The object of current research is a multi-section transport conveyor. The actual control problem of the flow parameters of a multi-section conveyor-type transport system with a given control quality criterion is solved. Algorithms for optimal control of the flow of material coming from the input accumulating bunkers into the collection section of the conveyor, ensuring the filling of the accumulating tank in the minimum time were synthesized. An admissible material flow from the accumulating bunkers is found, which allow filling the accumulating tank, taking into account the given distribution of the material along the section of the collection conveyor at the initial and final moments of the filling time with minimal energy consumption. The synthesis of algorithms for optimal control of the material flow from accumulating bunkers became possible due to the determination of differential constraints in the optimal control problem based on an analytical distributed model of a transport conveyor section. The distinctive features of the results obtained are that the allowable controls contain restrictions on the maximum allowable load of material on the conveyor belt and take into account the initial and final distribution of material along the collection conveyor section. Also, a feature of the obtained results is the consideration of variable transport delay in the transport conveyor control model. The application area of the results is the mining industry. The developed models make it possible to synthesize algorithms for optimal control of the flow parameters of the transport system for a mining enterprise, taking into account the transport delay in the incoming of material at the output of the conveyor section. The condition for the practical use of the results obtained is the presence of measuring sensors in the sections of the transport conveyor that determine the belt speed and the amount of material in the accumulating bunkers.

Keywords: PiKh model, speed control, transport delay, accumulating bunker, similarity criteria

1. Introduction

A transport conveyor serving a mining industry enterprise is a high-tech dynamic distributed system [1] that consists of a large number of sections separated from each other by accumulating bunkers [2–4]. With the normative value of the material loading factor of the conveyor section equal to 0.7, the specific transportation costs are up to 20 % of the material extraction cost [5]. A decrease in the material loading factor of the conveyor belt leads to a non-linear increase in specific transportation costs. These costs can constitute the main part of the material extraction cost, making mining unprofitable. Constant requirements for increasing the competitiveness of mining production substantiate the relevance of scientific topics devoted to building models of the transport system for the synthesis of algorithms for optimal control of the flow parameters of the transport conveyor. To reduce the transport costs of a conveyor section, algorithms have been developed for optimal control of the belt speed [6–8] and the input material flow [9–11] taking into account the energy management methodology [12–14]. The use of research results at existing enterprises allows reducing the cost of mining and decreasing carbon dioxide emissions into the atmosphere [15].

2. Literature review and problem statement

The scheme of a collection conveyor with three accumulating bunkers is quite often used when analyzing the flow of material in a transport system. At the same time, there are no models for this conveyor scheme that take into account the variable transport delay and the uneven distribution of material along the transport route. For example, in [15], a simulation model with a constant transport delay was developed for the power consumption control system. The model demonstrates the possibility of saving energy under the South Africa’s Demand Side Management program for conveyor sections with constant section belt speed and constant material flow from the accumulating bunker. In [16], a system dynamics model was developed to synthesize four material flow control policies for a three-bunker collection conveyor. The presented model does not take into account the transport delay, the main attention is focused on the problem of overfilling of bunkers without restrictions on the maximum allowable load on the conveyor belt. In [17], for a scheme of a 3-bunker collection conveyor, a model of a transport system was proposed, in which, to coordinate the filling level of the bunkers, constant transport delays were introduced into the control channels, which provide discrete control of the
material flow coming from the accumulating bunkers. The issue of modeling the transport system for the general case, assuming the presence of variable transport delays, remained unresolved. The use of the discrete element method [18], a neural network [19, 20], and specialized software [21] in modeling a three-bunker collection conveyor can be considered as suggested ways to overcome these difficulties. However, the proposed solutions to the problem require significant computing resources, which limits their practical use for the synthesis of control systems. This explains the lack of publications containing methods [18–21] for constructing models for a collection conveyor scheme with three accumulating bunkers. Another approach to solving the problem is to develop analytical models of a multi-section transport conveyor. The foundation of the study is the analytical model of the conveyor section [11]. The model is used to describe the transient operating modes of a transport conveyor section, does not require significant computing resources, and can be further developed to describe the state of the flow parameters of a multi-section conveyor. The first step in such a study is to build an analytical model for a three-bunker collection conveyor scheme. The presented analysis allows us to assert the practical necessity of constructing an analytical model of a three-bunker collection conveyor for the synthesis of algorithms for optimal control of flow parameters.

### 3. The aim and objectives of the study

The aim of the study is to improve the analytical methods for describing the transport system for a 3-bunker collection conveyor scheme. This will make it possible to synthesize algorithms for optimal control of the flow parameters of the transport system in order to reduce the energy consumption of the conveyor section.

To achieve the aim, the following objectives were set:

- to develop an analytical model of a 3-bunker collection conveyor, taking into account the transport delay;
- to synthesize an algorithm for optimal control of the flow parameters of the transport system for a 3-bunker collection conveyor scheme, based on the analytical model.

### 4. Materials and methods

The object of the present study is a multi-section transport conveyor. It is assumed that taking into account the transport delay makes it possible to increase control accuracy and reduce specific transport costs.

The foundation of the conducted research is the general provisions of the statistical theory of flow production control systems [22]. To describe the flow parameters of the transport conveyor section, a hydrodynamic model of the transport conveyor was used [23]. The application of similarity theory methods makes it possible to present the model of the transport system for the 3-bunker collection conveyor scheme in a universal form, expanding the scope of the model.

### 5. Results of the study of the transport system model for the 3-bunker collection conveyor scheme

#### 5.1. Construction of a 3-bunker conveyor model, taking into account the transport delay

A transport system consisting of one main (collecting) conveyor and three linear two-sectional conveyors with an intermediate bunker was considered (Fig. 1).

To describe the transport system, a dimensionless model of a conveyor section was used [11, 23]. Dimensionless parameters were introduced that characterize the state of the parameters of a separate section of the transport system:

\[
\gamma_n(t) = \frac{T_d}{S_{d0}[\chi]_{\text{max}}},
\]

\[
\gamma_{\text{min},n}(t) \leq \gamma_n(t) \leq \gamma_{\text{max},n}(t) = a_n(t) \frac{T_d}{S_{d0}}.
\]

\[
\theta_{\text{min},n}(t,\xi_{\text{min}}) = \frac{[\chi]_{\text{max}}(t,S_{\text{min}})}{[\chi]_{\text{max}} S_{d0} / T_d} = \theta_{\text{min},n}(t,\xi_{\text{min}}) S_{\text{min}}(t).
\]

\[
n_n(t) = \frac{N_n(t)}{S_{d0}[\chi]_{\text{max}}},
\]

\[
n_{\text{min}} = \frac{N_{n\text{min}}}{S_{d0}[\chi]_{\text{max}}},
\]

where \( S_{d0} \) is the length of the \( k \)-th section of the \( m \)-th input conveyor (\( m=1, 2, 3 \)); \( T_d \) is the characteristic time for which the material entering the collection conveyor of length \( S_{d0} \) reaches the exit point of the collection conveyor and enters the accumulating tank \( N_0(t) \). The flow of material from the output section (\( k=1 \)) of the \( m \)-th linear conveyor enters the main conveyor belt at point \( S_{d0} \).

![Fig. 1. Scheme of a 3-bunker collection conveyor](image-url)
To control the flow of material \[ X \] at the output conveyor, an accumulating bunker filled with material by the amount of \( N_m(t) \). \( N_{m_{min}} < N_m(t) < N_{m_{max}} \) is used. The material flow \[ X \] at the previous section \((k = 2)\) enters the accumulating bunker \( N_m(t) \). The material level is controlled by changing the flow intensity \( n_m\).

When constructing a model of the \( m \)-th linear conveyor, we use the assumptions [17, 18]:

1) section \( k = 1 \) of the \( m \)-th linear conveyor is a guide section. the length of the section \( S_{d2} \) is much less than the length of the section of the collection conveyor, \( \xi_{d\text{lin}} \rightarrow \theta \);

2) the value of the material flow \[ X \] at the guide conveyor \( S_{d0} \) is zero.

3) conveyor belt speed \( g_m(t) \) is constant.

Taking into account these assumptions, the equation determining the state of the flow parameters of the \( m \)-th linear conveyor takes the form [17, 18]:

\[
\frac{dn_m(t)}{dt} = \theta_{m_0}(t) - \gamma_m(t), \quad n_{m_0}(0) = n_{m_0},
\]

\[
n_{m_{min}} \leq n_m(t) \leq n_{m_{max}}.
\]

\[
\gamma_m(t) = \gamma_m(t, m = 1...3).
\]

To describe the state of the flow parameters of the collection conveyor, the equation was used:

\[
\frac{\partial \theta_0(\tau, \xi)}{\partial \tau} + g_0(\tau) \frac{\partial \theta_0(\tau, \xi)}{\partial \xi} = \sum_{n=1}^{3} \delta(\xi - \xi_{n0}) \gamma_0(\tau).
\]

\[
\theta_0(0, \xi) = \psi(\xi).
\]

\[
\theta_0(\tau, \xi) \leq \theta_{m_{max}}, \quad \theta_0(\tau, 0) = \theta_{m_{00}}(\tau, \xi_{00}), \quad \xi = \xi_{00}.
\]

\[
\xi_{d0} = \xi_{00} = 1, \quad \theta_0(\tau, \xi) = \theta_0(\tau, \xi) g_0(\tau).
\]

where \( \psi(\xi) \) is the linear dimensionless density of the material at time \( \tau = 0 \), \( \Delta(\xi) \) is the Dirac function.

The solution of equation (3) determines the linear density of the material at the point of the collection conveyor with the coordinate \( \xi \) at time \( \tau \) and has the following form [24]:

\[
\theta_0(\tau, \xi) = H(\xi - G(\tau)) \psi(\xi - G(\tau)) + \sum_{n=0}^{3} \left( H(\xi - \xi_{n0}) - H(\xi - \xi_{n0} - \xi_{in}) \right) \gamma_m(\tau, \xi_{in}) g_m(\tau, \xi_{in}).
\]

For a constant belt speed, \( g_0(t) = \frac{1}{\Delta t} \)

\[
\Delta t = \frac{\xi_{00}}{g_{0}}, \quad \Delta t = \frac{\xi_0}{g_0}, \quad \Delta t_1 = \frac{\xi_{10}}{g_0}, \quad \Delta t_2 = \frac{\xi_{20}}{g_0}
\]

\[
\theta_0(\tau, \xi) = H(\xi - \xi_{in}) \psi(\xi - \xi_{in}) + \left( \sum_{n=0}^{3} H(\xi - \xi_{n0}) - H(\xi - \xi_{n0} - \xi_{in}) \right) \gamma_m(\tau, \xi_{in}) g_m(\tau, \xi_{in}).
\]

\[
\tau_{in} = G^{-1}(G(\tau) - \xi_{in}).
\]

\[
\theta_0(\tau, \xi) = \psi(\xi - \xi_{in}), \quad 0 < \tau < \Delta t_{10}, \quad \Delta t_{10} = \Delta t_{00}.
\]

Thus, at time \( \tau = \Delta t_{10} \), the output flow of the material is determined by the rest of the material along the collection conveyor \( \psi(\xi) \):

\[
\theta_{m}(\tau) = \psi(1 - \tau), \quad 0 < \tau < \Delta t_{10}.
\]
For a time interval \( t < \Delta t_{11} \), it is impossible to change the flow value \( \theta_{10}(t) \) by controlling the intensity of the material flow \( \gamma(t) \). For the next time interval, the expression for the material flow is:

\[
\theta_{10}(t) = \psi(1-t) + \gamma(t - \Delta t_{11}), \quad \Delta t_{11} \leq t < \Delta t_{12}.
\]

The value of the output flow can be controlled by changing the value of the flow intensity \( \gamma(t) \). In this case, the change in the output flow value occurs with a delay \( \Delta t_{12} \). The minimum value of the output flow is limited by the value \( \psi(1-t) \). For the next time interval, the expression for the output flow value takes into account the incoming material from the second main conveyor with an intensity \( \gamma(t) \) and a transport delay \( \Delta t_{12} \):

\[
\begin{align*}
\theta_{10}(t) &= \psi(1-t) + \gamma(t - \Delta t_{11}) + \gamma(t - \Delta t_{10}) , \\
\Delta t_{12} &\leq t < \Delta t_{13}.
\end{align*}
\]

It is allowed to regulate the output flow value when changing the intensities \( \gamma(t) \). For the case of constant belt speed \( \Delta t_{13} \), the transport system operated in a transient mode, for which the possibility of regulating the output flow by changing the intensity \( \gamma(t) \) was limited or completely absent. At \( \Delta t_{13} < t \), the value of the output flow is expressed through the values of the intensities \( \gamma(t) \) of the material flow from the accumulating bunkers:

\[
\begin{align*}
\theta_{10}(t) &= \gamma(t - \Delta t_{11}) + \gamma(t - \Delta t_{12}) + \gamma(t - \Delta t_{10}) , \\
\Delta t_{13} &\leq t.
\end{align*}
\]

The output flow can be controlled over the entire range of allowable values of the function \( \theta_{10}(t) \). When constructing a control system for the output flow \( \theta_{10}(t) \), it is required to change the intensity values \( \gamma(t) \) of the material flow ahead of \( \Delta t_{13} \).

The linear density of the material on the conveyor belt \( \theta_0(t, \xi) \) must not exceed the maximum allowable value \( \theta_{\max} \). The restriction should be fulfilled for an arbitrary moment of time \( t \) and coordinates \( \xi = 1 - \xi_0 \) in which the material enters the collection conveyor from the \( k \)-th linear conveyor:

\[
\begin{align*}
\theta_0(t, 1 - \xi_0) &= H(1 - \xi_0 - G(t))\psi(1 - \xi_0 - G(t)) + \\
&+ \sum_{n=1}^{\infty} H(\xi_0 - \xi_0 - G(t))\gamma_n(t - \Delta t_{10} - \Delta t_{12}) \\
&\leq \theta_{\max}.
\end{align*}
\]

The latter inequality for the case of constant belt speed \( \xi_0 = 1 \) of the collection conveyor can be rewritten as:

\[
\begin{align*}
\gamma_1(t) &\leq \theta_{\max}, \\
\gamma_2(t) &+ \gamma_1(t - (\Delta t_{10} - \Delta t_{12})) \leq \theta_{\max}, \\
\gamma_3(\alpha) &= \psi(-\alpha) \quad \text{at} \quad \alpha < 0; \\
\gamma_4(t) &+ \gamma_3(t - (\Delta t_{12} - \Delta t_{11})) + \gamma_2(t - (\Delta t_{10} - \Delta t_{12})) \leq \theta_{\max}, \\
\gamma_5(\alpha) &= 0, \quad \gamma_5(\alpha) = \psi(1 - \Delta t_{12} - \alpha) \quad \text{at} \quad \alpha < 0.
\end{align*}
\]

The resulting expressions make it possible to determine the value of the output material flow of the collection conveyor with known values of the input material flows \( \gamma(t) \) from the accumulating bunkers.

5.2. Synthesis of the optimal control algorithm for the flow parameters of the transport system

The problem of the minimum loading time \( \tau_{\text{load}} \) of the accumulating tank \( n_0(t) \) with a capacity \( n_{\text{max}} \) is considered. The material flow speed is controlled by bunker gates for small-sized material or feeders for large-sized material. The performance of the shutter or feeder \( u_m(t) \) is set by changing the size of the bunker outlet or the operating parameters of the feeder. The problem of optimal control of the flow parameters of the transport system was formulated as follows: it is required to determine the optimal control of the intensity of the material flow \( u_m(t) = \gamma_m(t) \) coming from the bunker \( n_m(t) \) to the collection conveyor, which leads to a minimum:

\[
\tau_{\text{load}} \rightarrow \min, \quad (10)
\]

with differential constraints:

\[
\frac{dn_m(t)}{dt} = \theta_{\text{in}}(t) - u_m(t), \quad (11)
\]

and restrictions on phase variables:

\[
n_{\text{min}} \leq n_m(t) \leq n_{\text{max}}, \quad n_{\text{min}} = 0, \quad (12)
\]

control restrictions:

\[
\begin{align*}
\gamma_1(t) &\leq \theta_{\text{max}}, \\
\gamma_2(t) &+ \gamma_3(t - (\Delta t_{10} - \Delta t_{12})) \leq \theta_{\text{in}}, \\
\gamma_3(\alpha) &= \psi(-\alpha) \quad \text{at} \quad \alpha > 0;
\end{align*}
\]

where \( \gamma_3(\alpha) = \psi(-\alpha) \) at \( \alpha > 0 \):

\[
\begin{align*}
\gamma_2(t) &+ \gamma_3(t - (\Delta t_{12} - \Delta t_{11})) + \\
&+ \gamma_2(t - (\Delta t_{10} - \Delta t_{12})) \leq \theta_{\text{in}},
\end{align*}
\]

where \( \gamma_3(\alpha) = 0, \quad \gamma_3(\alpha) = \psi(1 - \Delta t_{12} - \alpha) \quad \text{at} \quad \alpha < 0 \), and initial conditions:

\[
n_0(t) = 0, \quad n_m(t) = n_{\text{max}}, \quad m = 1, 2, 3. \quad (13)
\]

The quality criterion (9) can be converted to an integral form:

\[
J = \int_0^{\tau_{\text{load}}} F(t) dt \rightarrow \min, \quad F_1 = 1. \quad (14)
\]

The condition of the maximum principle is obtained as a result of successive transformations. By definition of the quality functional (15):

\[
\frac{df}{dt} = F_1. \quad (15)
\]
On the other hand, considering $F_i$ and $J$ as functions of time $t$ and coordinates $n, n_0$ for admissible control:

$$\frac{df}{dt} = \frac{df}{dn} \frac{dn}{dt} + \sum_n \frac{df}{dn_n} \frac{dn_n}{dt} = \frac{\partial f}{\partial n} \frac{dn}{dt} + \sum_n \frac{\partial f}{\partial n_n} \frac{dn_n}{dt}$$

$$= \frac{\partial f}{\partial n} \frac{dn}{dt} + \sum_n \frac{\partial f}{\partial n_n} \frac{dn_n}{dt}$$

it follows that:

$$\frac{\partial f}{\partial n} = F_i - \psi_n \frac{dn}{dt} + \sum_n \psi_n \frac{dn_n}{dt}$$

(16)

Equation (16) is the Hamilton-Jacobi equation with the Hamilton function:

$$H = -F_i + \psi_n \frac{dn}{dt} + \sum_n \psi_n \frac{dn_n}{dt}$$

(17)

After substituting the differential constraint equation (11) into function (17), the Hamiltonian function with a control delay was obtained:

$$H = -F_i + \psi_n(t) \left( H(1 - t) \psi(1 - t) + \sum_n \psi_n(t) \left( \psi_n(t) + \sum_n (1 - H(\Delta \tau_{in} - \tau_n)) u_n(t - \Delta \tau_{in}) \right) \right) + \sum_n \psi_n(t) \left( \theta_n(t) - u_n(t) \right) \right)$$

(18)

Taking into account (18), the quality criterion (15) was written as:

$$J = \int_0^{t_{load}} \left( \psi_n(t) \left( H(1 - t) \psi(1 - t) + \sum_n \psi_n(t) \left( \psi_n(t) + \sum_n (1 - H(\Delta \tau_{in} - \tau_n)) u_n(t - \Delta \tau_{in}) \right) \right) + \sum_n \psi_n(t) \left( \theta_n(t) - u_n(t) \right) - H \right) dt \rightarrow \min.$$  

(19)

After replacing the variable $\beta = (t - \Delta \tau_{in})$ and allowing the representation for the function $\psi_n(t)$ outside the control area $t > t_{load}$ as $\psi_n(t) = 0$, the following was obtained:

$$\int_0^{t_{load}} \psi_n(t) \left( H(1 - t) \psi(1 - t) + \sum_n \psi_n(t) \left( \psi_n(t) + \sum_n (1 - H(\Delta \tau_{in} - \tau_n)) u_n(t - \Delta \tau_{in}) \right) \right) dt =$$

$$\int_{t_{load}}^{t_{load} + \Delta \tau_{in}} \psi_n(\beta + \Delta \tau_{in}) u_n(\beta) d\beta =$$

$$= \int_{t_{load}}^{t_{load} + \Delta \tau_{in}} \psi_n(\beta + \Delta \tau_{in}) u_n(\beta) d\beta.$$  

(20)

Substituting the result obtained into (19), the quality criterion for the control problem takes the form:

$$J = \int_0^{t_{load}} \left( \psi_n(t) H(1 - t) \psi(1 - t) + \sum_n \psi_n(t + \Delta \tau_{in}) u_n(t) + \sum_n \psi_n(t) \left( \theta_n(t) - u_n(t) \right) - H \right) dt \rightarrow \min.$$  

(21)

and, accordingly, the Hamiltonian (18) can be represented as:

$$H = -F_i + \psi_n(t) H(1 - t) \psi(1 - t) +$$

$$+ \sum_n \psi_n(t + \Delta \tau_{in}) u_n(t) +$$

$$+ \sum_n \psi_n(t) \left( \theta_n(t) - u_n(t) \right).$$  

(22)

Taking into account the restrictions on the phase variables, the optimality conditions for the control problem are written as:

$$L = H + \sum_n \mu_n(t) n_n(t) + \sum_n n_n(t) (n_{\text{max}} - n_n(t)).$$  

(23)

followed by the transformation of the Hamilton function to the form:

$$H = -1 + \psi_n(t) H(1 - t) \psi(1 - t) +$$

$$+ \sum_n \psi_n(t + \Delta \tau_{in}) - \psi_n(t) u_n(t) +$$

$$+ \sum_n \psi_n(t) \theta_n(t) \rightarrow \max.$$  

(24)

Since the Hamilton function is linear in the control $u_n(t)$, the maximum of the Hamilton function is reached at the ends of the control change segment, whence, the optimal control has the form:

$$u_n(t) \rightarrow u_{\text{max}}, \quad \psi_n(t + \Delta \tau_{in}) - \psi_n(t) \geq 0,$$

$$u_n(t) \rightarrow 0, \quad \psi_n(t + \Delta \tau_{in}) - \psi_n(t) < 0.$$

From the conjugate system, $\psi_0(t), \psi_n(t)$ can be found:

$$\psi_0(t) = \psi_0, \quad \psi_n(t) = \int_0^{\mu_n(\alpha) - \mu_n(\alpha)} d\alpha,$$

$$\psi_n(t_{load}) = 0.$$

Joint solution of equations (11), (15), (16), taking into account the restrictions on phase trajectories and control, allows us to determine the optimal program for controlling the intensity of the flow from the bunkers.

Of practical interest is the control program for a transport system containing large-capacity bunkers. To simplify the discussion of the results, let us dwell on the case when, at the initial moment of time, the material filling level of the input bunkers $n_0(0) = n_{\text{max}}$ allows, during the time $t_{load}$ of the process of loading the storage tank, not to reach the phase limitation $0 < n_{\text{max}}(t) < n_{\text{max}}$. Then from the solution of equations (23), it follows that:

$$\psi_0(t) = \psi_0 = 0,$$

$$\psi_n(t + \Delta \tau_{in}) = H(t_{load} - (\Delta \tau_{in} + t)) \psi_{bc},$$

$$\psi_n(t) = \text{const} = \psi_{bc},$$

$$H = -1 + \psi_0(t) H(1 - t) \psi(1 - t) +$$

$$+ \sum_n \psi_n(t + \Delta \tau_{in}) u_n(t) \rightarrow \max.$$  

(25)
The Hamilton function determines the control \( u_m(t) \) of the material flow from the \( m \)-th bunker (Table 1) under constraints (12), (13).

From a practical point of view, of interest is the operating mode of the transport system, for which the following requirements are met:

- at the initial time \( t = 0 \), the conveyor belt is empty;
- at the end time \( t = \tau_{\text{final}} \), the belt is empty;
- the energy costs for moving the material during the loading time \( \tau_{\text{load}} \) of the storage tank should be minimal.

These requirements are expressed in the form of equations:

\[
\begin{align*}
0 & = \int_0^{\tau_{\text{max}}} \psi_1(t) \, dt, \\
0 & = \int_0^{\tau_{\text{max}}} \psi_2(t) \, dt, \\
H &= -1 + \sum_{n=1}^3 \int_{\tau_{\text{max}}}^{\tau_{\text{load}}} \nu_n(t + \Delta t_m) \, dt \to \text{max}.
\end{align*}
\]

The solution of equation (28) for different maximum allowable control values was obtained in the form of four options. The results of a numerical experiment based on the developed model (3)–(8) of the transport conveyor are shown in Fig. 2.

Option 1: \( u_{1\text{max}} = u_{2\text{max}} = u_{3\text{max}} = \theta_{\text{max}} \).

The solution of equation (28) for this option has the form:

\[
n_n(t) = \sum_{n=1}^3 (1 - H(\Delta t_m - \beta)) u_n(\beta - \Delta t_m) \, d\beta.
\]

The presented solution is used to calculate the loading time of the accumulating tank:

\[
n_n(t_{\text{load}}) = \sum_{n=1}^3 (1 - H(\Delta t_m - \tau)) u_n(\tau - \Delta t_m) \, d\tau.
\]

The considered control mode can be adapted for the problem of filling the tank with a material of a given concentration (mixing the material in the required proportion). Equation (26) characterizes the state of the transport system, in which the conveyor belt is not filled with material at the initial and final time. The condition of minimum energy consumption for moving material along the collection conveyor determines the bunker control strategy in the case when the control problem has more than one solution. In accordance with this strategy, the maximum material flow intensity is achieved at the first bunker \( u_1(\tau) \to \text{max} \) (the closest bunker to the section output), then for the second bunker \( u_2(\tau) \to \text{max} \), and only then for the third bunker. In this case, conditions (12) limiting the maximum load on the belt must be observed. The value \( u_0(t) \) for the considered control algorithm takes either the maximum allowable value or a value equal to zero (Table 2).

### Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Time interval</th>
<th>( u_1(t) )</th>
<th>( u_2(t) )</th>
<th>( u_3(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \tau_{\text{load}} - \Delta t_1 \leq t \leq \tau_{\text{final}} )</td>
<td>From the control</td>
<td>From the control</td>
<td>From the control</td>
</tr>
<tr>
<td>2</td>
<td>( \tau_{\text{final}} - \Delta t_2 \leq t \leq \tau_{\text{final}} - \Delta t_1 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>( u_2(t) \to \text{max} )</td>
<td>( u_3(t) \to \text{max} )</td>
</tr>
<tr>
<td>3</td>
<td>( \tau_{\text{final}} - \Delta t_3 \leq t \leq \tau_{\text{final}} - \Delta t_2 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>( u_2(t) \to \text{max} )</td>
<td>( u_3(t) \to \text{max} )</td>
</tr>
<tr>
<td>4</td>
<td>( 0 &lt; t \leq \tau_{\text{final}} - \Delta t_3 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>( u_2(t) \to \text{max} )</td>
<td>( u_3(t) \to \text{max} )</td>
</tr>
</tbody>
</table>

To assess the level of material in the accumulator tank, the location of the bunkers is determined by the coordinates \( \xi_{11} = 1/3, \xi_{12} = 2/3, \xi_{13} = 1 \). With a constant belt speed \( g_b = 1 \) and the accepted location of the bunkers, the transport delay is constant \( \Delta t_1 = 1/3, \Delta t_2 = 2/3, \Delta t_3 = 1 \) and is proportional to the length of the transport route. The amount of material in the tank at an arbitrary point in time is determined by the equation:

\[
\frac{dn(t)}{d\tau} = \sum_{n=1}^3 (1 - H(\Delta t_n - \tau)) u_n(\tau - \Delta t_n). \quad n_0(0) = 0,
\]

the solution of which can be represented in the following form:

\[
n_n(t) = \sum_{n=1}^3 (1 - H(\Delta t_n - \beta)) u_n(\beta - \Delta t_n) \, d\beta.
\]

The storage tank is filled exclusively with the first bunker (Fig. 2, a, b).

### Table 2

<table>
<thead>
<tr>
<th>No.</th>
<th>Time interval</th>
<th>( u_1(t) )</th>
<th>( u_2(t) )</th>
<th>( u_3(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \tau_{\text{load}} - \Delta t_1 \leq t \leq \tau_{\text{final}} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \tau_{\text{final}} - \Delta t_2 \leq t \leq \tau_{\text{final}} - \Delta t_1 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \tau_{\text{final}} - \Delta t_3 \leq t \leq \tau_{\text{final}} - \Delta t_2 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>( u_2(t) \to \text{max} )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>( 0 &lt; t \leq \tau_{\text{final}} - \Delta t_3 )</td>
<td>( u_1(t) \to \text{max} )</td>
<td>( u_2(t) \to \text{max} )</td>
<td>( u_3(t) \to \text{max} )</td>
</tr>
</tbody>
</table>
The accumulating tank is filled with the first, second and third bunker.

Option 4: \( u_{\text{max}} = u_{2\text{max}} = u_{3\text{max}} \). \( u_{1\text{max}} + u_{2\text{max}} + u_{3\text{max}} < \theta_{1\text{max}} \).

The filling level of the accumulating tank and the loading time are determined by the equations:

\[
\begin{align*}
\eta_0(\tau) &= (1 - H(\Delta t_{11} - \tau))u_{\text{max}}(\tau - \Delta t_{11}) + \\
&+ (1 - H(\Delta t_{12} - \tau))u_{\text{max}}(\tau - \Delta t_{12}) + \\
&+ (1 - H(\Delta t_{13} - \tau))u_{\text{max}}(\tau - \Delta t_{13}), \\
\tau_{\text{load}} &= \min(\Delta t_{11} - \Delta t_{11}, \Delta t_{12} - \Delta t_{13})u_{\text{max}} \\
&+ 2u_{\text{max}}(\Delta t_{13} - \Delta t_{12}) + 3u_{\text{max}}(\tau_{\text{load}} - \Delta t_{13}).
\end{align*}
\]

The material flow control modes in the \( m \)-th bunker when calculating the loading time \( \tau_{\text{load}} \) of the storage tank are presented in Table 3.

At the initial moment of time, all bunkers are functioning. Since the Hamilton function is linear in control, the value for optimal control is expressed in terms of maximum or minimum control values, subject to a limit on the maximum allowable material density for the conveyor belt.

### 6. Discussion of the results of modeling the collection conveyor

An analytical model of the transport system is proposed, taking into account the features of material movement along the technological route of a multi-section collection conveyor (3)–(8). In contrast to the known models of a 3-bunker collection conveyor [16, 17], the analytical model takes into account the following fundamental features of existing conveyor-type transport systems. Firstly, the initial distribution of the material along the transport route, which, unlike existing models, makes it possible to obtain the value of the flow parameters, and, accordingly, form the optimal control during the initial movement of the belt. Secondly, the variable transport delay of material movement along the conveyor section, which makes it possible to synthesize optimal control for transient modes, which is impossible when using existing stationary models. Thirdly, the influence of the uneven distribution of material along the transport route on the flow characteristics of the transport system. At present, this is of particular importance due to the urgency of the problem of reducing the specific energy consumption in the extraction of material. Fourthly, restrictions on the maximum allowable linear density of the material along the belt, the speed of the belt and the intensity of the material flow coming from the accumulating bunkers make it possible to exclude controls that cause damage to the conveyor section.

The analytical model allows determining:
- linear density of the material at a given point of the transport route for an arbitrary point in time (4);
- the value of the transport delay of material movement between two points of the transport route for an arbitrary point in time (5);
- the value of the output material flow of the transport conveyor for an arbitrary moment of time (6) at known intensities of material flows from the accumulating bunkers.

It should be noted that the relative lengths of the transport conveyor sections necessary for the calculation, as well as the law of time variation of the belt speed of individual sections and the intensity values of the material flows from the accumulating bunkers, must be known. As a limitation of this study, we should mention the assumption of a constant belt speed of individual sections in the synthesis of optimal control algorithms (9). When synthesizing the control algorithm, the control quality...
The transport route. The advantages lie in the possibility of taking into account the initial/final distribution of material along models with constant transport delay and without taking into account the variable transport delay and during the conveyor operation in transient modes are not taken into account, which may make it impossible to ensure practical or theoretical expectations from the use of the research results.

The practical use of the proposed model consists in the possibility of synthesizing algorithms for optimal control of the flow parameters of the transport system of a mining enterprise for different control quality criteria. The prospect for further research is the synthesis of an optimal control system for a 3-bunker collection conveyor at a variable belt speed. This will increase the material loading factor of the collection conveyor section.

7. Conclusions

1. An analytical model of a 3-bunker collection conveyor was developed. Distinctive features of the presented model are taking into account the variable transport delay and the distribution of material along the transport route. This gave advantages over the known results obtained using models with constant transport delay and without taking into account the initial/final distribution of material along the transport route. The advantages lie in the possibility of describing not only stationary, but also transient operating modes of the transport system. The result obtained is explained by the fact that the basis for constructing the model of the transport conveyor was the statistical theory of the description of production systems, within which the transport conveyor is considered as a dynamic distributed system.

One of the comparative estimates of the result is the error of determining the estimated amount of material in the accumulating tank. For an extended collection conveyor with a small capacity tank, the quantitative error estimate can be a significant part of its rated capacity.

2. The analytical model of a 3-bunker collection conveyor was used to synthesize algorithms for optimal control of the flow parameters of a transport system containing a variable transport delay. Accounting for the variable transport delay allowed us to significantly expand the range of admissible optimal controls and improve control accuracy. The main feature of the obtained optimal controls is that the theory of systems with aftereffect is used for the synthesis of controls. This made it possible to use the Pontryagin maximum principle for systems containing transport delay in phase variables.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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