1 + 1},
\]

(25)

\[
d_{w2} = 2a_{x1} \cdot \frac{u_1}{u_1 + 1}.
\]

(26)

In (25) and (26), \( a_{x1} \) is the center distance of the helical gear provided by [26]:

\[
a_{x1} = k_x \cdot (u_1 + 1) \cdot \sqrt{\frac{k_{jg}}{T_{11} \cdot \left( \sigma_{x2} \right)^{2} \cdot u_1 \cdot X_{w1}}}.
\]

(27)

where, \( k_{jg} \) is the contact load factor for pitting resistance; \( k_{jg} = 1.16 \) for the helical gear set; \( \left[ \sigma_{x2} \right] \) is the allowable contacting stress of the first stage (MPa); \( k_{x} = 43 \) is the material coefficient for steel of gear materials; \( X_{w1} \) is the coefficient of wheel face width of the helical gear set; \( T_{11} \) is the torque on the pinion (N:mm):

\[
T_{11} = \frac{T_{ma}}{u_x \cdot \eta_x \cdot \eta_{ao} \cdot \eta_b}.
\]

(28)
Engineering technological systems: Reference for Chief Designer at an industrial enterprise

in which, $\eta_{\text{bg}}, \eta_b, \eta_w$ are the transmission efficiency of the helical gear unit, rolling bearing pair, and worm set, respectively. Choosing $\eta_{\text{bg}}=0.97; \eta_b=0.992; \eta_w=0.76$ and substituting them into (28) gives:

$$T_{11} = 1.3896 \frac{T_{\text{out}}}{u_s}$$  \hspace{1cm} (29)

where, $T_{\text{out}}$ is the torque on the outcome shaft of the system.

4.4. Calculating the mass of the worm
The mass of the worm can be determined by:

$$m_w = \rho_w \cdot \pi \cdot d_{w12}^2 \cdot \frac{l_w}{4}$$  \hspace{1cm} (30)

in which, $\rho_w=7.8 \text{ (kg/dm}^3\text{)}$ is the steel density used for the worm, \cite{[27]}; $l_w$ is the length of the worm. In practice, $l_w$ can be approximated by $l_w=2d_{w11}$. 

The pitch diameter of the worm $d_{w11}$ can be found by \cite{[27]}:

$$d_{w12} = \frac{d_{w22}}{u_s}$$  \hspace{1cm} (31)

with

$$d_{w22} = 2a_{w2} \sqrt{\frac{z_s}{z_1+q}}$$  \hspace{1cm} (32)

where, $a_{w2}$ is the center distance of the worm gear set determined as follows:

$$a_{w2} = \frac{K_s}{\sigma_{w2}} \left( \frac{K_{\text{HL}} - K_{\text{HB}}}{\sigma_{w2}} \right)^{\frac{1}{2}}$$  \hspace{1cm} (33)

where $K_s=610$ \cite{[34]}; $K_{\text{HB}}=1.2$ is the internal dynamic coefficient \cite{[34]}; $K_{\text{HL}}=1.2$ is the load concentration coefficient \cite{[26]}; $T_{22}$ is the wheel torque (Nm); $T_{22}=T_{\text{out}}$.

In (33), $[\sigma_{w2}]$ is the allowable contacting stress of the worm unit (N/mm$^2$); $[\sigma_{w2}]$ is influenced by the materials of the wheel. If tinless bronze or soft grey iron is adopted, $[\sigma_{w2}]$ is determined by the regression model below that is obtained by data in \cite{[26]} (with the determination coefficient $R^2=0.9906$):

$$[\sigma_{w2}] = 5.0515c_{w2}^3 - 49.742w_{d2} + 189.9$$  \hspace{1cm} (34)

wherein, $w_{d2}$ is the slip velocity, which is found by \cite{[26]}:

$$w_{d2} = 0.0088 \left( P_1 \cdot u \cdot n_s \right)^{\frac{1}{3}}$$  \hspace{1cm} (35)

where $P_1$ is the power on the worm shaft; $P_1$ can be found by:

$$P_1 = T_{12} \cdot \frac{n_w}{9.66 \times 10^5}$$  \hspace{1cm} (36)

in which, $n_w$ is the rotational speed of the worm; $T_{12}$ is the worm torque (Nm):

$$T_{12} = \frac{T_{\text{out}}}{u_s \cdot \eta_w \cdot \eta_b}$$  \hspace{1cm} (37)

Choosing $\eta_w=0.76; \eta_b=0.992$ and substituting them into (37) gives:

$$T_{12} = 1.3371 \frac{T_{\text{out}}}{u_s}$$  \hspace{1cm} (38)

In case of tin bronze materials, $[\sigma_{w2}]$ is determined by:

$$[\sigma_{w2}] = K_{\text{HB}} \cdot w_{d2} \cdot [\sigma_{w0}]$$  \hspace{1cm} (39)

where $[\sigma_{w0}]$ is the allowable contacting stress as its varying cycle is $10^7$:

$$[\sigma_{w0}] = (0.7 + 0.9) \sigma_t$$  \hspace{1cm} (40)

$\sigma_t$ is the tension stress (N/mm$^2$); the value of $\sigma_t$ depends on the slip velocity $v_{d2}$, $\sigma_t=260$ if $v_{d2}=5...8$; $\sigma_t=230$ if $v_{d2}=8...12$; and $\sigma_t=285$ if $v_{d2}=8...25$.

$K_{\text{HL}}$ is the servicing life ratio calculated by \cite{[26]}:

$$K_{\text{HL}} = \left( \frac{10^7}{N_{\text{HL}}} \right)^{\frac{1}{2}}$$  \hspace{1cm} (41)

where, $N_{\text{HL}}$ is the number of equivalent load cycles for the teeth of the wheel:

$$N_{\text{HL}} = 60 \cdot n_{w1} \cdot t_{w1}$$  \hspace{1cm} (42)

in which, $t_{w1}$ is the service lifetime (h); $n_{w1}$ is the wheel speed (rpm).

4.5. Calculating the mass of the worm wheel
The mass of the worm wheel is calculated by:

$$m_{w1} = \rho_{w1} \cdot \pi \cdot d_{w12}^2 \cdot \frac{b_{w1}}{4}$$  \hspace{1cm} (43)

in which, $\rho_{w1}$ is the worm wheel material density (kg/m$^3$), $\rho_{w1}=8.88$ (kg/m$^3$) for bronze worm wheel \cite{[25]}; $b_{w1}$ is the width of the worm wheel; $b_{w1}$ is determined by $b_{w1}=0.75d_{11}$ \cite{[26]}.

4.6. Calculating the mass of shafts
The mass of the shafts ($m_s$) can be determined as follows:

$$m_s = m_{s1} + m_{s2} + m_{s3}$$  \hspace{1cm} (44)

where:

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s12}^2 \cdot \frac{b_{s1}}{4}$$  \hspace{1cm} (45)

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s22}^2 \cdot \frac{b_{s2}}{4}$$  \hspace{1cm} (46)

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s32}^2 \cdot \frac{b_{s3}}{4}$$  \hspace{1cm} (47)

where $l_{s1}, l_{s2}, l_{s3}$ and $m_{s1}, m_{s2}, m_{s3}$ are the length and mass of the shaft 1, 2, 3, respectively; $\rho_s$ is the shaft material density (kg/m$^3$).

$$l_{s1} = L + 1.2 \cdot d_{s1} + S_G$$  \hspace{1cm} (48)

$$l_{s2} = L + l_{s1} - 0.5S_G$$  \hspace{1cm} (49)

$$l_{s3} = d_{s12} + 1.2 \cdot d_{s3} + 20$$  \hspace{1cm} (50)
where:
\[
d_{in} = \left[ \frac{T_{in}}{0.2[t]} \right]^{1/2},
\]
\[
d_{sb} = \left[ \frac{T_{sb}}{0.2[t]} \right]^{1/2},
\]
\[
d_{sh} = \left[ \frac{T_{sh}}{0.2[t]} \right]^{1/2},
\]

where \([t]=17 \text{ (MPa)}\) is the allowable shearing stress (MPa) [26].

4. 7. Bearing cost calculation

From the data in [6], a regression formula (with \(R^2=0.9877\)) to determine the cost of a medium-sized deep groove ball bearing on the \(i^{th}\) shaft with the inner diameter of 5 \(N_{i+}\) has been proposed:

\[
C_{bei} = 0.2707N_{i+}^2 - 1.2566N_{i+} + 3.3056,
\]

in which, \(C_{bei}\) is the maximum retail price of the bearing (USD); \(N_{i+}\) is the value of the last two digits of the bearing’s designation. \(N_{i+}\) can be easily determined from the value of the shaft diameter \(d_{i+}\).

The cost of a bearing on the \(i^{th}\) shaft \(C_{b,i}\) is determined by:

\[
C_{b,i} = k_{b} \cdot C_{bei},
\]

wherein, \(k_{b}\) is the bearing selling coefficient (\(k_{b}<1\)).

4. 8. Optimization problem

From the above analysis, the definition of the optimization issue follows:

\[
\text{Minimize } C_{gb}.
\]

And including the constraints below:

\[
1 \leq u_i \leq 9;
\]

\[
8 \leq u_i \leq 80;
\]

\[
a_{e,i} + \frac{d_{sh}}{2} + 2.5S_{e} < a_{e,i}.
\]

To evaluate the effects of the main design parameters or investigated parameters on the \(u_2\) partial gear ratio to get minimal gearbox cost, a screening experiment is conducted by using ten parameters. The investigated degrees of these parameters are listed in Table 1. To reduce the number of the required tests and still well evaluate the influence of the input parameters, we use partial experimental planning of \((2^5)^{11-4} = 128\) tests (a \(2^5(11-4)\) model and using 1/16 fractional model). The chosen experimental design is at 5 resolution degrees, and at this resolution no major factor or two-way interaction is coincident with any other major factor or 2-way interaction. The utilization of screen experiment can generate the largest number of experiments. Additionally, using this method can provide mathematical models that cannot be obtained by the Taguchi method.

To determine the experimental plan, Minitab®18 software is applied by \(2^5\) and the details of this design are described as presented in Table 2.

<table>
<thead>
<tr>
<th>The levels and denotation of input parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter name</td>
</tr>
<tr>
<td>Total gearbox ratio</td>
</tr>
<tr>
<td>Output torque</td>
</tr>
<tr>
<td>Coefficient of the face width of the first gear stage</td>
</tr>
<tr>
<td>Allowable contact stress of stage 1</td>
</tr>
<tr>
<td>Cost of gearbox housing</td>
</tr>
<tr>
<td>Cost of a kilogram of gears</td>
</tr>
<tr>
<td>Cost of the worm</td>
</tr>
<tr>
<td>Cost of the worm wheel</td>
</tr>
<tr>
<td>Cost of a kilogram of shafts</td>
</tr>
<tr>
<td>Bearing selling price coefficient</td>
</tr>
</tbody>
</table>

| Table 1 |

<table>
<thead>
<tr>
<th>The testing plans and calculated partial gear ratio of (u_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Order</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>81</td>
</tr>
<tr>
<td>117</td>
</tr>
<tr>
<td>86</td>
</tr>
<tr>
<td>88</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>107</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>93</td>
</tr>
</tbody>
</table>
It is noticed that the values of $u_2$ response corresponding to each test are displayed in the last column.

5. Results of research of finding the optimal set of the main design parameters

5.1. The effects of the main design parameters on the $u_2$ response

The effects of the main design parameters on the $u_2$ response can be evaluated by the results given in Fig. 4. It is visualized that $u_2$ is increased with increasing Total gearbox ratio ($u_g$) and Cost of a kilogram of gears ($C_g$), while $u_2$ is reduced when Output torque ($T_{out}$), Allowable contact stress of stage 1 ($AS_1$), and Cost of the worm wheel ($C_{wh}$) increase. Moreover, Coefficient of the face width of the first gear stage ($X_{ba}$), Cost of gearbox housing ($C_{gh}$), Cost of the worm ($C_w$), Cost of a kilogram of shafts ($C_s$) and Bearing selling price coefficient ($k_{cb}$) have a small effect on $u_2$.

The effects of input parameters on the $u_2$ response can be also presented by Pareto Chart as in Fig. 5. It shows both the impact of the main design parameters and interactions on the response $u_2$. The influence level of each parameter is exhibited by the blue column length. In this case, the parameters with exceeding magnitude, the red reference line, are those that have a significant effect on $u_2$ corresponding to the significance level $\alpha$ of 0.05. Specifically, it is seen that the influence of A factor ($u_g$) on $u_2$ is the largest. The parameters having a dominant influence on $u_2$ including interactions are: B ($T_{out}$), D ($AS_1$), F ($C_g$), H ($C_{wh}$) and interactions BD ($T_{out} \cdot AS_1$), BF ($T_{out} \cdot C_g$), BH ($T_{out} \cdot C_{wh}$), DF ($AS_1 \cdot C_g$), DH ($AS_1 \cdot C_{wh}$), FH ($C_g \cdot C_{wh}$), GK ($C_w \cdot k_{cb}$).

Taking into account the impacts of interactions and parameters on $u_2$ is crucial. These results can be seen in Fig. 6.
There is the fact that the Total gearbox ratio factor shows the dominant influence on $u_2$ even though this factor interacts with nine remaining parameters where all of these interactions, in the leftmost column of Fig. 6, have positive impacts on $u_2$.

To determine the tendency and impacts of the parameters and their interactions, one can refer to the normal distribution chart in Fig. 7.

It is seen that the parameters highlighted in red have a significant impact on $u_2$. The parameters further from the reference line are the ones having a greater impact on $u_2$. The parameters to the right of the reference line have a positive influence on $u_2$, including: A ($ug$), F ($Cg$), and interactions such as BD ($T_{out} \times AS_1$), BH ($T_{out} \times C_{wh}$), DH ($AS_1 \times C_{wh}$). The parameters to the left of the reference line have a negative influence on $u_2$, including: B ($T_{out}$), D ($AS_1$), H ($C_{wh}$) and interactions; BF ($T_{out} \times C_g$), DF ($AS_1 \times C_g$), FH ($C_g \times C_{wh}$), GK ($C_{wh} \times kcb$).

5.2. Determining the regression equation to calculate $u_2$

Proceed to removing the parameters that have no or very little influence on $u_2$. A regression model will be proposed for predicting $u_2$ by using Minitab®18 software. This is because of the large number of variables, so manual calculation is not feasible. The estimating coefficients of the main design parameters and their interactions are presented in Table 3. The summary of the model showed that R-square and R-square (adjusted) are higher than 99%.

![Fig. 6. Effects of interactions on $u_2$](image)

![Fig. 7. Histogram of the normal distribution showing the influence of parameters and interactions on $u_2$](image)
The coefficients of the regression model are obtained as follows:

\[ u_2 = 4.736 + 0.089125u_g - 0.000246T_{out} - 0.00688AS_1 + 0.2807C_g + 0.035C_w - 0.1931C_{wh} + 0.782k_{cb} + 0.000001T_{out} \cdot AS_1 - 0.000005T_{out} \cdot C_g + 0.000005T_{out} \cdot C_{wh} - 0.00515C_g \cdot C_{wh} - 0.1092C_w \cdot k_{cb}. \]

5.3. Evaluating the fitness of the proposed model

The model fitness is evaluated through the residual distribution chart to determine deviations between the experiments and prediction of \( u_2 \) in Fig. 8. It is revealed that on the Normal Probability Plot, comparing graph the probability distribution of errors (shown by points in blue) for the normal distribution (solid line), these deviations are distributed very near the normal distribution. The Versus fits graph discloses that the relation between the residual and fitted value of the model is random. This shows that the response \( u_2 \) is not impacted by any rule control factors other than the input variables. The histogram shows the frequency of residual values around the center of the distribution, which can be considered as a normal distribution. Finally, the Versus Order also exhibits the random relationship between residual and order of data point. This reveals that the response \( u_2 \) is not influenced by the time factor.

Fig. 8. Residual evaluation distribution charts: \( a \) — Normal Probability Plot; \( b \) — Versus Fits; \( c \) — Histogram; \( d \) — Versus Order

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F-Value</th>
<th>P-Value</th>
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<tbody>
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<td>Linear</td>
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<td>917.226</td>
<td>131.032</td>
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<td>0.000</td>
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<tr>
<td>( u_g )</td>
<td>1</td>
<td>915.064</td>
<td>915.064</td>
<td>36082.17</td>
<td>0.000</td>
</tr>
<tr>
<td>( T_{out} )</td>
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<td>0.594</td>
<td>23.42</td>
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<tr>
<td>( AS_1 )</td>
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<td>0.498</td>
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</tr>
<tr>
<td>( C_g )</td>
<td>1</td>
<td>0.510</td>
<td>0.510</td>
<td>20.11</td>
<td>0.000</td>
</tr>
<tr>
<td>( C_w )</td>
<td>1</td>
<td>0.035</td>
<td>0.035</td>
<td>1.38</td>
<td>0.242</td>
</tr>
<tr>
<td>( C_{wh} )</td>
<td>1</td>
<td>0.510</td>
<td>0.510</td>
<td>20.11</td>
<td>0.000</td>
</tr>
<tr>
<td>( k_{cb} )</td>
<td>1</td>
<td>0.015</td>
<td>0.015</td>
<td>0.59</td>
<td>0.445</td>
</tr>
<tr>
<td>2-Way Interactions</td>
<td>7</td>
<td>3.290</td>
<td>0.470</td>
<td>18.53</td>
<td>0.000</td>
</tr>
<tr>
<td>( T_{out} \cdot AS_1 )</td>
<td>1</td>
<td>0.498</td>
<td>0.498</td>
<td>19.62</td>
<td>0.000</td>
</tr>
<tr>
<td>( T_{out} \cdot C_g )</td>
<td>1</td>
<td>0.510</td>
<td>0.510</td>
<td>20.11</td>
<td>0.000</td>
</tr>
<tr>
<td>( T_{out} \cdot C_{wh} )</td>
<td>1</td>
<td>0.510</td>
<td>0.510</td>
<td>20.11</td>
<td>0.000</td>
</tr>
<tr>
<td>( AS_1 \cdot C_g )</td>
<td>1</td>
<td>0.421</td>
<td>0.421</td>
<td>16.60</td>
<td>0.000</td>
</tr>
<tr>
<td>( AS_1 \cdot C_{wh} )</td>
<td>1</td>
<td>0.421</td>
<td>0.421</td>
<td>16.60</td>
<td>0.000</td>
</tr>
<tr>
<td>( C_g \cdot C_{wh} )</td>
<td>1</td>
<td>0.510</td>
<td>0.510</td>
<td>20.11</td>
<td>0.000</td>
</tr>
<tr>
<td>( C_w \cdot k_{cb} )</td>
<td>1</td>
<td>0.421</td>
<td>0.421</td>
<td>16.60</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>113</td>
<td>2.866</td>
<td>0.025</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>923.382</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
This result is also clearly shown in the error frequency graph. The near zero error dominates for the majority (ranging from −0.4 to 0.4). It is observed that the graph showing the relationship between the errors and the calculated values of the proposed model shows that the data are arbitrarily distributed.

6. Discussion of the results of finding the optimal set of the main design parameters

Due to the simulation experiments instead of physical one where there is no limiting of the cost, it is desired to perform all tests as possible in this study. However, the available function given by Minitab®18 could not execute as expected. For this reason, the authors have selected a 2^(10−3) model and using 1/16 fractional model, maximum ability to conduct the number of tests (128 tests). Furthermore, the screening experiments are adopted to reduce the number of parameters, which has a minor influence on the response. This is a simple method to get the mentioned aims. Compared to using Taguchi technique, for the model of 2^11 corresponding to L32 or 32 tests. This is not satisfied the maximum tests as previously mentioned (128 tests). Additionally, Taguchi design could not provide the regression equation. Hence, the method used in this study is better than simply using Taguchi method. For example, it is seen that there are seven single factors and seven interaction factors having a significant influence on the response. P-value of each parameter mentioned in Table 3 if higher than significant of ($\alpha = 0.05$) will be eliminated because of its minor effect on the response.

The authors wish to thank the Thai Nguyen University of Technology for supporting this work.

The aim of this study is to investigate the influence of the main design parameters on the optimal gear ratio, which is the constraint of the objective function. Hence, the influence of the assumptions is beyond the target of this study. Moreover, the cost of a given gearbox is decided by some cost factors such as the cost of manufacturing, the cost of the assembly process, the cost of operation, the cost of bearing, the cost of wear and reliability, and the cost of materials. Besides, there are other factors related to the cost such as the kind of surface treatment, the shape of the gear body, etc. In this study, the cost of materials is only considered, but others are beyond the scope here. The cost of material of the studied gearbox is indicated herein by $C_{gb}$. For future research, the previously mentioned factors should be considered in the gearbox optimization problems.

7. Conclusions

1. Total gearbox ratio has the biggest effect on the $u_2$ response, while the remaining parameters have minor. These results are both shown in Main Effects Plot and Pareto Chart of the Standardized Effects. The interactions between input parameters also have a significant impact on the response. In particular, the interaction between Total gearbox ratio and the remaining factors strongly influences the partial gear ratios.

2. The proposed regression model for $u_2$ is consistent with validated data with the coefficients of adjusted $R^2$ and $R^2$ both greater than 99%.

3. The fitness of the proposed model was reliably confirmed in Normal Probability Plot, Versus Fits, Histogram, and Versus Order, and ANOVA. For this reason, it can be concluded that the proposed optimization process can be applied to minimally calculate gearbox cost.

Acknowledgments

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References


