1. Introduction

Ensuring the safety of gas supply stimulates the development of gas storage facilities to ensure against the risks of interruptions in gas supply and fluctuations in gas demand. There are 662 underground storage facilities in the world with a total volume of 421 billion cubic meters. Their maximum volume capacity of gas withdrawal is 7.3 billion cubic meters per day. Among the first three major global players in the gas storage market are the United States, Russia, and Ukraine [1]. Natural gas storage facilities have become an economic component of the global natural gas supply chain. They ensure the efficiency of commercial cycles in the gas industry and improve the economy of gas supply.

During the operation of UGSF, problems of optimal UGSF operation under gas compression modes and tasks of ensuring maximum UGSF performance at projected time intervals arise. Classical methods for solving them are not effective or unsuitable. They usually require continuity, often monotony (convexity, concavity), multiple differentiation of...
Industry control systems

the characteristics of processes, etc. The characteristics being investigated are nonlinear, with possible breaks and jumps. The processes of UGSF operation are influenced by predicted, poorly predicted, discrete, and continuous factors. To solve the tasks, it is necessary to develop universal combinatorial algorithms – methods of combinatorial optimization [2, 3].

The complexity of solving optimization tasks depends on the complexity of the simulation. Mass transfer processes in UGSF objects (Fig. 1) are described by gas-dynamic models on network-type structures and filtration models in heterogeneous porous multilayered environments with geological faults (equations of mathematical physics). Taking into consideration the work of each object in technological connection with others provides the ability to manage processes to achieve the specified optimality criteria. Representation in terms of the theory of graphs of the model of the structure of UGSF provide variants of UGSF operation, as well as initial and boundary conditions.

Fig. 1. Schematic representation of the piping and instrumentation diagram of an underground gas storage facility

The UGSF model is described by a system of equations with a different type of mathematical representation and a set of algorithms (simulation models). Methods of implementation of such models require the development of special analytical-numerical and combinatorial methods.

During periods of sharp increase in gas consumption, the reliability of GTS operation depends on its maximum productivity. The maximum productivity of each storage facility is influenced by many factors, including the volume of active gas in the reservoir and pressures in gas pipelines-branches. The process of managing the processes of gas withdrawal, especially under the conditions of insufficiency of gas in UGSF and the lack of imports due to its high cost, is significantly complicated. Effective management of UGSF in such cases requires the development of optimization methods. Methods of UGSF optimal management ensure at once the saving of fuel and energy resources and, accordingly, reduce the volume of harmful emissions into the environment.

The current global trend is the sharp increase in energy and the problems generated by the unbalanced development of multi-energy systems. One of the directions of balanced development of multi-energy systems is hydrogen energy. For the accumulation of produced hydrogen, it is planned to use underground storage facilities. Adaptation is carried out using the developed mathematical and software to control the processes of storage of hydrogen and its mixtures with natural gas.

Some problems that have arisen during the operation of UGSF and GTS are characteristic of many UGSF and GTS [4]. Despite the surplus of UGSF power during the cold snap in 2012, European UGSF could not cope with the uneven consumption of gas. The main issue is that there is no single control center for the operation of UGSF. After those events, sixteen major European operators joined forces to reserve the capacity of gas transmission systems.

The safety of gas supply depends on the stable operation of GTS. Additionally, the stable operation of GTS, under unstable foreign gas markets, can only be ensured by the effective operation of UGSF. The complexity of UGSF and GTS as a single hydraulic complex, as well as the complexity of process management at their facilities can only be provided by the relevant software. Therefore, the development of mathematical support and software for optimal planning of operational modes of UGSF is a relevant task.

2. Literature review and problem statement

The tasks under consideration are a continuation of work of optimal planning of the operating modes of UGSF and GTS. The reason for their emergence is a sharp increase in gas price at world hubs [3] and, as a result, problems with the volume of its imports. The developed methods to solve them are based on the results given in works [6–9]. Work [6] reports the development of an integrated mathematical model of an underground gas storage facility. The model describes the filtration and gas-dynamic processes that occur as a result of UGSF operation. A reservoir nonstationary filtration model of UGSF was proposed, whose component is stationary models of gas inflow to working wells. The cited paper does not provide a justification for the correctness of such a model. Work [7] is a continuation of the previous work. It tackles the development of methods for implementing UGSF models for the construction of an estimation complex. It should be noted that:

– the developed iterative method for solving nonlinear systems with different mathematical representations of equations ensures guaranteed convergence;
– the developed discrete-continuous algorithmic (simulation) model of multi-shop compressor stations (CSc) and the developed method of combinatorial optimization ensured the calculation of its optimal modes;
– the proposed approach to the combination of numerical and analytical methods for analyzing filtration processes in reservoirs ensured a high speed of obtaining results.

As real measured data have shown, a stationary model of gas inflow to wells does not guarantee enough accuracy in calculating their debits at significant time intervals.

The optimality of planning the operating modes of UGSF-GTS is influenced by the volume of accumulated gas in the GTS and its distribution in the system of main gas pipelines. In the autumn-winter period, it happens that up to 75% of the volume of gas in MG comes from UGSF. Therefore, the optimal and reliable operation of GTS mainly depends on the operation of UGSF.
Many works addressed the issues of building effective methods of optimal planning of GTS operation modes with a complex piping and instrumentation diagram (P&ID). Work [8] deals with the problem of assessing the potential of GTS optimization and real opportunities for its implementation. To ensure optimal mode, resources are needed both in terms of power and sources of change, if necessary, the volume of accumulated gas in the pipe part of GTS. There are no methods in the cited work for realizing the optimization potential, which requires the construction of a transitional control system.

Work [9] considers the tasks of optimizing the GTS according to the energy criterion and without changing the volumes of accumulated gas in the GTS. The optimization of work of both individual objects and individual subsystems of the GTS is investigated. The estimated results of the impact of optimization of individual subsystems on the optimality of the system as a whole are not given. The calculation of the parameters of flow distribution in the gas transmission system under the conditions of its operation under nonstationary modes of operation are given in [10]. The systems of differential equations with partial derivatives of large dimensionality were solved by a finite-element method at dynamic boundary conditions. In the process of solving the problem, technical and technological limitations and restrictions on pressures at controlled points of the system are taken into consideration. The proposed algorithm for editing (simplifying) P&ID provided an increase in the stability of solving equation systems, as well as significantly reduced the calculation time. The cited paper does not provide the results of the impact of simplification of P&IDs on the accuracy of the results obtained.

An overview of the most relevant studies related to stochastic optimization methods of stationary operation of gas pipelines is given in [11]. This method of optimizing the operation of gas pipelines is a method of mixed integer nonlinear programming, including continuous, discrete, and integer variable optimizations. It is claimed that many stochastic algorithms show better performance in solving optimization problems compared to classical deterministic optimization algorithms. The demonstration of the method is carried out on very simplified P&IDs with several CSs. Actual technological circuits often contain tens of thousands of objects with a mathematical model and dozens of active objects. Estimates of the dependence of the complexity of optimization algorithms on the dimension of GTS P&IDs are not given.

Paper [12] considers the implementation of the stochastic method of a set of evolution strategies for solving a wide class of stationary optimization problems. One of the main results of the cited work is the effective reduction of a large number of design parameters and replacing them with a smaller optimal number. The results were tested using a dynamic programming algorithm because it guarantees the achievement of a global optimum. The cited paper does not provide estimates of the impact of simplifying the system model on the degree of achievement of a global optimum.

The most relevant research work that needs to be carried out to solve the problem of gas transmission through pipelines are given in [13]. Optimization methods for both stationary and transitional modes of gas flow are considered. There are no results of the use of optimization methods for existing pipeline gas systems.

Suites for modeling technological chains, which include all objects in the path of gas flow from the reservoir to the gas pipeline, are mainly operated on gas fields. For this purpose, various software packages are used (PIPEPHASE by Aveva Software, LLC, USA [14], OLGA by Scandpower Petroleum Technology (SPT), Norway [15], PIPESIM by Schlumberger, USA [16]), which simulate the operation of individual subsystems – wells, gas collection networks, reservoirs, etc. These packages have not yet been brought to the state of modeling underground storage facilities as a single hydraulic system, which take into consideration all the features of its operation [17]. The main issues that arise in the process of developing estimation procedures are the complexity of the models and the incompleteness of their information support. There are significant difficulties in adapting models under conditions of uncertainty of their parametric descriptions, especially reservoir systems. Practically absent in the composition of the mentioned integrated software packages are compressor station models. This makes it impossible to set operational tasks in optimization statement.

There is a tendency to integrate the main directions of interpretation and modeling of data “seismic interpretation – geological modeling – hydraulic modeling” (examples of products that combine these areas include Petrel and ECLIPSE (Schlumberger), IRAP RMS (Roarax) [18]).

The demand for integrated systems (software segment for the SKUA-GOCAD geophysical service (Paradigm) [19]) is growing due to the possibility of a comprehensive solution to the problems of geophysical modeling at once. This trend will continue in the future due to the growing complexity of the tasks for the development of new fields.

The problem of the integrity of gas supply systems is tackled in [20], which explores the infrastructure of an underground gas storage facility for the energy reliability of its functioning. The authors of the cited work combined the process of monitoring the integrity and modeling of UGSF operation into a decision support system.

In work [21], an integral mathematical model of the process of nonstationary filtration of the gas-liquid mixture is constructed and the solutions to the obtained related equations are given. The resulting analytical formula makes it possible to determine the dynamics of the knocking pressure and the performance of a reservoir as a function of the system parameters. For modeling filtration processes in reservoirs with a complex geological structure, analytical models are not always suitable.

A relevant problem is discussed in [22]. It reports the results of economic feasibility of options for transporting and storing gas in underground storage facilities to balance the load between thermal and electric energies. The problem is multifactorial and its solution is complicated due to instability in the gas and electricity markets.

There are no generally accepted recommendations for the creation of UGSF models. The complexity of the development of models of reservoirs is to take into consideration the geological and physical features of reservoirs, water factor, information uncertainty, the multi-phase nature of filtration processes, etc. Some recommendations for building UGSF models are formulated in [23].

Most publicly available studies address partial problems of design and operation of UGSF, or problems of adaptation of well-known commercial packages that have been operated in hydrocarbon fields for decades. In the joint operation of UGSF and GTS as a single integrated complex, a significant synergistic effect is manifested.

To implement such an effect, it is necessary to ensure the further development of existing mathematical support and
software [6, 7] towards a more complete realization of the optimization potential, which is manifested in the case of planning UGSF and GTS modes as a single hydraulic complex. The above substantiates the expediency of conducting systematic research.

3. The aim and objectives of the study

The purpose of this study is to develop algorithms for the minimum complexity of solving the problems of optimal planning and operation of UGSF as part of GTS under the modes of gas pumping and gas withdrawal. This will ensure:

– operational and predictive planning of optimal modes of operation of UGSF for fuel gas and the required compatible maximum UGSF performance at projected time intervals;
– more stable operation of technological equipment that will prolong its service life;
– the process of assessing the quality of operating modes and the formation of parameters for their adjustment;
– economical consumption of fuel and energy resources;
– higher quality of dispatching services.

To accomplish the aim, the following tasks have been set:

– to build the functions of maximum productivity of gas storage facilities as a function of reservoir pressure and pressure in a gas pipeline-branch in the area of design and real modes of UGSF operation;
– for the projected volumes of gas storage, find such a distribution between storage facilities in order to ensure the total maximum productivity in the withdrawal process at the projected time intervals;
– for the projected volumes of gas withdrawal, to find such a distribution between storage facilities in order to maintain the maximum total productivity of storage facilities over the entire time of gas withdrawal;
– to implement a strategy for optimal management of gas storage facilities to ensure a balanced criterion, which includes the necessary productivity and costs of fuel and energy resources.

4. The study materials and methods

The object of our research is underground gas storage facilities as part of the gas transmission system. The subject of this research is the effective operation of underground storage facilities as part of the gas transmission system.

The hypothesis of the study assumed the following. It is necessary to distinguish between the effective operation of individual and joint operation of UGSF. Optimal joint operation is not always ensured by optimal operation of individual or all UGSF together. It is necessary to investigate the potential of effective joint operation of UGSF and propose algorithms for maximum realization of the identified potential under the real conditions of UGSF and GTS operation.

Some of the assumptions and simplifications were taken into consideration in the software packages [6, 7], which are involved in the process of solving the tasks set in the work. They relate mainly to ensuring the guaranteed convergence of iterative processes, the minimum complexity of sorting algorithms, and the process of modeling filtration and gas-dynamic processes under the existing uncertainty in many respects – the absence, incompleteness, and non-guaranteed accuracy of the measured data. The construction of hydraulic equivalents of complex subsystems of UGSF, averaging characteristics, not taking into consideration many insignificant factors of influence on processes in UGSF, simplification of P&IDs provided the necessary speed and quality of the results obtained.

The development of the functionality of mathematical support and, accordingly, the software GIMS and GTS Calculation [6] allowed us to calculate the peak characteristics of UGSF in the entire area of manufacturability of UGSF and GTS. The calculation was preceded by the process of updating mathematical support (calculation of adaptive parameters of models) according to operational regime data over 5–10 seasons of gas injection and withdrawal. The adequacy of gas flow models was ensured at controlled points of the UGSF P&ID. In the process of adaptation, factors of influence on hydraulic parameters have manifested themselves, which at daily time intervals are in the area of uncertainty of the existing metrological support.

The calculation of pressures in in gas pipelines-branch (the point of connection of UGSF to the main gas pipeline) was provided by the GTS Calculation software package [8, 9] for calculating the stationary and non-stationary mode of GTS with a complex P&ID.

Planning the operation of UGSF in the seasons of gas pumping and gas withdrawal is significantly different. The gas withdrawal season is closely related to the heating season. The volume of gas withdrawal is influenced by weather conditions, the reliability of forecasting which is quite approximate and often changes significantly. And, therefore, at the same time, they develop a long-term forecast of UGSF operation and operational for the current day.

As for research methods [7, 8]. The processes of gas flow in UGSF objects are described by gas-dynamic models in network-type structures and by filtration models in heterogeneous porous multi-layer environments with geological faults (equations of mathematical physics). Taking into consideration the work of each gas pumping unit (GPU) of UGSF is ensured by the representation of booster compressor station (BCS) in the form of simulated discrete models of the algorithmic type.

The model of the structure of UGSF and GTS, which are represented in terms of graph theory, depends on the mode of its operation and, therefore, constantly changes, which leads to a change in the model of the system, initial and boundary conditions (Fig. 2). Numerical methods and algorithms for solving such systems, which would reliably and quickly work in the entire field of process efficiency, are absent or not studied enough. Therefore, to find the parameters of the distribution of gas flows, adaptive numerical iterative methods for solving problems of mathematical physics are proposed.

All UGSF together with GTS form a single thermal-hydraulic complex. UGSF and GTS technologically combine gas pipeline branches. Gas pressure in gas pipeline branches is calculated at known volumes of withdrawal from GTS MG or pumping gas into UGSF. The amount of pressure in the gas pipeline branch affects the operation of UGSF components and, accordingly, the consumption of fuel gas under the mode of CS operation. In such cases, the calculation of the optimal operation of UGSF is carried out by known volumes of gas (withdrawal/injection), pressure distribution in the reservoirs and pressure in the gas pipeline branch. The optimal operation of UGSF is influenced by hydraulic losses in the components of the P&ID and the mode of CS operation.
5. Results of solving problems of planning the operational modes of underground gas storage facilities as part of the gas transmission system

5.1. Calculation of the peak characteristics of underground gas storage facilities

Problem 1. The initial values are known: $P_{mi}$ is the pressure in the main gas pipelines (on the gas pipeline branch of the $i$-th gas storage facility), $P_{ri}$ is the distribution of pressure in the reservoirs of gas storage facilities. It is required to find the maximum performance $V_i(P_{ri}, P_{mi})$ of gas storage facilities as a function of reservoir pressure and the pressure in the gas pipeline branch in the region of design and real modes of UGSF operation.

The software package “GTS Calculation” was used [6], which allows the calculation of gas volumes in the process of its withdrawal according to the input data – pressure or consumption at a gas gathering station (GGS). After calculating the unknown flow rate or pressure (what is not assigned), the gas parameters at the input of a BCS were obtained, which allows the calculation of the operating mode of BCS. Maximum gas withdrawal is provided by the maximum productivity of BCS. The choice of pressure in the technological range of changes at a gas collection point (GCP) (the existing pressure limit below for BCS GPU) allowed the calculation of the volume of gas withdrawal from the reservoirs at the maximum performance of BCS.

The scheme of the algorithm for calculating the maximum performance is shown in Fig. 3. If it is necessary to find the maximum performance of UGSF, the parameters at the input and output of BCS are calculated as follows (Fig. 3):

- the initial value of the pressure at GCP is set – the minimum gas pressure at the BCS input at which gas pumping units can still operate at input pressure (this is about 15 atm);
- the consumption at GCP is calculated (steps 5, 6);
- using the consumption found, the pressure is calculated at the output of BCS (steps 1, 2) and the pressure at the input of BCS (step 4);
- according to the calculated boundary conditions, the optimal mode of BCS operation is calculated;
- provided that the operating mode of BCS involves the consumption found at GCP that is of the maximum productivity for UGSF, otherwise it is necessary to increase the pressure on GCP by a specified value and it is necessary to repeat the cyclic process until there is a BCS mode with a given degree of stability.

The calculation process takes into consideration all technical, technological, and geological limitations. Maximum productivity should take into consideration the minimum withdrawal from the reservoirs at the maximum performance of BCS.

Fig. 2. Underground gas storage facility on a fragment of the piping and instrumentation diagram of the gas transmission system
distance from the pumping zone of workstation on the consolidated characteristics of GPUs. In the maximum UGSF performance, fuel gas volumes are not taken into consideration. To refine the calculation of gas withdrawal volumes, an iterative procedure is also needed to take into consideration the volumes of fuel gas. It is unknown at the initial stage of searching for the maximum performance.

The calculation of maximum productivity is given in the simplest case — for a single-reservoir system, with one gas gathering station and for BCS with gas turbine drives and centrifugal superchargers.

The optimal UGSF operation in the gas compression mode is ensured by BCS. Most UGSF employ multi-shop CSs. In multi-shop CSs, each workshop is equipped with the same type of GPU; there are different interchangeable impellers VN. The calculation of multi-shop CS implies such a distribution of gas flows between workshops, which makes it possible to achieve the total minimum energy costs. Suppose that the compressor station consists of \( n \) groups connected in parallel, each of which has \( m_i \) sequentially operating workshops \((i = 1, n)\). Workshops differ in different types of units, and, therefore, their limitations are also different. They differ in the following parameters:

- the region of regulation by the speed of centrifugal superchargers (CSs);
- the maximum and minimum permissible limits for the performance of units;
- limits on the volumetric performance of CS;
- limits on the efficiency for fuel gas of GPU drives and polytropic efficiency of CS.

For such CSs, it is necessary to establish additional restrictions

\[
\sum_{i=1}^{n} q_i = Q_i, \quad i = 1, 2, ..., n.
\]

\[
\sum_{i=1}^{n} Q_i = Q; \quad (1)
\]

\[
\sum_{j=1}^{k_i} \Delta P_{j} = P_{in} - P_{out} = \Delta P_i, \quad \Delta P_1 = \Delta P_2 = ... = \Delta P_s, \quad (2)
\]

where \( q_i \) is the value of the flow in the \( k_i \)-th supercharger of the \( j \)-th shop of the \( i \)-th group,

\( Q_i \) is the productivity of the \( i \)-th group,

\( \Delta P_{s_i} \) is the increase in pressure on the \( s_i \)-th stage of the \( j \)-th shop,

\( \Delta P_i \) on the \( i \)-th group of \( m_i \) sequentially operating workshops,

\( k_i \) is the number of stages of compression of gas in the \( j \)-th shop of the \( i \)-th group.

The functionality to minimize fuel and energy costs, in this case, will take the form

\[
F(\bar{r}, \bar{p}) = \sum_{j=1}^{s} \sum_{p=1}^{k} F_j(\bar{r}, \bar{p}). \quad (3)
\]
where $F_i$ is the consumption of fuel gas in the $i$-th shop of the $i$-th group;

$\mathbf{f}$ is the control vector, the components of which are parameters: the number of GPUs and the number of revolutions of CS, the position of valves on the P&ID of UGSF, the scheme of the system for collecting and preparing gas for operation;

$r$ is the mode vector, which is determined by the gas parameters at the input and output of UGSF.

To calculate the operating modes of CS, an algorithm for combinatorial optimization of minimal complexity is proposed. It provides a choice of GPU types, a minimum number of gas compression degrees, a minimum number of GPUs at each stage of gas compression, and the distribution of the compression value of the gas between the stages. Among the modes, one can identify stable (at a distance from the pumping zone of CS); energy efficient (for the consumption of fuel and energy resources), and those with weighted consideration of both criteria.

During the operation of UGSF, competing criteria for the quality of their operation arise – maximum performance and optimality. The optimality is ensured by such a distribution of $V(t)$ volumes of gas withdrawal between storage facilities $V_i(t)$ (for each distribution, pressures are calculated in gas pipeline branches) so that the total volumes of fuel gas

$$V = \sum_i V_i = \left( V_{\text{in}}, P_{\text{in}}, P_{\text{out}} \right)$$

are minimal. An optimal operation of UGSF over a considerable time can have a significant impact on the total peak intensity of UGSF at the predicted time intervals. At predicted time intervals that do not require the operation of UGSF in peak modes, UGSF operating modes are balanced by a combination of their optimality and maintaining the total productivity “close” to peak one.

Below is an example of calculating the maximum performance of one of the gas storage facilities [7]. If the compressor station is equipped with a GPU with gas turbine drives, then the characteristic, the maximum performance, can have both breaks and jumps (Fig. 4). The reservoir pressure $P$ is set in atm., and the volumes of gas withdrawal $Q$ – in millions of m$^3$/day.

With maximum UGSF productivity, energy costs per unit of selected gas increase significantly, so it is important to have an estimate of the consolidated energy (production) costs at different stages of UGSF operation. In the process of calculating peak characteristics, one also finds consumption energy values $V_{\text{in}}(V^*, P_{\text{in}}, P_{\text{out}})$. That makes it possible to obtain

$$A(P_{\text{in}}, P_{\text{out}}) = V_{\text{in}}(V^*, P_{\text{in}}, P_{\text{out}}) : V^*(P_{\text{in}}, P_{\text{out}}),$$

where $A(P_{\text{in}}, P_{\text{out}})$ is the reduced minimum fuel gas consumption per unit of gas volume;

$\bar{A}(Q_{\text{in}})$ is the average value of fuel gas consumption, provided that all active gas is taken away in minimal time.

Numerical experiment. It is required to assess the dependence between the volume of gas injection and fuel gas consumption at a reservoir pressure in the region of working wells of 40.5 atm and a pressure at the gas gathering station of 60.0 atm. The results, given in Table 1, demonstrate that the mode of UGSF operation with a daily consumption of 14.7 million m$^3$/d is optimal in terms of fuel gas consumption per unit of gas pumped volume.

![Fig. 4. Maximum productivity of Dashavskyi underground gas storage facility [24] under a gas withdrawal mode](image)

**Table 1**

<table>
<thead>
<tr>
<th>Gas volume, million m$^3$/day</th>
<th>Fuel gas volume, million m$^3$/day</th>
<th>Total gas consumption</th>
<th>Fuel gas consumption coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>0.058</td>
<td>0.0058</td>
<td>1.318</td>
</tr>
<tr>
<td>14.7</td>
<td>0.065</td>
<td>0.0044</td>
<td>1</td>
</tr>
<tr>
<td>21.0</td>
<td>0.116</td>
<td>0.0055</td>
<td>1.250</td>
</tr>
<tr>
<td>27.7</td>
<td>0.127</td>
<td>0.0045</td>
<td>1.022</td>
</tr>
</tbody>
</table>

The calculation of the optimal mode is illustrated in Fig. 5, 6 for the assigned volume of gas withdrawal – 1202,458 m$^3$/h. For the assigned volume of 28,859 m$^3$/day, reservoir pressure in the region of well operation and pressure at the input to BCS are calculated (Fig. 5). Based on the pressures at the inlet and outlet (pressure in a gas pipeline branch) of BCS and the withdrawal volume of 1202,458 m$^3$/d, the operating modes of BCS are calculated (Fig. 6). The list of possible modes of operation is given in the upper and lower left windows (Fig. 6). Among the operating modes of BCS, there is a mode that requires minimal fuel gas consumption (mode 01). To implement the regime, two-stage gas compression is required by four gas pumping units (GPUs) of the specified type – two GPUs at each stage with the corresponding revolutions of centrifugal superchargers.

UGSF operates CSs with different types of GPUs. For UGSF with reservoir pressures, in particular more than 100 atm, piston-type GPU is used. Most often, many-shop CSs with centrifugal superchargers with gas turbine drives work on UGSF. To ensure the required pressure difference between the CS input and output, several de-
grees of compression of the gas (up to three stages) are used. Fig. 6 shows the results of the calculation of modes with two-stage gas compression.

The maximum performance of UGSF is calculated for each day. The results of calculating the maximum performance of UGSF as of 25.12.2021 are shown in Fig. 7.

Fig. 5. Interface for solving direct and inverse optimization mode problems. Calculation of the mode of operation of the technological chain – reservoir – gas gathering station.

Fig. 6. Interface for calculating the operating modes of a multi-shop booster compressor station.
5.2. Algorithm for the distribution of projected volumes of gas storage among storage facilities

Problem 2. It is required that the projected volume of gas storage \( Q_a \) is distributed among \( i \) storage facilities \( Q_{ai} \) in such a way as to ensure their maximum total peak at the specified gas withdrawal time intervals. For each interval of time of operation in off-peak modes of operation of UGSF, their total volumes of gas withdrawal are known.

The steps of the algorithm for solving problem 2 are given.

Among \( n_{ugs} \) gas storage facilities, it is necessary to distribute the projected volume of active gas \( dQ_{sum} \) for injection so that during \( n_{day} \) days of peak withdrawal from all gas storage facilities at known measured main pressures \( P_{gm} \), the maximum volume of gas can be obtained.

1. For each \( i \) gas storage facility, the limits of the volume of active gas \( aQ_{min} \) (the initial volume of active gas) are established \( aQ_{max} \).

2. Calculate the step of breaking the volume of the active gas \( \left( \frac{aQ_{max} - aQ_{min}}{k} \right) \) for each UGSF, provided that the interval of the active gas is divided into \( k \) parts.

\[
dQ = \frac{aQ_{max} - aQ_{min}}{k}
\]

3. For each value of active gas \( Q_{aj} = Q_{min} + j \cdot dQ \) and measured pressures in the main gas pipeline, the volume of peak withdrawal for \( n_{day} \) days is calculated.

\[
peak_{\text{sum}}(Q_{min} + j \cdot dQ, P_{gm})
\]

4. It is necessary to sort out different options for the distribution of active gas among gas storage facilities:

\[
(m^i \cdot dQ^i),
\]

where \( m^i = 0...k \), which meet the following conditions:

\[
\sum_{i=1}^{n_{ugs}} m^i \cdot dQ^i < dQ_{sum},
\]

\[
dQ_{sum} - \sum_{i=1}^{n_{ugs}} m^i \cdot dQ^i < \max \{dQ^i\}.
\]

5. The unallocated volume of gas

\[
\left( dQ_{sum} - \sum_{i=1}^{n_{ugs}} m^i \cdot dQ^i \right)
\]

which will ensure the condition of equality \( \sum dQ^i = dQ_{sum} \), must be distributed among storage facilities by the method of coordinated descent.

It is necessary to take a certain small amount of the volume of active gas \( dq_a \) and add it to the active gas of the UGSF so as to get the maximum increase in the total peak activity

\[
\text{peak}_{\text{sum}}(Q^i + dq_a, P_{gm}^i) - \text{peak}_{\text{sum}}(Q^i, P_{gm}^i)
\]

This procedure must be repeated until all the assigned active gas is distributed \( \sum dQ^i = dQ_{sum} \).

Among all distributions \( \{dQ_i^i\} \), it is necessary to choose the distribution of active gas with maximum peak activity. The nature of the peak functions implies the pres-
ence of several local extremes of the total peak function. The above algorithm finds the distribution of active gas, in the vicinity of which, with a certain accuracy, it can be argued that there is an absolute extreme of the function of total peak activity. Repetition of the procedure in the vicinity of the found distribution of active gas will make it possible to approach the absolute extremity of the function of total peak activity.

An example of the above algorithm.

It is required to ensure the distribution of the projected volume (7 billion m$^3$) of gas among storage facilities in such a way as to ensure the maximum performance of UGSF during the assigned time interval. The results obtained for two time intervals are 20 and 50 days. For 20 days, the results are given in Table 2 and Fig. 8; for 50 days – in Table 3 and in Fig. 9. For the first case, the maximum productivity over 20 days varies in the range of 139–110, and in the second, within 50 days, it changes within 132–85 million m$^3$.

**Table 2**

<table>
<thead>
<tr>
<th>Underground gas storage facility</th>
<th>UGSF projected maximum active volume, million m$^3$</th>
<th>Initial active gas, million m$^3$</th>
<th>UGSF calculated volume of active gas, million m$^3$</th>
<th>includng:</th>
<th>Distributed process active gas, million m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uhers'ke UGSF</td>
<td>2,150</td>
<td>250</td>
<td>276</td>
<td>250</td>
<td>26</td>
</tr>
<tr>
<td>Bil'che-Volys'tk-Uhers'ke UGSF</td>
<td>20,750</td>
<td>3,700.00</td>
<td>7,860</td>
<td>3,700</td>
<td>4,160</td>
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<td>7,326</td>
<td>3,700</td>
<td>3,626</td>
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<td>2,700</td>
<td>0</td>
<td>534</td>
<td>0</td>
<td>534</td>
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<tr>
<td>Dashav'ske UGSF</td>
<td>2,772</td>
<td>622</td>
<td>742</td>
<td>622</td>
<td>120</td>
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<tr>
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<td>1,920</td>
<td>0</td>
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<td>80</td>
</tr>
<tr>
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<td>2,257</td>
<td>0</td>
<td>2,257</td>
</tr>
<tr>
<td>Olyshivs'ke UGSF</td>
<td>400</td>
<td>90</td>
<td>90</td>
<td>90</td>
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</tr>
<tr>
<td>Chervonopartyzans'ke UGSF</td>
<td>1,500</td>
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<td>96</td>
<td>0</td>
<td>96</td>
</tr>
<tr>
<td>Solohvs'ke UGSF</td>
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<td>113</td>
<td>0</td>
<td>113</td>
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<tr>
<td>Proletars'ke UGSF (M-7)</td>
<td>1,000</td>
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<td>80</td>
<td>0</td>
<td>80</td>
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<tr>
<td>Kehychivs'ke UGSF</td>
<td>700</td>
<td>0</td>
<td>58</td>
<td>0</td>
<td>58</td>
</tr>
<tr>
<td>Krasnopopivs'ke UGSF</td>
<td>420</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>35,212</td>
<td>4,662.00</td>
<td>11,662</td>
<td>4,662</td>
<td>7,000</td>
</tr>
</tbody>
</table>

**Fig. 8.** Maximum productivity of underground gas storage facilities within 20 days of their operation
5.3. Distribution of active gas storage volumes among storage facilities to ensure their maximum average peak activity

Problem 3. It is required that the projected volume of gas storage $Q_a$ is distributed among storage facilities in such a way as to ensure their maximum integral peak activity during the minimum operating time of all storage facilities (at the complete withdrawal of active gas).

The algorithm for solving a problem is based on such an obvious statement. The maximum average integral peak activity over the entire interval of UGSF operation is achieved if, with the maximum performance of all storage facilities, the withdrawal of active gas will be completed simultaneously (over the same time of withdrawal). The task of the algorithm is to distribute $Q_a$ among storage facilities so that the formulated condition is satisfied as accurately as possible. To ensure it, the algorithm at $k$ iterative step adds the quantity $\Delta Q = Q_a/n$ to $Q_a^{k-1} + \Delta Q$ – active gas in the $i$ storage facility if the maximum difference between the times of complete withdrawal of the active gas is minimal.

5.4. Optimal operation of underground gas storage facilities

Problem 4. Find $Q_{op} = \sum Q_{op} = Q_o$ to achieve the lowest total volume of fuel gas for the operation of UGSF $I_{op}(t)$ and integral peak activity $IV(t)$ not less than the assigned value at the time interval $[0, t]$,

$$I_{op}(t) = \sum_{x=1}^{n} [V_x(t), P_x(t), P_{op}(t)] dt \rightarrow \min,$$

$$IV(t) = \sum_{x=1}^{n} [V_x(t)] dt.$$  

The algorithm for solving the problem is to find, at each $k$ iterative step, such $i \left( Q_a^{k} = Q_a^{k-1} + \Delta Q \right)$, which would ensure the maximum increase in the peak activity $IV^k(t) - IV^{k-1}(t)$ with a slight increase in fuel gas $IV^k(t) - IV^{k-1}(t)$ until the required level of integral peak activity is reached. If the withdrawal of gas from one of the storage facilities under a peak mode reaches the time $t$, then it must be excluded from participation in the iteration process.

Table 3

<table>
<thead>
<tr>
<th>Underground gas storage facility</th>
<th>UGSF projected maximum active volume, million m$^3$</th>
<th>Initial active gas, million m$^3$</th>
<th>UGSF calculated volume of active gas, million m$^3$</th>
<th>including:</th>
</tr>
</thead>
<tbody>
<tr>
<td>including: unpaid buffer gas, million m$^3$</td>
<td>unpaid buffer gas, million m$^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uhers'ke UGSF</td>
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<td>250</td>
<td>318</td>
<td>250</td>
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<tr>
<td>Bil'che-Volyts'ko-Uhers'ke UGSF</td>
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<td>3,700.00</td>
<td>5,523</td>
<td>3,700</td>
</tr>
<tr>
<td>Bil'che-Volyts'ky deposit UGSF</td>
<td>18,050</td>
<td>3,700.00</td>
<td>4,600</td>
<td>3,700</td>
</tr>
<tr>
<td>Uhers'ky deposit UGSF</td>
<td>2,700</td>
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<td>923</td>
<td>0</td>
</tr>
<tr>
<td>Dashavsky UGSF</td>
<td>2,772</td>
<td>622</td>
<td>2,158</td>
<td>622</td>
</tr>
<tr>
<td>Oparske UGSF</td>
<td>1,920</td>
<td>0</td>
<td>210</td>
<td>0</td>
</tr>
<tr>
<td>Bohorodchans'ke UGSF</td>
<td>2,300</td>
<td>0</td>
<td>2,257</td>
<td>0</td>
</tr>
<tr>
<td>Olyshivs'ke UGSF</td>
<td>400</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Chervonopartyzans'ke UGSF</td>
<td>1,500</td>
<td>0</td>
<td>250</td>
<td>0</td>
</tr>
<tr>
<td>Solohivs'ke UGSF</td>
<td>1,300</td>
<td>0</td>
<td>312</td>
<td>0</td>
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<tr>
<td>Proletars'ke UGSF (M-7)</td>
<td>1,000</td>
<td>0</td>
<td>367</td>
<td>0</td>
</tr>
<tr>
<td>Kehychivs'ke UGSF</td>
<td>700</td>
<td>0</td>
<td>156</td>
<td>0</td>
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<tr>
<td>Krasnopopivs'ke UGSF</td>
<td>420</td>
<td>0</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>35,212</td>
<td>4,662.00</td>
<td>11,665</td>
<td>4,662</td>
</tr>
</tbody>
</table>

Fig. 9. Maximum productivity of underground gas storage facilities within 50 days of their operation

Table 3

Distribution of gas injection volumes (7 billion m$^3$) among storage facilities to achieve the assigned maximum performance at the projected time interval (50 days)
Industry control systems

\[
\frac{IV^1(t) - IV'^1(t)}{IV^2(t) - IV'^2(t)} \rightarrow \text{min.} \tag{7}
\]

Another important problem is given. Its proper statement requires numerical experiments and analysis of its results. A preliminary meaningful statement is as follows. With the known projected volumes of gas withdrawal, it is necessary to find the minimum volumes of gas storage and its distribution among storage groups to ensure the assigned reliability of GTS operation. It should also be noted that the distribution of gas among storage facilities is influenced by the volume of transit and the placement of points for the transfer of imported gas to the gas transportation system.

To simplify the solution to such a problem, it is necessary to represent GTS as an association of subsystems that are adjoining certain groups of gas storage facilities. After that, it is necessary to form a predictable dynamic gas balance in each subsystem. Then one can state an optimization problem about the volume of gas storage, its distribution among groups of storage facilities, and analyze how to distribute the projected volumes of gas transit and import among entry points optimally. To solve the problem, one can use the functionality of the software package "GTS Calculation" [6–10]. The list of tasks, a set of developed and implemented known methods and algorithms to solve them, as well as the programming environments of mathematical support and software, are shown in Fig. 10 [25].

**INFORMATION SUPPORT**

- Piping and instrumentation diagrams in terms of graph theory
- Information models of technological objects
- Technical condition of technological equipment
- Workstations - piping and instrumentation diagrams, cranes, control log, "fire works", support of the process of operation of technological equipment during the life cycle
- Metrological support and information databases
- Base of gas flow models in technological equipment

**GAS TRANSPORT SYSTEM**

Tasks:
- modeling of stationary and non-stationary modes of GTS operation with a complex piping and instrumentation diagram and variable boundary conditions;
- identification of model parameters;
- optimal planning of modes;
- formation of control parameters for non-stationary operating modes;
- dynamic optimal balancing of gas flows under conditions of non-stationary operation;
- modeling and optimization of design solutions, reconstruction, development, promising solutions;
- actualization of information support of tasks in real time;
- analysis of the tightness of the pipeline system and identification of leaks

**UNDERGROUND GAS STORAGE 1**

Tasks:
- identification of geophysical, filtration, hydraulic and thermal parameters; modeling of filtration processes in inhomogeneous porous media; calculation of the mode of operation of booster compressor stations with different types of gas pumping units;
- calculation of flow distribution parameters in distribution networks (bottom hole zone-gas gathering station);
- research of capacity of technological objects; research of optimization potential;
- assessment of gas losses in case of abnormal situations;
- displacement in gas-gas and gas-water reservoirs;
- assessment of economic feasibility of replacement or modernization of facilities

**UNDERGROUND GAS STORAGE 2**

Tasks:
- optimal planning of modes at the stages of gas injection and extraction;
- planning of operational modes with the maximum productivity; calculation of peak characteristics of underground storage in the modes of gas injection and extraction and assessment of "bottlenecks" in terms of capacity; direct and inverse regime problems in the project areas of parameter changes and beyond

**Programming environment**

DBMS Oracle, Delphi, Angular Framework, Ionic Framework, Phalcon framework, DBMS MySQL, Apache Http Server

**METHODS AND ALGORITHMS**

- Finite element method
- Fundamental algorithms on weighted graphs
- Methods of combinatorial optimization of minimum complexity
- Method of contour equations
- Algebra of operations on graphs
- Fast-converging methods of non-gradient type are adapted for solving systems of contour nonlinear equations
- Algorithmic representation of the operation of many shop compressor stations
- Methods for solving systems with different mathematical representations of nonlinear equations

**REPRESENTATION, ANALYSIS, AND INTERPRETATION OF SIMULATION RESULTS**

Fig. 10. List of tasks, methods, and algorithms for solving them
Fig. 11–13 demonstrate real examples of practical use of the developed algorithmic support and software.

At the same time, for operational and predictable operating modes of UGSF, the volume of production and technological needs of gas that go to work of gas pumping units is calculated.

The results of the work are found in the industrial operation. Some publicly available results are shown at the official website of JSC “Ukrtransgaz”: Free commercial capacities [24].

[Graph showing projected volumes of gas withdrawal under predicted weather conditions]

06.03.22
- U herske UGS 1.40
- Bilche-Volysky reservoir 25.62
- U hersky reservoir 15.41
- D ashavske UGS 9.03
- O parske UGS 6.00
- B ohrodchanske UGS 4.44
- O lyshivske UGS 0.00
- M rynske UGS 2.11
- S olotivske UGS 7.20
- P roletarske UGS (M-7) 5.30
- P roletarske UGS (B-5+B-9) 0.00
- K ehychivske UGS 5.50
- C hervonopopivske UGS 0.00
- T OTAL 82.01

Fig. 11. Projected volumes of gas withdrawal under predicted weather conditions

[Graph showing volumes of gas withdrawal: real, calculated according to operational projected data (at the time interval – 15.10.21–19.01.22), and projected volumes of gas withdrawal (at the time interval – 20.01.22–01.04.22)]

Other results are submitted to dispatching services – calculated operating modes of each UGSF with a detailed scheme for enabling GPU (the number of gas compression stages used by the GPU at each stage, the revolutions of centrifugal superchargers, fuel gas consumption, the degree of distance from the pumping zones) (Fig. 7). In addition, there is the calculation of the distribution of pressure in the reservoirs, the flow rate of wells, and the loss of pressure on technological chains – reservoir – gas pipeline branch.
6. Discussion of methods of planning the modes of operation of underground gas storage facilities

The results given in this work were obtained using the developed integrated mathematical, software, and information support for the operation of gas transmission systems with underground gas storage facilities [9]. An integral part of this complex is the software modules for solving the problems given in our work. This complex is in a state of constant development and is an effective tool for dispatching and geological services in the process of planning operational and perspective operating modes of individual and UGSF in general, taking into consideration the operating modes of GTS.

The software module for the implementation of the developed algorithm for finding the maximum productivity of underground gas storage facilities under variable conditions at their inputs and outputs has an independent importance, which ensures the calculation of the daily maximum performance of UGSF of all UGSF under the modes of gas pumping and withdrawal. It informs all gas owners and those who plan to store gas in UGSF about the available free capacities [24]. This module is used to solve the problem of planning gas injection according to the maximum performance criterion at the projected time intervals. The results of the developed algorithm are demonstrated in Fig. 8, 9 and Tables 2, 3.

The regime characteristics that are considered in the tasks set are influenced by discrete, discontinuous, and continuous processes. And so they are variable, not always monotonic and continuous in time, at times with jumps, which does not allow the use of classical methods of mathematical analysis. The only approach is combinatoric methods. Therefore, to solve the considered extreme problems for optimality and maximum productivity, combinatorial methods are pro-

Fig. 13. Dynamics of change in the free capacities of underground gas storage facilities: green — free withdrawal capacities; blue — free injection capacities
posed that do not impose special requirements for the nature of the change in the processes under consideration.

Regarding the use of existing sorting methods of combinatorial optimization [26, 27]. In many cases, taking into consideration the physics and nature of the behavior of the parameters of processes make it possible to build sorting methods that are not inferior in computational complexity to the known but may be better.

In addition to the development of existing software, the tasks of preparing for the modernization of UGSF, taking into consideration the operation in the near future, of UGSF in the storage of hydrogen mixtures with natural gas, are set.

The main shortcomings and efficiency of the developed algorithmic support are manifested in the process of its operation under real conditions. The parameters of managing the computational complexity of individual sorting algorithms are constantly being refined. An important test will be the 2021–2022 operating season under conditions of critical lack of the volume of gas for pumping, which will make it possible to assess the quality of the developed algorithmic support in the maximum way.

7. Conclusions

1. We have built functions of the maximum productivity of gas storage facilities as functions that depend on the distribution of reservoir pressure and pressure in gas pipeline branch in the region of design and real modes of operation that are piecewise-continuous with jumps and breaks. The nature of peak functions is influenced by discrete and continuous processes. Among the discrete influences is a change in the scheme of work of CS, the number of stages of gas compression, the number of GPUs in the stages of gas compression; among the continuous is a change in the capacity of GPU and pressure in the main gas pipeline. The rupture of peak functions for individual input sets indicates its absence – in such cases, there is no technologically acceptable mode of CS operation. During UGSF operation, the reservoir pressure changes slowly since the filtration flows are quite inert and pass in reservoirs whose surface has tens of square kilometers while pressure in the main gas pipelines changes several times during the day. The form of maximum performance functions, respectively, also changes. And, in order to ensure the stable operation of UGSF at maximum productivity, it is necessary to implement the mode of CS operation with the assigned stability – to ensure the necessary remoteness of working points of its centrifugal superchargers from the pumping zone.

The calculated functions of maximum productivity of gas storage facilities are relevant during UGSF operation season. For the next season of gas withdrawal, they must be recalculated after the re-identification of the parameters of CS models and reservoir systems of all storage facilities.

Our functions provide a quick assessment of the maximum peak activity for individual UGSF in technologically possible intervals for changing measured boundary conditions. The speed of obtaining the result is ensured by the numerical complexity of the algorithms for calculating the optimal modes of CS operation and the rapid numerical integration of the filtration equation, which is derived by a specially developed method for solving systems with sparse matrices.

2. The developed algorithm for planning the distribution among active gas storage facilities at the pumping stage to ensure the total maximum performance in the withdrawal process at projected time intervals can be used in the case of solving the problem of planning the distribution of gas withdrawal volumes under the modes of non-peak loading of UGSF according to the optimality criteria.

Our algorithm for solving the problem provides the necessary computational complexity by selecting the value of the step of the sorting algorithm. In this case, the complexity of the algorithm and the realization of the potential for achieving an extreme result are interconnected. In the process of conducting numerical experiments, one finds a satisfactory step of the sorting algorithm, which depends on the nature of the change in the functions of the maximum productivity of gas storage facilities.

3. The need to solve the problem, for the projected volumes of gas withdrawal to find such a distribution among storage facilities that would maintain the maximum total productivity of storage facilities throughout the entire time of gas withdrawal, occurs in several cases. In the case of significant uncertainty of weather conditions over a considerable time interval, the distribution of pumping volumes among storage facilities is advisable to carry out according to the criterion of maximum average peak activity under the mode of its withdrawal.

Often at the final period of gas withdrawal, the strategy for distributing the withdrawal of the required volumes of gas is built according to the criterion of maintaining the residual distribution of active gas of the maximum average peak activity until the active gas is completely withdrawn. In such cases, it is necessary to be insured, on average, for the case of unpredictable sudden weather changes and the need to ensure sufficiently high, perhaps not maximum, total productivity. In addition, based on the time of UGSF operation under the mode of maximum average peak activity, the sufficiency of active gas volumes in storage facilities is estimated until the end of the heating season under the most adverse weather conditions.

A simple and effective algorithm for solving the problem is based on an obvious statement – the maximum average peak activity of UGSF operation before the completion of gas withdrawal is achieved only in the case of simultaneous completion of the selection of active gas at the maximum performance of all UGSF.

The computational complexity of the algorithm is negligible and allows, if necessary, its multiple use.

4. The operating criteria – optimal and peak – are competing. The strategy of optimal management of gas storage facilities has not yet been fully implemented to ensure a balanced criterion, which includes ensuring the necessary maximum performance of UGSF operation at some time intervals and minimal consumption of fuel and energy resources at others. Each of these issues is separately effectively solved while the strategy of combining them requires combinatorial optimization methods with appropriate computational complexity. And, therefore, the effect of managing UGSF according to a weighted criterion is not yet fully appreciated.

The available tools make it possible to quickly evaluate one or another variant of the UGSF operation plan under the withdrawal mode. It should be noted that it is impossible to build a conditional trajectory of the optimal process for each UGSF over significant time intervals in advance – the boundary conditions for numerical UGSF models are constantly changing. All planning of regimes is carried out on the basis of the rated boundary conditions agreed in advance with the GTS operator.
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