This paper considers the processes of cost management and the duration of road maintenance projects. A conceptual model is proposed to describe the relationship between time, cost, and quality of projects, which is based on the theory of the «silver triangle». Based on the results of studying the influence of factors, a model has been built for determining the cost and duration of road maintenance projects. Multiple optimization approaches to the components of the «silver triangle» of projects were used. The model constructed at the stage of initiating projects for the maintenance of roads, in contrast to the previous ones, is based on the pre-project level of quality indicators.

Owing to the introduction of additional parameters and coefficients, the proposed model of cost management, quality, and duration of road projects was calibrated. The model graph was constructed and optimization was carried out on the example of a long-term road operation project. The algorithm of application of cost and duration management model of highway projects under conditions of uncertainty has been developed, which was checked for adequacy at an estimated error of about 3%. This makes it possible to assert the effectiveness of the devised model for solving tasks related to project management for maintenance.

The model was used to perform calculations based on a real example of the project, which showed that the optimization of the parameters of long-term contracts for the maintenance of highways produces a multiplier effect. This is expressed by reducing the administrative costs of the customer, reducing the responsibility of road services, creating prerequisites for stable financing of road works, the satisfaction of road users, and building strong partnerships between the customer and contractor.

Keywords: cost and duration management, maintenance project, long-term maintenance, management model, silver triangle

1. Introduction

The key to the successful implementation of road maintenance projects is preventive consideration of possible factors of influence on this process. Important parameters of projects in the road industry are the time of their implementation, the cost of future contracts, and quality indicators that will be achieved as a result. Thus, there is a so-called «silver triangle», which reflects the ratio of «time-cost-quality» of projects. The interdependence of the components of this relationship is complex, and rather uncertain since each of them is influenced by various factors at different stages of the project implementation. Unfortunately, the consideration of cost management processes and the duration of road maintenance projects, as well as the impact of a set of factors on them, are given insufficient attention in the scientific environment.

The use of modeling processes to determine the cost and duration of the road organization in accordance with the level of complexity and quality of maintenance work makes it possible to adequately substantiate the production capacity of the contractor’s organization. This will contribute to creating a free competitive market environment and ensuring the quality of work and services for the future. However, the main problem in the process of determining the cost and duration of projects is the difficulty of determining the impact of factors on the project implementation process. Thus, the relevance of consideration of this issue is due to the need to build an effective model for managing the cost and duration of road projects under conditions of uncertainty.

2. Literature review and problem statement

The classic way of determining the cost of a road maintenance project is absolutely not suitable for determining the cost of long-term contracts that require taking into consideration quality indicators (service levels). In the scientific com-
munity, the latter conducted research in the direction of determining the cost and duration of road maintenance projects based on road maintenance levels. In particular, [1] proposed a model that was tested to determine the cost of guarantees for the project of operation of the road network. However, the process of maintaining highways has several cycles of repair and wear of the coating during the warranty period, which was not taken into consideration in this model [1].

In study [2], to estimate the cost of a short-term guarantee on the project of road operation, a structural method based on reliability was applied. To determine the quantitative value of the cost of guarantees, a methodology for system analysis was devised. However, as the approbation of the results of the study has shown [2], the proposed method is ineffective when planning for the long term.

Several authors [3] emphasize that in order to stimulate contractors in the framework of long-term road operation projects, it is advisable to use the «cost+time» and «incentive/containment» scheme.

In particular, in [3], the researchers draw attention to the fact that the scheme of «incentive/containment» contributes to the development of the project and encourages contractors to complete the tasks assigned to it, indicated by the contract, earlier.

Study [4] draws attention to the problem of overspending of road agencies for the maintenance of highways, especially taking into consideration the need for a higher level of traffic comfort and the problem of underfunding of this sector. However, the cited study does not specify how the term of the contract affects overruns and how to optimize the cost in order to avoid overspending.

Studying costs based on the approach of public-private partnership in road maintenance projects [5] can make it possible to more accurately predict the probability and volume of overspending and thus determine the optimal cost of the project. However, one of the limitations of the cited research [5] is the assumption (usually due to lack of data) that cost overruns, along with other cost characteristics, behave equally under different methodical approaches.

According to the results reported in [6], it is determined that the process of implementing road maintenance projects depends on such three main components as time, cost, and quality. Thus, there is a so-called «silver triangle», whose components are directly related. However, the cited study does not give an unambiguous assessment of how to optimize these characteristics of the project.

Study [7] determined that focusing or solving one goal of a triangle affects the other two tricks. However, the cited study was not used for road maintenance projects.

Somebody’s analysis of scientific research [1–6] reveals insufficient validity of mechanisms for managing the cost and time of road maintenance projects.

3. The aim and objectives of the study

The aim of this study is to build a model for managing the cost and duration of road projects, which makes it possible to optimize the cost-time ratio in long-term road maintenance contracts.

To achieve the set aim, the following tasks have been solved:
- to construct a mathematical model for determining the cost and duration of road maintenance projects;
- to test the devised model on the example of the long-term maintenance project of the highway.

4. The study materials and methods

The object of our research is the processes of cost management and the duration of projects for the maintenance of highways.

The model of cost management and duration of road projects is based on the main provisions of graph theory, structural method, and theoretical aspects of determining the interaction of the components of the «silver triangle». The main hypothesis of the study assumes that with long-term operational maintenance of highways, it becomes necessary to justify the main parameters of the project, such as time and cost with the quality of work. The main assumption of our study is to take into consideration the uniqueness of each individual project by determining the likely impact of different groups of factors on cost, duration, and quality. This makes it possible to perform modeling on the principle of pairwise comparison. The devised model was implemented in the Excel (USA) environment in the form of a separate optimization module. The proposed model makes it possible to compile individual tables of interdependence of the components of the «silver triangle» of the project for making managerial decisions. Testing of the model is carried out on the example of a long-term maintenance project of the highway. The completed calculations on the real example according to the model were tested for adequacy using the Fisher criterion with a level of significance of 0.05. At the same time, the estimated error according to the model was about 3 %.

5. Results of building a cost and duration management model for highway projects

5.1. Constructing a conceptual model for managing the cost and duration of road maintenance projects

According to somebody’s research, it is determined that the process of implementing road maintenance projects depends on such three main components as time, cost, and quality, which, in turn, constitute the so-called «silver triangle» (Fig. 1) [6]. Given the complexity of determining the relationship of these components, it becomes necessary to identify and take into consideration the main groups of factors that influence the implementation of projects in the contract phase.

The main aspects of managing the components of the «silver triangle» according to [6, 8] are:
1. Time management (T) – a multi-stage process that is necessary for the timely fulfillment of contractual obligations.
2. Cost management (C) – a process that ensures monitoring and control of the contract budget and consists of many successive inalienable stages (determining the necessary resources, determining the cost, budget planning, analyzing resource deviations, etc.).
3. Quality management (Q) – the process of checking compliance with the quality levels specified in the contract.

Thus, the conceptual model of cost management and duration of the project of long-term maintenance of the highway, taking into consideration the quality of the triangle TCQ will include a number of factors that affect its components (Fig. 2).
5.2. Constructing a mathematical model for determining the cost and duration of road maintenance projects

When constructing a mathematical model for determining the cost and duration of road maintenance projects for each project resource, it is necessary to determine the most important indicators (factors).

To solve the problem of optimizing the components of the TCQ triangle, it is advisable to describe the long-term maintenance of roads with a network model activity-on-node:

\[ G = \{(A, P), \ A \in \{0, ..., n+1\}\}, \]

\[ P \in \{P\}, \]

where \( n \) is the number of nodes (actions, assets); \( P \) is the sum of all paths in the activity-on-node network, ranging from activity 0 to \( n+1 \); \( P_L \) is the sum of actions (works, assets) that are along the way \( L \).

The components of the triangle can be represented as a model using the structural method (Table 1).

According to Table 1, the influence of the interaction of certain factors on the labor force can be described by the formula:

\[ L_{\text{PREQ}} = L_{\text{PREQ}} - K_{LQ}(Q_{LQ} - Q_{LQ}^\text{min}). \]

where \( L_{\text{PREQ}} \) is productivity; \( Q_{LQ} \) is the actual level of quality of work \((i)\); \( Q_{LQ}^\text{min}, Q_{LQ}^\text{max} \) – minimum and maximum level of quality of work \((i)\) involved in activities \((i)\); \( K_{LQ} \) is the coefficient of the ratio of factors affecting the labor force, which is determined from the formula:

\[ K_{LQ} = \left(\frac{L_{\text{PREQ}}^\text{max} - L_{\text{PREQ}}^\text{min}}{L_{\text{PREQ}}^\text{max} - L_{\text{PREQ}}^\text{min}}\right) / (Q_{LQ}^\text{max} - Q_{LQ}^\text{min}). \]

The main characteristics of the "silver triangle"

The triangle shows that quality, cost, and time are interrelated

Focusing or solving one goal of the triangle affects the other two focuses

The factors of the triangle are balanced by the goals and challenges of the management task

Many triangle inductions have been developed

Most often in the solution of management problems one of the factors is fixed, and the other two are variable in inverse proportion to each other

Fig. 1. The basic characteristics of the «silver triangle»

Fig. 2. Conceptual model of cost management and duration of the long-term maintenance project of the highway

The relationship of the components of the triangle TCQ is very complex and ambiguous, this is due to the presence of various kinds of factors that under different circumstances can affect one or more components at the same time. Certain groups of factors have a variable character, which depends on the uniqueness and environment of the project, the activities of the organization-performer, and the external environment. Thus, there is a need to take them into consideration in the mathematical model of determining the cost and duration of road maintenance projects.

Table 1

<table>
<thead>
<tr>
<th>Model structure (adapted from [9])</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity (Contract)</strong></td>
</tr>
<tr>
<td><strong>Activity (i)</strong></td>
</tr>
<tr>
<td><strong>Resources</strong></td>
</tr>
<tr>
<td>Human resources ((L))</td>
</tr>
<tr>
<td><strong>Main indicators (factors) of resources</strong></td>
</tr>
<tr>
<td>Productivity; cost; labor intensity of works; quality of work</td>
</tr>
<tr>
<td><strong>Time-Cost-Quality</strong></td>
</tr>
<tr>
<td>Time ((i))</td>
</tr>
<tr>
<td>Cost (L_i)</td>
</tr>
<tr>
<td>Quality (L_i)</td>
</tr>
</tbody>
</table>
The relationship between the quality of materials and their cost can be described by the formula:

\[ C_{M(i)} = C_{M(i)}^\text{min} - K_{M(i)} \left( Q_{M(i)} - Q_{M(i)}^\text{min} \right) \]  

(4)

where \( Q_{M(i)} \) is the actual level of quality of materials \( i \); \( Q_{M(i)}^\text{min}, Q_{M(i)}^\text{max} \) are the minimum and maximum level of quality of materials \( i \); \( K_{M(i)} \) – the ratio of factors affecting the quality of materials, which is determined from the formula:

\[ K_{M(i)} = \left( C_{M(i)}^\text{max} - C_{M(i)}^\text{min} \right) / \left( Q_{M(i)}^\text{max} - Q_{M(i)}^\text{min} \right). \]  

(5)

where \( C_{M(i)}^\text{min}, C_{M(i)}^\text{max} \) are the minimum and maximum cost of materials involved in the activities \( i \), \( C_{M(i)} \) – the actual cost of materials involved in the activities \( i \), \( C_{M(i)}^\text{min}, C_{M(i)}^\text{max} \).

Since construction equipment is a factor that exerts a significant influence on the TSQ triangle, it is necessary to introduce a modification coefficient taking into consideration the impact of this equipment on the duration of the project:

\[ P_{E(i)} = P_{RD(i)} \cdot D_{E(i)}. \]  

(6)

where \( P_{RD(i)} \) is the actual performance of the activity \( i \); \( D_{E(i)} \) is a modification factor in productivity \( i \) that takes into consideration the parameters of machinery and equipment.

The modification coefficient can be derived from the relationship between the quality of construction equipment and labor productivity:

\[ D_{E(i)} = D_{E(i)}^\text{min} + K_{E(i)} \left( Q_{E(i)} - Q_{E(i)}^\text{min} \right). \]  

(7)

where \( D_{E(i)} \) is a modification factor in performance \( i \); \( Q_{E(i)} \) – the actual level of quality of construction machinery and equipment \( i \) in the activities \( i \), \( Q_{E(i)}^\text{min}, Q_{E(i)}^\text{max} \); \( K_{E(i)} \) – the ratio of factors affecting the performance of construction equipment:

\[ K_{E(i)} = \left( D_{E(i)}^\text{max} - D_{E(i)}^\text{min} \right) / \left( Q_{E(i)}^\text{max} - Q_{E(i)}^\text{min} \right). \]  

(8)

where \( D_{E(i)}^\text{min}, D_{E(i)}^\text{max} \) – the minimum and maximum quality of construction machinery and equipment \( i \) in activities \( i \).

To determine the relationship between the quality of equipment and its cost, it is worth using the same linear dependence as for materials and products:

\[ C_{E(i)} = C_{E(i)}^\text{min} - K_{E(i)} \left( Q_{E(i)} - Q_{E(i)}^\text{min} \right). \]  

(9)

\( K_{E(i)} \) is the coefficient of the ratio of factors affecting the quality of construction equipment:

\[ K_{E(i)} = \left( C_{E(i)}^\text{min} - C_{E(i)}^\text{max} \right) / \left( Q_{E(i)}^\text{max} - Q_{E(i)}^\text{min} \right). \]  

(10)

where \( C_{E(i)}^\text{min}, C_{E(i)}^\text{max} \) are the minimum and maximum costs of construction machinery and equipment \( i \) in activities \( i \); \( C_{E(i)} \) – the actual cost of construction machinery and equipment involved in the activities \( i \), \( C_{E(i)}^\text{min}, C_{E(i)}^\text{max} \).

To objectively determine the cost of overtime work, it is necessary to use the coefficient of change in the cost of construction equipment and equipment for overtime work (associated with an increase in the number of used construction equipment and machinery):

\[ \alpha_i = \left[ 1 + \left( K_{D(i)} - 1 \right) K_{E(i)} \right]. \]  

(12)

\( K_{D(i)} \) is the coefficient of decrease in productivity of work (we take equal to 20%).

The relationship between the quality and cost of work management can be described by the formula:

\[ C_{A(i)} = C_{A(i)}^\text{min} - K_{A(i)} \left( Q_{A(i)} - Q_{A(i)}^\text{min} \right). \]  

(13)

where \( Q_{A(i)} \) is the actual level of quality of work administration \( i \) in activities \( i \), \( Q_{A(i)}^\text{min}, Q_{A(i)}^\text{max} \); \( K_{A(i)} \) is the minimum and maximum level of quality of work \( i \) administration in activities \( i \); \( K_{A(i)} \) is the coefficient of the ratio of factors influencing management:

\[ K_{A(i)} = \left( C_{A(i)}^\text{min} - C_{A(i)}^\text{max} \right) / \left( Q_{A(i)}^\text{max} - Q_{A(i)}^\text{min} \right). \]  

(14)

\( C_{A(i)}^\text{min}, C_{A(i)}^\text{max} \) are the minimum and maximum cost of administration of work \( i \) in activities \( i \); \( C_{A(i)} \) – the actual cost of administering work \( i \) in activities \( i \), \( C_{A(i)}^\text{min}, C_{A(i)}^\text{max} \).

It is determined that during overtime work the cost of operation of construction equipment and machinery for relevant works increases. The cost of administering works, in this case, will also increase. Therefore, to objectively determine the cost of administering work overtime, it is necessary to introduce an appropriate coefficient of increase in the cost \( \beta \):

\[ \beta_i = \left[ K_{A(i)} + 1 \right] / K_{A(i)}. \]  

(16)

where \( K_{A(i)} \) is the hourly coefficient of administration of overtime work (we accept equal to 2.0).

The time required for the implementation of certain activities is determined based on the volume of work, actual productivity, and factors that determine overtime:

\[ D_{UR(i)} = Q_{UR(i)} / \left( P_{RD(i)} \cdot D_{UR(i)} \right). \]  

(17)

where \( D_{UR(i)} \) is the time of activity \( i \); \( Q_{UR(i)} \) is the scope of activity \( i \); \( P_{RD(i)} \) is the actual performance in activities \( i \); \( K_{D(i)} \) is the overtime rate, \( K_{D(i)} \in [1.0; 1.5] \) if overtime work is performed from 0 to 4 hours per day (since the working day is 8 hours per day), \( K_{D(i)} = 1 \) if there is no overtime work.

The cost of labor is determined according to the cost of the activity \( i \) and overtime (if any):

\[ C_{L(i)} = C_{L(i)}^\text{min} \cdot \left( K_{D(i)} - 1 \right) / \left( K_{A(i)} + 1 \right) \left[ C_{L(i)}^\text{max} \cdot \left( K_{D(i)} - 1 \right) / K_{A(i)} \right] \times \left( \left[ 1 - \left( K_{D(i)} - 1 \right) K_{A(i)} \right] / \left( -K_{A(i)} \right) \right)^{\left( K_{A(i)} + 1 \right)} \times \left( K_{A(i)} + 1 \right) \left( -K_{A(i)} \right). \]  

(18)
where \( C_{LE_i} \) is the cost of labor in activities \((i)\); \( C_{LE0} \) - the cost of work per unit of time (example, day) of activity \((i)\); \( R_{LE(i)} \) is the coefficient of consideration of the cost of work \((i)\) activities in overtime (we take equal to \(2.0\)).

Having determined the main relationships of indicators (factors) of resources, it is possible to definitively describe the model proposed in [9]. In this case, the total time \((T_{W(i)})\) is determined from the formula:

\[
T_{W} = \max_{n\in\mathbb{N}} \left( EST_{0(i)} + D_{iR(i)} \right). \tag{19}
\]

where \( EST_{0(i)} \) is the early commencement of activity \((i)\), which corresponds to the end of the previous work.

The total cost \((C_{W})\):

\[
C_{W} = \sum_{i=1}^{n} \left( C_{LE(i)} + C_{M(i)} + C_{E(i)} + C_{A(i)} \right). \tag{20}
\]

where \( n \) is the volume of operations. The overall quality \((Q_{W})\):

\[
Q_{W} = \sum_{i=1}^{n} \left[ W_{i} \cdot Q_{PW(i)} \right]. \tag{21}
\]

where \( W_{i} \) is an indicator of the quality of each activity \((i), \sum_{i=1}^{n} W_{i} = 1.0; Q_{PW(i)} \) is the quality of activity \((i)\), calculated taking into consideration the quality of work, materials, products, construction machinery and equipment, administration:

\[
Q_{PW(i)} = L_{WT}, Q_{4(i)} + M_{WT}, Q_{5(i)} + E_{WT}, Q_{6(i)} + A_{WT}, Q_{7(i)}, \tag{22}
\]

\( L_{WT}, M_{WT}, E_{WT}, A_{WT} \) - the indicators of quality of works, materials, and products, construction machinery and equipment, administration of activities \((i)\); \( Q_{4(i)}, Q_{5(i)}, Q_{6(i)}, Q_{7(i)} \) - the quality of works, materials, and products, construction machinery and equipment, administration of activities \((i)\).

The project can be considered successfully implemented only when it is executed in the shortest possible time in accordance with the requirements for the quality of work and without cost overruns, that is, within the framework of the budget specified in the contract [10, 11].

Thus, the proposed model of cost management, the quality and duration of road projects, formulated on the basis of graph theory, can be calibrated by introducing additional parameters and coefficients [12, 13]:

\[
\begin{align*}
X' &\in f \left\{ T_{W(i)}; T_{W(i)}; X; k_{G}; k_{W(i)}; k_{f(i)} \right\} \\
Y' &\in f \left\{ C_{W(i)}; C_{P(i)}; C_{R(i)}; C_{A(i)}; C_{T} \right\} \\
Z' &\in f \left\{ Q_{W(i)}; Q_{M(i)}; Q_{E(i)} \right\} \tag{23}
\end{align*}
\]

where \( X' \) is an optimized time value (\( T' \)); \( Y' \) is the optimized cost value (\( C' \)); \( Z' \) is the optimized quality value (\( Q' \)); \( T_{W(i)} \) is the duration of the main (planned) work; \( T_{W(i)} \) is the duration of unplanned and auxiliary work; \( k_{G} \) - a coefficient that takes into consideration delays due to the peculiarity of climatic conditions; \( k_{W(i)} \) - a coefficient that takes into consideration the productivity of working personnel; \( k_{A(i)} \) - a coefficient that takes into consideration other factors that affect the time of work; \( C_{W(i)} \) is the cost of work \((C_{LE(i)}; C_{M(i)}; C_{E(i)}; C_{A(i)}; C_{T})\); \( C_{P} \) is the profit; \( C_{T} \) - the general production costs; \( C_{R(i)} \) is the amount (cost) of fines and incentive payments; \( k_{f} \) - a coefficient that takes into consideration the risks of all parties to the contract; \( k_{W(i)} \) - a coefficient that takes into consideration the level of inflation; \( k_{A(i)} \) - a coefficient that takes into consideration the level of complexity of work; \( k_{P(i)} \) - a coefficient that takes into consideration the priority of financing the contract; \( Q_{W(i)} \) - the level of quality of work; \( Q_{M(i)} \) - the level of quality of materials and products; \( Q_{E(i)} \) - the level of quality of equipment and machinery; \( Q_{A(i)} \) - the level of administrative quality.

The model graph is described by formula (24) with the structure of the nodes from Table 2 [14]:

\[
G(K) = \left\{ T_{W(i)}; T_{M(i)}; T_{W(ii)} \right\} \tag{24}
\]

where \( K = \{0, ..., n+1\} \) is the number of nodes (activities, assets).

| Characteristics of the nodes of the graph of the built model [14] |
|------------------|------------------|------------------|------------------|------------------|
| Project (Contract) | Work (W) | Work resources (R_W) | Time-Cost-Quality |
|\( W(1) \) | \( W(i) \) | \( R_{W(1)} \) | \( R_{W(i)} \) | \( T_{W(i)} \) | \( T_{W(ii)} \) | \( n/e \) | \( \Sigma T_{W} \) | \( n/e \) | \( \Sigma C \) |
| Cost \( C_{L(i)} \) | Cost \( C_{M(i)} \) | Cost \( C_{E(i)} \) | \( C_{T} \) | \( \Sigma C_{W} \) | \( C_{A(i)} \) | \( \Sigma Q \) |
|\( Q_{E(i)} \) | \( Q_{M(i)} \) | \( Q_{E(i)} \) |

Note: \( n/e \) – the indicator is not taken into consideration; \( C_{T} \) - general production costs; \( C_{A} \) - administrative expenses of road services; \( C_{P} \) – profit; \( R_{W(1)} \); \( R_{W(i)} \); \( R_{W(ii)} \) – human, material, machinery, and equipment resources, respectively.
Based on research, an algorithm for applying the cost and duration management model of highway projects in conditions of uncertainty has been constructed (Fig. 3).

Optimization of model parameters is performed using a hybrid (multi-object) genetic algorithm, based on the approach reported in [15]. At the same time, the objective optimization functions were determined, taking into consideration the approach of multipurpose optimization of this problem by two methods [13]:

- by the method of constants:

\[
Z = \omega_1 \left[ \frac{T - T_{\text{min}} + \gamma}{T_{\text{max}} - T_{\text{min}} + \gamma} \right]^+ + \omega_2 \left[ \frac{C - C_{\text{min}} + \gamma}{C_{\text{max}} - C_{\text{min}} + \gamma} \right]^+ + \omega_3 \left[ \frac{Q - Q_{\text{min}} + \gamma}{Q_{\text{max}} - Q_{\text{min}} + \gamma} \right]^+ \\
\rightarrow \min, \quad (25)
\]

where \( \omega_1, \omega_2, \text{and } \omega_3 \) are coefficients (adaptive values, respectively, of time, cost, and quality); \( \gamma \) is a constant (determined empirically according to the statistical data of such projects, that is, by the method of analogs);

- by the method of a sum of squares of deviation:

\[
U = \frac{1}{2} \left[ \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \right]^2 + \omega_2 \left[ \frac{C - C_{\text{min}}}{C_{\text{max}} - C_{\text{min}}} \right]^2 + \omega_3 \left[ \frac{Q - Q_{\text{min}}}{Q_{\text{max}} - Q_{\text{min}}} \right]^2 \\
\rightarrow \min, \quad (26)
\]

where \( T, C, Q \) are, respectively, the total time, cost, and quality of the project, which are determined taking into consideration formulas (19)–(21), graph (24), and the algorithm for applying the cost and duration management model of road projects (Fig. 3):

\[
T = T_w \cdot \left( 1 + \sum_{i=1}^{s} (k_{a(i)} + k_{b(i)} + k_{d(i)}) \right) + T_{\text{AB}};
\]

\[
C = C_w \cdot \left( 1 + \sum_{i=1}^{s} (k_{a(i)} + k_{b(i)} + k_{d(i)}) \right) + \sum_{i=1}^{s} C_{\text{EI}(i)} + C_p + C_T;\]

\[
Q = Q_w.\]

The main conditions (boundaries) of the model are formulated for objective functions (25), (26):

\[
\begin{align*}
Z_1 &= T \rightarrow \min; T \neq 0; \\
C \cdot X \leq C_{\text{max}}; & \quad S \cdot C \geq (C - C_p) \cdot C \rightarrow \min; C \neq 0; \\
Q \cdot X \geq Q_{\text{min}}; & \quad Q \rightarrow \max; Q \neq 0,
\end{align*}
\]

where \( C_{\text{max}} \) is the maximum cost indicator; \( Q_{\text{min}} \) – the minimum quality requirements in the project (service levels);

\[
X = \left[ K_{\text{prot}}(1) - K_{\text{prot}}(4); Q_{\text{min}}(1) - Q_{\text{max}}(1); Q_{\text{min}}(2) - Q_{\text{max}}(2); Q_{\text{min}}(3) - Q_{\text{max}}(3) \right]^T
\]

is the vector of all variables of the model.

Having determined all the initial parameters and specifying constraints and assumptions, a genetic algorithm is chosen as a method by which a multipurpose problem of optimizing time, cost, and quality can be solved.

Based on the results of this study, the author’s software module DCtcq (Ukraine) was born to run the model on the Eclipse Java platform (USA) with further integration into MS Excel (USA). The process of running the model occurs until the required response is obtained, that is, compliance with the objective function, and includes the following main stages:

1. Creating a random solution for a parameter (the first generation of the genetic algorithm \( G=1 \)):

\[
d^0_{ij} = d^{(1)}_{ij} + r^0_{ij} (d^{(1)}_{ij} - d^{(1)}_{ij}), \quad \forall i = 1...s, \; \forall j = 1...n,
\]

where \( r^0_{ij} \) is the evenly distributed random values in the range from 0.0 to 1.0; \( d^{(1)}_{ij} \) and \( d^{(1)}_{ij} \) are the upper and lower limits of the variable \( d_{ij} \) (symbol of the parameters that change in the TCQ model).

2. The stage of determining the mutated (weighted) vector \( (w_i) \), which is determined by the product of the mutation coefficient \( F \) and the differential vector, determined by the difference from the two randomly selected vectors from the set:

\[
w^C_{ij} = F (d^{C}_{ij} - d^{C}_{ij}), \quad \forall i = 1...n, \; \forall j = b, \quad (30)
\]

where \( F \) is the mutation coefficient, which is usually accepted in the range from 0.4 to 1.0; \( a, b \) is the variable number, \( a \in [1; p], \; b \in [1, p]; \; p \) is the population size.

Fig. 3. Algorithm of application of cost management model and duration of highway projects [14].

Note: \( Q_{\text{min}} \) – minimum quality requirements in the project (service levels)
Control processes

3. Build population \( p(G+1) \):

\[
d^{(G+1)}_{i,j} = \begin{cases} 
   d^{(G)}_{i,j} + m^i_j & \text{if } r_{i,j} \leq \xi, \forall j = 1 \ldots n, \\
   d^{(G)}_{i,j} & \text{otherwise}, 
\end{cases}
\]

(31)

where \( \xi \) is the cross-coding constant that contributes to the creation of a new generation, \( \xi \in [0;1] \).

4. Create a new generation \( p(G+1) \) population that meets conditions (28) of objective function (25) or (26):

\[
x^{(G+1)}_{i,j} = \begin{cases} 
   x^{(G)}_{i,j} & \text{if } f(d^{(G)}_{i,j}) \in Z \text{ or } f(d^{(G)}_{i,j}) \in U, \\
   x^{(G)}_{i,j} & \text{otherwise}. 
\end{cases}
\]

(32)

The use of the mathematical model and the algorithm (Fig. 3), which were the basis for the development of the relevant DCtceq software (Ukraine), will in practice make it possible to simplify the process of determining the duration and cost of road maintenance projects at a given limitation, at a certain level of quality.

5. 3. Testing the built model on the example of the long-term maintenance project of a highway

The implementation of the project of maintenance of the highway is preceded by a visual examination of its elements. In particular, as a result of the survey of the experimental section of a highway, the destruction and deformation of the existing road surface are registered. These are potholes of crumbling, track, a grid of cracks, common transverse cracks, destruction of the edges of roadbeds, and subsidence of roadbeds (Fig. 4).

The next step is to evaluate the indicators and optimize by using the model built. The data for optimizing the operational maintenance project are given in Table 3. The cost indicators of the initial data \( C_{Q(i)}, C_{Q(NO)}, C_{A(i)}, C_{A(NO)} \) were calculated on the basis of the estimated cost of the work performed \((A-F)\) per unit of the specified amount of damage elimination work \( Q_{NO} \). Quality indicators \( Q \) are set in the form of dimensionless value (coefficient) according to the standards of road maintenance.

Fig. 4. Results of visual inspection of the road section (example): a – a grid of cracks with pit repairs; b – transverse cracks

![Fig. 4](image)

Data to optimize the cost and duration of the maintenance project (example)

<table>
<thead>
<tr>
<th>N</th>
<th>ID</th>
<th>( Q_{NO} )</th>
<th>( L_{PDN} ), min-max</th>
<th>Workforce ((L))</th>
<th>Materials and products ((M))</th>
<th>Construction machinery and equipment ((E))</th>
<th>Administration ((A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>260 m³</td>
<td>110–160 m³/day</td>
<td>770</td>
<td>0.8–1.0</td>
<td>3520–4900</td>
<td>660–720</td>
</tr>
<tr>
<td>2</td>
<td>120 m³</td>
<td>11–13 m³/day</td>
<td>330</td>
<td>0.8–1.0</td>
<td>880–1220</td>
<td>122–132</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>3</td>
<td>18 t</td>
<td>0.7–0.9 t/day</td>
<td>660</td>
<td>0.7–1.0</td>
<td>6100–7950</td>
<td>144–158</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>4</td>
<td>57 m³</td>
<td>17–22 m³/day</td>
<td>825</td>
<td>0.9–1.0</td>
<td>20100–25600</td>
<td>165–198</td>
<td>0.9–1.0</td>
</tr>
<tr>
<td>5</td>
<td>83 m³</td>
<td>9–13 m³/day</td>
<td>1870</td>
<td>0.8–1.0</td>
<td>17100–22000</td>
<td>1650–1980</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>6</td>
<td>66 m³</td>
<td>22–27 m³/day</td>
<td>473</td>
<td>0.8–1.0</td>
<td>7100–9400</td>
<td>132–163</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>7</td>
<td>132 m³</td>
<td>9–12 m³/day</td>
<td>495</td>
<td>0.7–1.0</td>
<td>13200–15800</td>
<td>230–255</td>
<td>0.9–1.0</td>
</tr>
<tr>
<td>8</td>
<td>3.5 t</td>
<td>0.9–1.3 t/day</td>
<td>880</td>
<td>0.7–1.0</td>
<td>12300–15800</td>
<td>282–310</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>9</td>
<td>53 m³</td>
<td>8–11 m³/day</td>
<td>1100</td>
<td>0.7–1.0</td>
<td>18600–24200</td>
<td>220–264</td>
<td>0.9–1.0</td>
</tr>
<tr>
<td>10</td>
<td>68 m³</td>
<td>5–7 m³/day</td>
<td>825</td>
<td>0.8–1.0</td>
<td>14400–18300</td>
<td>1380–1638</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>115 m³</td>
<td>9–12 m³/day</td>
<td>500</td>
<td>0.7–1.0</td>
<td>1210–1760</td>
<td>198–220</td>
</tr>
<tr>
<td>2</td>
<td>3.1 t</td>
<td>0.9–1.3 t/day</td>
<td>890</td>
<td>0.7–1.0</td>
<td>11500–13800</td>
<td>246–277</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>3</td>
<td>47 m³</td>
<td>8–11 m³/day</td>
<td>1150</td>
<td>0.7–1.0</td>
<td>16500–21350</td>
<td>198–230</td>
<td>0.9–1.0</td>
</tr>
<tr>
<td>4</td>
<td>60 m³</td>
<td>4.5–7 m³/day</td>
<td>830</td>
<td>0.8–1.0</td>
<td>12800–16100</td>
<td>1210–1450</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>5</td>
<td>105 m³</td>
<td>9–12 m³/day</td>
<td>320</td>
<td>0.7–1.0</td>
<td>1210–1760</td>
<td>198–230</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>6</td>
<td>2.4 t</td>
<td>0.9–1.3 t/day</td>
<td>890</td>
<td>0.7–1.0</td>
<td>8500–11900</td>
<td>192–220</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>17</td>
<td>41 m³</td>
<td>8–11 m³/day</td>
<td>1130</td>
<td>0.7–1.0</td>
<td>14600–18800</td>
<td>193–210</td>
<td>0.9–1.0</td>
</tr>
<tr>
<td>18</td>
<td>55 m³</td>
<td>4.5–7 m³/day</td>
<td>830</td>
<td>0.8–1.0</td>
<td>11900–14600</td>
<td>1100–1320</td>
<td>0.8–1.0</td>
</tr>
<tr>
<td>19</td>
<td>146 m³</td>
<td>19.5–26 m³/day</td>
<td>340</td>
<td>0.7–1.0</td>
<td>3550–4200</td>
<td>44–55</td>
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</tr>
<tr>
<td>20</td>
<td>66 m³</td>
<td>44–55 m³/day</td>
<td>830</td>
<td>0.7–1.0</td>
<td>7700–9400</td>
<td>110–132</td>
<td>0.8–1.0</td>
</tr>
</tbody>
</table>

Note: \( N \) – groups of works; \( ID \) – works (activities) \( Q_{NO} \) – volume of work; \( A \) – elimination of blockage; \( B \) – restoration of signs; \( C \) – work on the elimination of effusion of binders; \( D \) – elimination of cracks; \( E \) – elimination of pits; \( F \) – markup.
To determine the quality of work, indicators of groups of works were used (Table 4).

<table>
<thead>
<tr>
<th>N</th>
<th>GWT</th>
<th>ID</th>
<th>WT</th>
<th>LWT(0)</th>
<th>MWT(0)</th>
<th>EWT(0)</th>
<th>AWT(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.06</td>
<td>1</td>
<td>0.04</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.24</td>
<td>2</td>
<td>0.06</td>
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</tr>
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<td>6</td>
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<tr>
<td>C</td>
<td>0.18</td>
<td>7</td>
<td>0.04</td>
<td>0.1</td>
<td>0.7</td>
<td>0.15</td>
<td>0.05</td>
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<td>D</td>
<td>0.22</td>
<td>12</td>
<td>0.05</td>
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<td>0.05</td>
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<td>14</td>
<td>0.04</td>
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<tr>
<td>E</td>
<td>0.17</td>
<td>15</td>
<td>0.04</td>
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<td></td>
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<td></td>
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<td>16</td>
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<td></td>
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<td></td>
<td></td>
<td>18</td>
<td>0.05</td>
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<tr>
<td>F</td>
<td>0.13</td>
<td>19</td>
<td>0.06</td>
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</tr>
<tr>
<td>Σ</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: GWT – a quality indicator for work groups; WT – the level of quality of work; LWT(0) – a quality indicator for the workforce; MWT(0) – a quality indicator for materials and products; EWT(0) – a quality indicator for construction machinery and equipment; AWT(0) – quality score for administration

With the help of the optimization module DCtcq (Ukraine), developed by the authors, which was built on the basis of the algorithm for applying the cost and duration management model of road projects and implemented by the MS Excel macro studio (USA), the selection of parameters of this project was carried out (Table 5). The criteria for the optimization process that are accepted are the probability of a crossover (εr=0.5) and a mutation coefficient (F=0.8).

Thus, if one sets the quality level to 0.98 and above, and the cost is 15 million monetary units, the project implementation time will be 172 days.

Table 5 makes it possible to determine the numerical dependence of the elements of the triangle YCQ for a given project, taking into consideration the specified factors. After analyzing the calculation data given in a given table, one can clearly determine over which most optimal time and at lower cost a contract can be implemented in compliance with the required level of quality. For example, if a project manager needs to implement a contract in about T=160–170 days, and the quality level should not be lower than Q=0.8, he can use this model to calculate the approximate cost of implementing the contract, which for a given example will be C=8.0–11.0 million monetary units. In the same way, one can determine the time if the cost is fixed, or, based on time and cost, find out with at what level of quality the road maintenance project will be implemented.

The next step was to perform a test of the adequacy of the model according to Fisher’s criterion. The data that were obtained on the quality level indicator were compared with the project data (Table 6).

Table 6

<table>
<thead>
<tr>
<th>No. of entry</th>
<th>Estimated value, Q(x)</th>
<th>Project value Q(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.74</td>
<td>0.70</td>
</tr>
<tr>
<td>2</td>
<td>0.76</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>0.75</td>
</tr>
<tr>
<td>4</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>5</td>
<td>0.82</td>
<td>0.80</td>
</tr>
<tr>
<td>6</td>
<td>0.84</td>
<td>0.80</td>
</tr>
<tr>
<td>7</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>8</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>9</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>10</td>
<td>0.92</td>
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<td>11</td>
<td>0.94</td>
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<td>12</td>
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</tr>
<tr>
<td>13</td>
<td>0.98</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The check was performed by the built-in functions of Excel (Table 7).
Thus, the model for determining the cost and duration of road maintenance projects was tested for adequacy using the Fisher criterion with a significance level of 0.05.

### 6. Discussion of results of testing the simulation model for assessing the effectiveness and risks of long-term contracts

Analysis of the experience in determining the cost and duration of projects showed the need to build an original model for managing the cost and duration of road projects, which is based on the main characteristics of the «silver triangle» (Fig. 1). According to the results of the theoretical study, a conceptual model for managing the cost and duration of projects for the maintenance of highways was built (Fig. 2). In this case, the factors of influence are grouped into 4 classes.

A mathematical model of cost management and duration of projects on maintenance of highways (24), (25) has been constructed, which, unlike the traditional approach, is based on the main provisions of graph theory and structural approach (Table 2). The main purpose of the model development is to determine the main parameters of the project of long-term maintenance of roads, such as time and cost at a given quality of work (Fig. 3). This will take into consideration the uniqueness of each individual project by determining the likely impact of different groups of factors on the cost, duration, and quality on the principle of pairwise comparison. The built model harmoniously combines the mathematical method and the heuristic approach to solve the problems of optimizing the parameters of the «silver triangle».

The cost and duration management model of road maintenance projects is implemented in the Excel environment in the form of a separate optimization module. When testing the model according to a real project, all works and services were divided into 6 groups (Table 3). As a result of modeling, an individual matrix of interdependence of the components of the «silver triangle» for this project was built (Table 5). The adequacy of modeling the parameters of the project, which was tested by Fisher’s criterion with a level of significance of 0.05, showed a deviation of 3% from tabular values. This confirms the reliability of the achieved actual data by calculations. Thus, the level of quality of modeling makes it possible for project managers to perform effective management of road maintenance projects.

The main limitation of the use of the devised model is the problem of determining the parameter «quality» in the triangle TCQ. In the further development of this study, it is proposed to pay attention to the optimization of the time-cost ratio when varying the «quality» parameter.

### 7. Conclusions

1. A conceptual model for managing the cost and duration of road maintenance projects was built, which is based on the main provisions of the «silver triangle» theory. In contrast to the existing approaches, it was proposed to apply the main provisions of the «silver triangle» to the processes of cost, duration, and quality management of projects in the road industry.

2. A mathematical model for managing the cost and duration of projects for the maintenance of highways based on the allocation of resources for their implementation, which is based on graph theory and structural method, has been constructed. The mathematical model of project cost and duration management and the algorithm for determining cost and duration in the aggregate were the basis for the development of the author’s software module on the Eclipse Java platform (USA) with further integration into MS Excel (USA). This makes it possible to simplify the process of determining the duration and cost of road maintenance projects at specified restrictions, at a certain level of quality. Determining the parameters of the road maintenance project, such as cost, duration, and quality, has a multiplier effect, which is expressed in reducing the administrative costs of the customer, reducing the responsibility of road services, creating prerequisites for stable financing of road works, the satisfaction of road users, creating strong partnerships between the customer and the contractor.

3. The model built was tested on the example of a real project of long-term maintenance of a section of the highway. The calculated values of the model parameters were compared with the design values and tested for adequacy using the Fisher criterion with a significance level of 0.05. According to the results of the test, it was determined that the estimated error according to the model was about 3%, and the standard deviation was 5%, which confirms the reliability of the data obtained. This means that our studies are of practical value and can be used by road organizations at the stage of designing services and works on road maintenance using an automated calculation based on the developed DCtcq software (Ukraine).

### References


