1. Introduction

The efficiency of the electrical grid operation is determined by the level of electricity losses. Known studies make it possible to estimate technological losses [1]. At the same time, non-technological losses, including theft, fraud, non-payment of bills, deficiencies in electricity metering, are difficult to take into account, although in some countries their value can reach 15–18 % [2]. The first three of the listed causes of losses can be minimized by organizational measures, which are being worked on by regional energy companies. Increasing the efficiency of metering units is determined by the metrological and computer capabilities of metering equipment. The main sources of measurement uncertainty are errors in measuring current transformers operating in the zone of non-normalized, according to [3], relative error. Also, the accuracy of measurement decreases due to small currents of secondary circuits of current transformers, the values of which approach the sensitivity limit of the meter.

The lack of methods for estimating non-technological losses of electricity in distribution networks due to shortcomings of the measuring equipment functioning results in lower metering efficiency. Energy supply companies have no arguments to convince consumers with low consumption to update the equipment of metering units, in particular, to replace measuring current transformers. A scientifically based estimation of the electricity measurement uncertainty at a reduced load will be an effective tool for stimulating consumers to ensure the normal operation of metering units. So, the
The application of the possibility theory provisions involving fuzzy sets makes it possible to overcome the weaknesses of the statistical approach when processing measurement data, especially when the amount of experimental material is limited. The appropriate mathematical apparatus, which is a generalization of interval arithmetic, allows you to correctly describe the systematic component of measurement uncertainty.

Thus, most studies of the uncertainty of electricity measurement by meters use statistical methods for analyzing experimental data. This approach is justified when studying the functioning of the metering unit in the normalized load range. At the same time, the study of the uncertainty of electricity measurement in the reduced load mode requires significantly more time to conduct each of the experiments, which reduces the number of experimental points. As literature analysis shows, the non-random measurement uncertainty can be estimated using the fuzzy set theory. This will increase the accuracy of commercial electricity metering in the reduced load mode. The application of the results of electricity measurement uncertainty obtained through mathematical calculations will increase the correctness of financial calculations between consumers and suppliers of electricity.

3. The aim and objectives of the study

The aim of the study is to increase the accuracy of electricity measurement by the metering unit in the reduced load mode based on mathematical modeling of measurement uncertainty. This enables to improve financial settlements between energy supply companies and electricity consumers.

To achieve the aim, the following objectives must be accomplished:
- to substantiate the indicator that characterizes the uncertainty of electric energy measurement when the metering unit is operating in the reduced load mode;
- to develop a methodology for estimating the uncertainty of electricity measurement for one channel of the metering unit under the condition of reduced load when the current level changes;
- to improve the mathematical model for estimating, using a fuzzy interval, the uncertainty of electricity measurement during the operation of a three-phase metering unit at a reduced load, taking into account the phase current values;
- to carry out an empirical evaluation of the parameter values of the mathematical model for the metering unit of a specific configuration;
- to assess the adequacy of the mathematical model with experimental data.

4. Materials and methods

The object of the study is a three-phase commercial electricity metering unit for 380 V electrical grids. Such a unit consists of a three-phase electricity meter P11 and measuring current transformers TAА, TAВ, TAС of electromagnetic type, Fig. 1.

The subject of the study is the process of measuring the consumed electricity during user operation in the reduced load mode. To estimate the measurement uncertainty of the specified value, it is necessary to have an actual value of the active energy consumed for a certain time. This value should be the most accurate, under experimental conditions, approximation to the true value of the active energy consumed. To measure the actual value, it is suggested to use a direct connected meter P12, the current windings of which are connected in series to the primary windings of the current transformers of the metering unit under investigation, Fig. 1.
To analyze the uncertainty of electricity measurement by the metering unit, as a measuring converter, the following assumptions are made:

1. In the reduced load mode, the current of each phase is in the range from 0 to $2I_{\text{min}}^*$. Meanwhile, $I_{\text{min}}^*$ – the minimum relative primary current of the current transformer for which, according to [3], the relative error is normalized. In particular, for measuring current transformers of the accuracy class 0.5S, $I_{\text{min}}^* = 1\%$, that is, the specified mode occurs at a phase current of up to 2%.

2. The case of active load of each phase is considered. This is explained by the operation of only lighting, technical means of signaling, video surveillance, etc. during the downtime of the main equipment.

3. The effect of changes in the load and connecting wires resistance due to temperature changes on the values of phase currents is not taken into account.

4. The root mean square values of phase voltages are considered as the realization of a random stationary process. The probability of finding the root mean square value of the phase voltage in the permissible, according to [14], range (0.9 ÷ 1.1) p.u. is assumed to be no lower than 0.95 for an arbitrary moment.

5. It is assumed that during operation in the reduced load mode, the random change of phase currents is stationary and is caused solely by the action of unaccounted disturbances. This allows us to consider the root mean square values of phase currents unchanged during measurements.

6. A linear dependence of the active energy, measured by any measuring channel of the meter, on the measurement duration is assumed. This follows from the assumption of constant voltage and current during the measurement.

The method of uncertainty estimation of the PI1 readings deviation from PI2 for one measuring channel is based on the approach to estimating non-random uncertainty using the fuzzy set theory [15]. The influence of the measuring channel current on the boundary of the fuzzy interval, which characterizes the measurement uncertainty, is supposed to be estimated by the L-R type fuzzy function [16] for a fixed confidence level. The mathematical model for estimating the uncertainty of electricity measurement at reduced load is proposed to be represented by a fuzzy function of phase currents.

Experimental studies were carried out in laboratory 509 of the National University of Water and Environmental Engineering. The wiring diagram of the laboratory facility corresponds to Fig. 1. Digital electricity meters of the following types were used (Fig. 2): transformer connected PI1 – NIK2307 ART T.1600.M2.21 (Ukraine); direct connected PI2 – NIK2307 ARP3 T.1600.M2.21 (Ukraine). The accuracy class of both meters when measuring active energy is 0.5S. Measuring current transformers T-0.66-600/5 (Ukraine) of accuracy class 0.5S were used to connect PI1. Incandescent lamps with a power of 100 W, 200 W, 300 W, 500 W were used as a load. They were connected in different numbers using a microprocessor control board. This enables to form the resistive load of each phase from 0 to 2,500 W in steps of 100 W.

During the processing of experimental data, the following numerical methods are used: the Nelder-Mead simplex method for minimizing a function with several variables; the method of finding the roots of a polynomial based on calculating the eigenvalues of the associated matrix; the least squares method for approximation by a nonlinear curve, which involves the confidence region method. The MATLAB system (USA) is used for numerical calculations.

Assessing the adequacy of the mathematical model with experimental data is carried out according to the actual value of the confidence level. This value is found in accordance with the membership function of the metering unit. The function is calculated for specific phase currents according to the experimental value of the relative deviation of the meter readings. The hypothesis of the adequacy of the mathematical modeling results with empirical data is accepted if the actual value exceeds the marginal confidence level.
5. Results of the study of electricity measurement uncertainty at reduced load

5.1. Substantiation of the indicator for estimating electricity measurement uncertainty in the reduced load mode

The meter P11 includes three measuring channels P11, corresponding to the phases ζ={A,B,C} of the power grid. Each of the channels measures the active energy \( W_{p11\zeta} \) as a time integral of the active power consumed by the load \( Z_\zeta \). The dependence \( I_{zm}=Z_\zeta (I_{z}) \) of the secondary current \( I_{zm} \) on the primary current \( I_{z} \) is a static characteristic of the measuring current transformer \( TA_\zeta \). Generally, the static transformation function for the \( \zeta \) measuring channel of the metering unit is:

\[
W_{p11\zeta} = W_{p11\zeta} [t, I_{z}(t)] U_{z}(t) \cos \varphi_{z}(t) = \\
= \int_{0}^{t} I_{z}(t) \cdot U_{z}(t) \cdot \cos \varphi_{z}(t) \, dt.
\]  

(1)

Taking into account the accepted assumptions, the static transformation function (1) takes the form:

\[
W_{p11\zeta} = W_{p11\zeta} [t, I_{z}(t)].
\]

(2)

If the consumer operates in the reduced load mode during the time interval \( \Delta t = t_2 - t_1 \), then the active energy measured by the \( \zeta \) channel is equal to:

\[
W_{p11\zeta} (\Delta t, I_{z}) = W_{p11\zeta} [t_2, I_{z}(t_2)] - W_{p11\zeta} [t_1, I_{z}(t_1)].
\]

(3)

Suppose that during \( \Delta t \), the current flowed only through the \( \zeta \) channel, and the currents of the other two channels were equal to zero. Denote the readings of the meter P11 at moments \( t_{1(2)} \) as \( W_{p11\zeta\omega}(t_{1(2)}) \). Then the value of the static transformation function at moments \( t_{1(2)} \) is:

\[
W_{p11\zeta} (t_{1(2)}, t_{1(2)}) = \frac{I_{z}(t_{1(2)})}{I_{zm}(t_{1(2)})} - W_{p11\zeta\omega}(t_{1(2)}).
\]

(4)

where \( \frac{I_{z}(t_{1(2)})}{I_{zm}(t_{1(2)})} \) is the \( TA_\zeta \) ratio.

The active energy on the \( \zeta \) channel of the metering unit from (3) is:

\[
W_{p11\zeta} (\Delta t, I_{z}) = \frac{I_{z}(t_{1(2)})}{I_{zm}(t_{1(2)})} \left[ W_{p11\zeta\omega}(t_2) - W_{p11\zeta\omega}(t_1) \right].
\]

(5)

The active energy measured by the three-phase metering unit is equal to:

\[
W_{p11} (\Delta t, I_{A}, I_{B}, I_{C}) = \sum_{\zeta \in \{A,B,C\}} W_{p11\zeta} (\Delta t, I_{z}).
\]

(6)

The properties of the measuring circuits of the digital meter for each phase, as well as the parameters of the measuring current transformers, have a significant impact on the dependence \( W_{p11}(\Delta t, I_{A}, I_{B}, I_{C}) \).

The uncertainty of electricity measurement on the \( \zeta \) measuring channel in the reduced load mode can be estimated by the relative deviation \( \delta W_{\zeta} \) of the active energy, measured for the time interval \( \Delta t \), between the readings of meters P11 and P12. At the same time, it is considered that the current of the other two channels is equal to zero. The absolute deviation of the meter readings is:

\[
\Delta W_{\zeta} (\Delta t, I_{z}) = W_{p11\zeta} (\Delta t, I_{z}) - W_{p12\zeta} (\Delta t, I_{z}).
\]

(7)

where \( W_{p12\zeta}(\Delta t, I_{z}) \) – the active energy calculated on the \( \zeta \) measuring channel of the direct connected meter equal to:

\[
W_{p12\zeta} (\Delta t, I_{z}) = W_{p12\zeta\omega}(t_1) - W_{p12\zeta\omega}(t_2).
\]

(8)

Then the relative deviation can be defined as:

\[
\delta W_{\zeta} (\Delta t, I_{z}) = \frac{W_{p11\zeta}(\Delta t, I_{z}) - W_{p12\zeta}(\Delta t, I_{z})}{W_{p12\zeta}(\Delta t, I_{z})} - 1.
\]

(9)

Assumption No. 6 allows us to exclude \( \Delta t \) from the arguments of function (9). Then, given (5) and (8), from (9) we have:

\[
\delta W_{\zeta}(I_{z}) = \frac{I_{z}}{I_{zm}(I_{z})} \frac{W_{p11\zeta\omega}(t_1) - W_{p12\zeta\omega}(t_1)}{W_{p12\zeta\omega}(t_1) - W_{p12\zeta\omega}(t_2)} - 1.
\]

(10)

The measuring channels of the transformer and direct connected meters operate mutually independently. Errors accompanying measurements can be considered random. Therefore, the relative deviation of the transformer connected meter result from a fixed channel current, an assumption of the normal nature of the measurement results, there exists such a value of \( \delta W_{\zeta} \) at which \( \mu(\delta W_{\zeta}) = 1 \). The membership function \( \mu(\delta W_{\zeta}) \) is considered as a set of left \( \mu_1(\delta W_{\zeta}) \) and right \( \mu_2(\delta W_{\zeta}) \) branches separated by the true value of the measured parameter \( \delta W_{\zeta} \): \( \mu(\delta W_{\zeta}) = \begin{cases} \mu_1(\delta W_{\zeta}), & \text{as } \delta W_{\zeta} \leq \delta W_{\zeta}; \\
\mu_2(\delta W_{\zeta}), & \text{as } \delta W_{\zeta} > \delta W_{\zeta}. \end{cases} \)

(13)
The sample values of $\delta W_k$ are obtained experimentally, sorted by increasing $\delta W_k < \delta W_{k+1}$, $k = 1,(n-1)$ [15]. The lengths of intervals $\Delta_k = \delta W_{k+1} - \delta W_k$ between adjacent values $\delta W_k$ and $\delta W_{k+1}$ are calculated. The largest $\Delta_{\text{max}} = \max [\Delta_k]$ and the smallest $\Delta_{\text{min}} = \min [\Delta_k]$ intervals are determined. Assume that the width of the interval $\Delta_k$ is inversely proportional to the number of corresponding values. Then the frequencies of the measured values in each interval are:

$$m_k = 1 - \frac{\Delta_k - \Delta_{\text{min}}}{\Delta_{\text{max}}}, \quad k = 1,(n-1).$$

Since at $\Delta_k = \Delta_{\text{min}}$ the frequency is $m_k = 1$, the corresponding value of the measured parameter $\delta W_k$ is taken as the closest to the true value. The value $\delta W_k$ divides the vector $\delta W^*$ into two parts corresponding to the branches of the membership function: the left branch includes the elements $\delta W_k^*, \ldots, \delta W^*_{n-1}$; the right branch – the elements $\delta W^*_{n-1}, \ldots, \delta W^*$, Fig. 3.

![Fig. 3. Assignment of the measured values of the relative deviation $\delta W$, \% by branches of the membership function](image3)

Using the frequencies $m_k$ for each branch, the values of the membership function are calculated [15]:

$$\mu_{k,1} = \frac{m_k - m_{\text{min1}}}{m_{\text{max1}} - m_{\text{min1}}},$$

where $m_{\text{min1}} = \min \{m_1, \ldots, m_n\}$, $m_{\max1} = \max \{m_1, \ldots, m_n\}$, $k = 1, u$;

$$\mu_{k,2} = \frac{m_{\text{max2}} - m_{k,2}}{m_{\text{max2}} - m_{\text{min2}}},$$

while

$$m_{\text{min2}} = \min \{m_{n-1}, \ldots, m_n\},$$

$$m_{\text{max2}} = \max \{m_{n-1}, \ldots, m_n\}, \quad k = 1,(n-u).$$

The transition from absolute $\delta W_k^*$ to relative $\tau_k$ values of the measured parameter is carried out as follows:

$$\tau_{k,1} = \frac{\delta W_k - \delta W_k^*}{\delta W_{\text{max}} - \delta W_{\text{min}}}, \quad k = 1, u;$$

$$\tau_{k,2} = \frac{\delta W_{k-1} - \delta W_k^*}{\delta W_{\text{min}} - \delta W_{\text{min}}}, \quad k = 1,(n-u).$$

The obtained experimental points in [15] are proposed to be approximated by polynomials using the maximum norm method. For the left branch, the polynomial has the form (Fig. 4):

$$f_1(\tau_k) = 1 + \sum_{j=1}^3 a_j \tau_k^j,$$

and for the right branch (Fig. 5):

$$f_2(\tau_k) = 1 + \sum_{j=1}^3 b_j \tau_k^j.$$

The relative value of the fuzzy interval $U_{\tilde{\xi}} = \tilde{\xi}_1 + \tilde{\xi}_2$, describing the uncertainty of measurement results, is defined as the sum of the relative values of the subranges $\tilde{\xi}_1$ and $\tilde{\xi}_2$. The value $\tilde{\xi}_1$ corresponds to the root of the function $f_1(\tilde{\xi}_1) = \lambda$, where $\tilde{\xi}_2$ is determined from the expression $f_2(\tilde{\xi}_2) = \lambda$, where $\lambda$ is the confidence level.

![Fig. 4. Approximating polynomial for the left branch of the membership function $\mu_1$ of the relative values $\tau_1$ of the measured parameter and determining the left relative boundary of the fuzzy interval](image4)

![Fig. 5. Polynomial approximation of the right branch of the membership function $\mu_2$ for the relative values $\tau_2$ of the measured parameter and determining the relative value of the right boundary of the fuzzy interval](image5)

The boundaries of the fuzzy interval in units of the measured parameter are:

$$\delta W_k = \delta W_k - \tilde{\xi}_1 (\delta W_{\text{max}} - \delta W_{\text{min}}).$$

$$\delta W_k = \delta W_k + \tilde{\xi}_2 (\delta W_{\text{min}} - \delta W_{\text{min}}).$$

Dependencies (21) and (22) make it possible to estimate the uncertainty of electricity measurement by the metering unit at a fixed channel current. Such an estimate is provided by the left $\delta W_k$ and right $\delta W_k$ boundaries of the fuzzy interval, which at $\lambda$ includes the true value of the measured quantity.
5.2.2. Estimating the influence of the channel current on the boundaries of the fuzzy interval for the measurement result

Suppose that for the \( \zeta \) channel at a fixed current \( I_{\zeta} \) (\( \gamma \) is the number of the current level), sample values of the relative deviations of the meter readings \( \delta W_{\zeta} \{ I_{\zeta} \} \) are obtained in accordance with (10), where \( \iota \) is the number of the sample value. At the same time, the currents of the other two channels were zero. Let’s establish the set of confidence levels \( \{ \lambda_{\iota} \} \), while \( \lambda_{\iota} > \lambda_{\iota^*} \), \( j = 1, \Lambda \), where \( \Lambda \) is the total number of confidence levels under consideration. Using the above method for each current level \( I_{\zeta} \), we can find: the relative deviation of the meter readings \( \delta W_{\zeta} \) closest to the true value; L-R boundaries of the fuzzy interval for relative deviations of the meter readings at a given confidence level \( \lambda_{\iota^*} \):

\[
\delta W_{\zeta} = \begin{bmatrix} \delta W_{\zeta L} \{ I_{\zeta} \} ; \delta W_{\zeta R} \{ I_{\zeta} \} \end{bmatrix}.
\]

(23)

For a graphical presentation of the obtained fuzzy boundaries (23) and the \( \delta W_{\zeta} \) value, it is suggested to use a fuzzy box plot, which is formed similarly to a box-and-whisker plot for sample statistical characteristics of a random variable [17]. In this case, it is proposed to represent the right and left boundaries of the fuzzy parameter for the marginal confidence levels by equilateral trapezoids. Meanwhile, the smaller base of each trapezoid corresponds the value of the fuzzy parameter at \( \lambda_{\iota} \), and the larger base – at \( \lambda_{\iota^*} \). Fig. 6. The centers of the smaller trapezoid bases are connected by a vertical line corresponding to the value of the influence factor (load current). The horizontal line located between the smaller trapezoid bases corresponds to the value closest to the true one.

![Fig. 6. The set of boundaries of the fuzzy function characterizing the metering uncertainty for the \( \zeta \) channel as a dependence of the relative deviation \( \delta W \) on the channel current \( I_{\zeta} \) for confidence levels \( \lambda_{\iota} \) and the proposed fuzzy box plot at a specific current value.](image)

The obtained L-R boundaries of fuzzy intervals for each of the \( \gamma \) current levels at the confidence level \( \lambda_{\iota} \) can be approximated by the dependencies \( \delta W_{\zeta L} \{ I_{\zeta} \} \) and \( \delta W_{\zeta R} \{ I_{\zeta} \} \). Such dependencies represent the boundaries of the fuzzy function that in case of \( \lambda_{\iota} \) describes the metering accuracy for the \( \zeta \) measuring channel (Fig. 6):

\[
\delta W_{\zeta} \{ I_{\zeta} \} = \begin{bmatrix} \delta W_{\zeta L} \{ I_{\zeta} \} ; \delta W_{\zeta R} \{ I_{\zeta} \} \end{bmatrix}
\]

(24)

Suppose that for the approximation of the left and right boundaries of the sets of fuzzy functions, the dependencies \( F \) are used. The parameters of these dependencies form the following sets: for the left boundaries \( \{ L_{\zeta} \} \), for the right boundaries \( \{ R_{\zeta} \} \), namely:

\[
\delta W_{\zeta} \{ I_{\zeta} \} = F \{ I_{\zeta} \} \{ L_{\zeta} \}.
\]

(25)

\[
\delta W_{\zeta} \{ I_{\zeta} \} = F \{ I_{\zeta} \} \{ R_{\zeta} \}.
\]

(26)

Similarly, a set of parameters \( \{ Y_{\zeta} \} \) of the dependence \( F \) can be determined, which approximates the values of deviations closest to the true ones:

\[
\delta W_{\zeta} \{ I_{\zeta} \} = F \{ I_{\zeta} \} \{ Y_{\zeta} \}.
\]

(27)

Thus, estimation of the \( \zeta \) channel current influence on the uncertainty of measurement results can be represented by the dependence \( F \), which approximates the empirical points with the smallest error, and by sets of its parameters. The set \( \{ Y_{\zeta} \} \) determines, in accordance with (27), the values of deviations of the meter readings of the metering unit that are closest to the true values. The sets \( \{ L_{\zeta} \} \) and \( \{ R_{\zeta} \} \) characterize, according to (25) and (26), respectively, the left and right boundaries of the fuzzy function at a certain confidence level.

5.3. Mathematical model of the uncertainty of electricity measurement by a three-phase metering unit at a reduced load

Representation of the characteristics of measuring channels by fuzzy functions (24) makes it possible, in accordance with the dependence (12), to obtain a fuzzy function, which characterizes the uncertainty of electricity measurement by a three-phase metering unit for given values of the phase currents \( I_A, I_B, I_C \):

\[
\delta W(I_A, I_B, I_C) = \frac{\sum \delta W_{\zeta} \{ I_{\zeta} \}}{\sum \delta W_{\zeta} \{ I_{\zeta} \}}.
\]

(28)

The left L (right R) boundary of such a fuzzy function for a set of confidence levels \( \{ \lambda_{\iota} \} \) is defined as follows:

\[
\delta W_{\zeta L}(I_A, I_B, I_C) = \frac{\sum \delta W_{\zeta L}(I_{\zeta})}{\sum \delta W_{\zeta L}(I_{\zeta})}.
\]

(29)

The relative deviation of the meter readings closest to the true value when current flows through three measuring channels in the reduced load mode is:

\[
\delta W_{\zeta L}(I_A, I_B, I_C) = \frac{\sum \delta W_{\zeta L}(I_{\zeta})}{\sum \delta W_{\zeta L}(I_{\zeta})}.
\]

(30)

The values of the boundaries of the fuzzy function \( \delta W(I_a, I_b, I_c) \) for a set of confidence levels, calculated according to (29), and the values of \( \delta W_{abc}(I_A, I_B, I_C) \) make it possible to construct (by approximation) the membership function \( \mu_{abc} (\delta W) \) for \( \delta W_{abc}(I_A, I_B, I_C) \). This function corresponds to the specified values of the currents of measuring channels, Fig. 7, and includes the left \( \mu_{abc L}(\delta W) \) and right \( \mu_{abc R}(\delta W) \) branches:
5.4. Empirical evaluation of the parameter values of the mathematical model for estimating the electricity measurement uncertainty

To evaluate the uncertainty of deviation of the readings of the transformer connected meter from the direct connected meter, 67 tests were carried out on the measuring channel of phase A, 66 tests – phase B, 67 tests – phase C. The duration of each test was from 2 to 3 hours. During each test, the load of the studied channel was kept constant from the interval \( I_e = 0 \pm 2 \% \), the other two phases were not loaded. Tests on different phases and load levels were conducted randomly. It was not possible to maintain a stable load level for different tests conducted on different days due to the influence of disturbances (in particular, fluctuations in the supply voltage, different room temperatures). Therefore, actual current levels were recorded in the protocols. During further analysis, the load current change interval of each phase was divided into 7 ranges: the 1st range of 0 to 9.04 \%, the 2nd range of 9.04 to 13.02 \%, the 3rd range of 13.02 to 17.4 \%, the 4th range of 17.4 to 21.8 \%, the 5th range of 21.8 to 26.2 \%, the 6th range of 26.2 to 30.6 \%, the 7th range of 30.6 to 35 \%. The boundaries (21), (22) of fuzzy intervals (23), including the true value, were estimated for each current range \( \gamma \) of each channel \( \zeta \). The estimation was carried out for \( \Lambda = 13 \) confidence levels: \( \lambda^*_9 = 0.8, \lambda^*_7 = 0.75, \ldots, \lambda^*_1 = 0.2 \). Straight lines were used to approximate the experimental points of the membership function branches according to (19) and (20). In particular, for the range \( \gamma = 4 \) of current values in phase B (Fig. 8), the left branch of the membership function was approximated by the dependence \( f_1(\tau) = 1 - 1.47 \tau \), the right branch by \( f_2(\tau) = 1 - 5.36 \tau \); the value closest to the true value was \( \delta W^*_{RB} = 0.4 \% \); at the confidence level \( \lambda^*_9 = 0.4 \), the boundaries of the fuzzy interval were \( \delta W^*_{RB, 0.4} = -1.91 \% \), \( \delta W^*_{RB, 0.4} = 1.03 \% \).

Fig. 7. General view of the membership function \( \mu_{abc}(\delta W) \) for the relative deviation \( \delta W(I_a, I_b, I_c) \) of the meter readings, as a fuzzy quantity, at the specified phase currents with the indicated empirical value \( \delta W_e \) and the corresponding actual confidence level \( \lambda_e \).

Fig. 8. Approximation of the branches of the membership function \( \mu \) for the relative deviations \( \delta W \), by straight lines along the experimental points for the range \( \gamma = 4 \) of phase B current values, determination of the value closest to the true one and the boundaries of the fuzzy interval for the measured value at the confidence level \( \lambda^*_9 = 0.4 \); additional axes \( \tau_1, \tau_2 \) (p.u.) are shown for the relative values of the measured quantity corresponding to the left and right branches of \( \mu \).

To approximate the boundaries (25), (26) of the fuzzy functions (24), which characterize the dependence of the measuring uncertainty on the channel current value, at each
of the determined confidence levels $\lambda_0^*$, $\lambda_1^*$, $\lambda_2^*$, the following dependencies were used:

$$F(x, \{K\}) = K^{(i)} \exp \left( -x / K^{(i)} \right) + K^{(i)} \exp \left( -x / K^{(i)} \right) + K^{(i)},$$

where $\{K\} = \{K^{(1)}, \ldots, K^{(5)}\} = \{K \}$ – set of parameters.

In particular, for phase B at the confidence level $\lambda^*_0 = 0.8$, the sets of parameters of approximating dependencies (35) for the left and right boundaries of the fuzzy function have taken the following values, respectively:

$$\{L_B\} = \left\{ -1.78 \times 10^{-3}, -9.84 \times 10^{-4}, 2.33 \times 10^{-2}, -1.09 \times 10^{-3}, 9.91 \times 10^{-1} \right\},$$

$$\{R_B\} = \left\{ -2.56 \times 10^{-3}, -9.84 \times 10^{-4}, 2.33 \times 10^{-2}, -1.09 \times 10^{-3}, 9.91 \times 10^{-1} \right\}.$$

The boundaries of the fuzzy function describing the measurement uncertainty on the phase B channel at reduced load are plotted in Fig. 9.

The ordinate axis in Fig. 9 was scaled using the «safe log». This transformation $y = \text{sign}(\delta W) \cdot \lg(1 + |\delta W|)$ allows using negative numbers as an argument [18]. To illustrate the boundaries of fuzzy intervals, which characterize the measurement uncertainty at the averaged current value of each of the ranges, the proposed fuzzy box plots were used.

### 5.5. Assessment of the adequacy of the mathematical model with experimental data

The obtained numerical estimates of the fuzzy functions (24) parameters for each channel make it possible to estimate the measurement uncertainty of the laboratory metering unit as the boundaries of the fuzzy interval (34). The adequacy of the mathematical model with experimental data is suggested to be evaluated by the actual value of the confidence level $\lambda^*_e$ determined by (32). The actual value $\lambda^*_e$ exceeding the limit level of 0.4, which is typical for samples with a volume of up to several hundred elements, is taken as the adequacy criterion [15].

In particular, the experimental value of the relative deviation of the meter readings at currents $I^*_e = 1.02\%$, $I^*_e = 0.22\%$ was $\delta W_e = -5.6\%$. According to (30), $\delta W_e = -2.54\%$ was calculated. The points corresponding to the left branch of the membership function $\mu_{abc}$ were approximated by a third-degree polynomial (Fig. 10):

$$\lambda_{\mu_{abc}} = 1.60 \times 10^{-3} \cdot \delta W^2 + 4.30 \times 10^{-3} \cdot \delta W^2 + +3.94 \times 10^{-3} \cdot \delta W + 1.75.$$

The points related to the right branch (31) were approximated by a straight line:

$$\lambda_{\mu_{abc}} = -2.28 \times 10^{-1} \cdot \delta W + 4.20 \times 10^{-1}.$$

The actual confidence level was:

$$\lambda^*_e = \mu_{abc}(\delta W_e) = \mu_{abc}(-5.6) = 0.61.$$

The test results are listed in Table 1 (row 1).

![Fig. 9. Experimental points $\delta W_{\mu_{abc}}$ % divided into 7 ranges by current $I^*_e$, %, of phase B with fuzzy box plots and limits of the fuzzy function at confidence levels $\lambda^*_0 = 0.8$ and $\lambda^*_e = 0.2$, which approximate the corresponding fuzzy intervals for the measured parameter](image)

![Fig. 10. Approximation of the branches of the empirical membership function $\mu_{abc}$ for the relative deviation $\delta W$, %, of the meter readings of the laboratory metering unit at $I^*_e = 1.02\%$, $I^*_e = 0.182\%$, $I^*_e = 0.220\%$; the left branch is approximated by a third-degree polynomial, the right — by a first-degree polynomial](image)

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<tbody>
<tr>
<td>$I^<em>_0, I^</em>_1, I^*_2$</td>
<td>$\delta W_{\mu_{abc}}^0, \delta W_{\mu_{abc}}^1, \delta W_{\mu_{abc}}^2$</td>
<td>$\delta W_e$</td>
<td>$\mu_{abc}$</td>
</tr>
<tr>
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<td>1.020</td>
<td>0.182</td>
<td>0.220</td>
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<tr>
<td>2</td>
<td>0.746</td>
<td>0.186</td>
<td>0.217</td>
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<tr>
<td>3</td>
<td>0.610</td>
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<tr>
<td>4</td>
<td>0.605</td>
<td>0.593</td>
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<tr>
<td>8</td>
<td>0.224</td>
<td>0.220</td>
<td>0.147</td>
</tr>
</tbody>
</table>
Similarly, 7 more tests on the operation of the metering unit in the reduced load mode with asymmetric channel currents were conducted and the results were processed. Meanwhile, the values of channel currents in each test were chosen randomly (Table 1).

The analysis of Table 1 enables to establish that the actual value of the confidence level was $\lambda \geq 0.54$ in all tests. According to the formulated criterion, this gives grounds to accept the hypothesis about the adequacy of the results of mathematical modeling with empirical data.

6. Discussion of the results of electricity metering uncertainty study for reduced load mode

The obtained expression (12) makes it possible to estimate the uncertainty of electric energy measurement as a function of load phase currents. At the same time, the relative deviation $5\%$ in the reduced load mode corresponds to the weighted average, by phase currents, relative deviations of the readings of the transformer connected meter from the direct connected meter for each measuring channel. This approach, in contrast to [4], makes it possible to take into account the influence of load asymmetry on the accuracy of electricity metering.

The method of estimating the electricity measurement uncertainty by one channel using a fuzzy function (24), compared to the known approach [7], has the following advantage. The metrological characteristics of each measuring channel of the metering unit are taken into account when asymmetric currents flow, which increases the measuring accuracy in the reduced load mode.

The mathematical model of the electricity measurement uncertainty, which for specific values of the phase currents is given by the fuzzy function (28) with the membership function (31), takes into account the characteristics of each measuring channel. The advantage of this approach, compared to [10], lies in taking into account the influence of the operating mode of each load phase on the measuring uncertainty. Refinement of the marginal confidence level for measuring equipment of a specific configuration in accordance with (33) makes it possible to estimate the boundaries of the fuzzy interval of measurement results in terms of (34). Accepting the left boundary of such an interval as the amount of electricity underestimation in the most unfavorable conditions opens the way to the practical application of the proposed model.

It was found for the laboratory metering unit that straight lines are acceptable for approximating the branches of the membership function (13) for relative deviations $5\%$, Fig. 8. This is explained by the limited number (approximately 5–6) of experimental points corresponding to each branch. Approximation of the fuzzy functions (24) boundaries (25), (26) is carried out using the function (35), which is the sum of two exponents. The choice of such dependence is explained by the significant nonlinearity of $5\%$ in the range of currents up to 0.5 %. Fig. 9. The specified dependence approaches the linear one at higher values of currents. Analysis of graphs in Fig. 9 allows us to establish that for phase B of the laboratory metering unit, the current level of 0.2 % is the limit of sensitivity. Power consumption at lower current levels cannot be measured. The level of underestimation in the most unfavorable conditions can be from $-32\%$ to $-8\%$ for currents from 0.2 % to 0.8 %. As the current value increases, the largest underestimation decreases, reaching 3 % at a current of 2 %.

A similar situation is observed for other measuring channels. The underestimation for the channel increases when the phase current decreases (Fig. 9) can be explained by the error rising of the current transformer and by the accuracy decreasing of the analog-to-digital signal conversion by the digital meter circuit.

Approximation of the experimental points corresponding to the left and right branches of the membership function (31) was carried out using polynomials of the third (36) and first (37) degrees, respectively. The empirical membership function was plotted (Fig. 11) for specific phase currents. At other currents, the numerical characteristics of the function will change. However, it was found that polynomials of the specified degrees for the branches of the membership function are characterized by the smallest approximation errors. On the basis of experimental data, it was found that for different values of phase currents, there is a tendency towards asymmetry of the membership function. The length of the $5\%$ interval corresponding to the left branch significantly (approximately 3–4 times) prevails over the interval of the right branch (Fig. 10). This corresponds to the shift of the channels characteristics (Fig. 9) into the negative half-plane. This feature is explained by a decrease in the sensitivity of the measuring equipment of the metering unit in the reduced load mode. The consequence of this is a predominant underestimation of electricity in the reduced load mode. This circumstance is significant from the point of view of financial losses of energy supply companies.

The refined mathematical model of the electricity measurement uncertainty at reduced load provides an analytical basis for assessing the amount of electricity underestimation in real conditions. The application of this approach allows the energy supply company to move from general appeals to consumers regarding the normalization of the operation mode of metering units to reasoned dialogue. Implementation of the necessary technical (for example, replacement of measuring current transformers, selection of metrological equipment of a higher accuracy class, etc.) or other measures will increase the accuracy of electricity measurement.

The developed mathematical model solves the issue of reducing non-technological losses of electricity caused by deficiencies in the functioning of metering units. Estimation of the uncertainty of electricity metering with a fuzzy interval (34), the boundaries of which are obtained by applying the proposed mathematical model, increases the measurement accuracy due to the following. The energy supply company clarifies monthly electricity consumption, calculated from the meter readings of the metering unit taking into account the unaccounted energy during the reduced load mode. Accordingly, the payment amount is specified. In the case of unacceptable, from the energy supplier’s point of view, level of underestimated energy, the measuring equipment is replaced, which excludes the operation of the latter in an undefined mode.

Application of the proposed mathematical model for estimating the uncertainty of electricity measurement at reduced load is limited to the use in low-voltage power grids. In high-voltage grids, the meter is connected, in addition to current transformers, also using measuring voltage transformers. Such a case was not considered during the research.

The main drawback of the proposed approach to estimating the uncertainty of electricity measurement in the reduced load mode is the need for a preliminary assessment of the measuring channel characteristics of the metering unit. This
increases the time to obtain the final result and requires some organizational effort.

Planning, conducting and analyzing the results of a full factorial experiment to estimate the limit confidence level (33) for a specific electricity metering unit are planned to be carried out in further research.

7. Conclusions

1. It is proposed to use the relative deviation of the active energy measured by the metering unit from the actual value as an indicator characterizing the uncertainty of electricity measurement in the reduced load mode. In contrast to known studies, such an indicator is considered as a function of the measuring channel currents corresponding to the phases of the power grid. This enables to take into account the impact of load asymmetry on metering uncertainty.

2. The method of estimating the uncertainty of electricity measurement over one channel of the metering unit by a set of boundaries of the L-R fuzzy function for a set of confidence levels is substantiated. The intersections of the specified set of fuzzy function limits for fixed channel current values correspond to the membership function, obtained by approximating experimental data for the relative deviation of the measured energy. A feature of the method is the possibility to determine the impact of the load phase current on the measurement uncertainty with a limited amount of empirical data.

3. The mathematical model for estimating the uncertainty of electricity measurement at reduced load was refined, which involves estimating the parameters of the membership function for the relative deviations of the meter readings of the metering unit from the actual value. The model differs from the known models by taking into account the influence of load values over each phase of the metering unit on the measurement uncertainty.

4. The sensitivity limit of the laboratory metering unit, consisting of a digital meter of the NIK2307 ART type and measuring current transformers 600/5 of the 0.5S accuracy class, was estimated at the level of 0.2 %. For currents up to 0.8 %, the level of electricity underestimation under the most unfavorable conditions can reach 32 %, for currents up to 2 % – it can be up to 3 %.

5. The adequacy of mathematical modeling results is confirmed by experimental data. Comparing the analytically obtained membership function for the relative deviations of the metering unit readings with the empirically obtained value of such a deviation made it possible to determine the actual value of the confidence level. This value was not less than 0.54 in randomized tests. The obtained result is satisfactory at the limit value of 0.4 of the adequacy criterion.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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