Based on the analysis of scientific developments in the application of project management methodology for educational environment management, the application of the vector paradigm to the construction of project management methodology in academic environments is substantiated. The methodological concept of project management in the educational environment is proposed, and the conceptual basis of the methodology of project-vector management of educational environments is created. The theoretical foundations of project-vector management of educational environments are developed. A mathematical model for managing projects in educational environments has been developed, the originality of which is ensured by representing the entities of projects, products, tools and subjects of educational environments as the objects of project-vector space, moving from the starting point (project origination) to its completion. The notion of the potential movement of objects in the project-vector space (PVS) is introduced and used in the mathematical model. The definition of unforced resistance of PVS is given, the source of which is the regularity in the influence of a significant number of objects and subjects of the project-vector space on each other, which is present in most projects of educational environments and hinders their implementation. The proposed methodology creates a modern scientific and methodological basis for the construction of project management systems in educational organizations. The practical implementation of the results of the research creates a basis for improving the efficiency and quality of educational organizations due to a well-defined organization of management procedures.

Keywords: project-vector methodology, project-vector space, project management, educational environment

1. Introduction

The successful solution of the problems that developing countries face on their path from integration into the world economy is impossible without specialists with a high qualification level. The changes necessary for education system consist of organizational, technological, and functional improvements that will allow reaching the level of Europe's leading educational institutions. Moreover, not only educational institutions require improvement; all educational environments formed by legislative, executive, and territorial bodies of state administration, including education management and various educational, scientific, scientific-methodological and other institutions need to be improved.
Within the methods and means existing in the traditional system of management of educational environments, it is impossible to solve this strategic problem. In order to change the methods and means of managing educational environments, it is necessary to use modern concepts and methodologies of managing complex organizational and technical systems, primarily the methodology of project management.

Thereby, the problem arises of linking the experience accumulated by mankind in managing complex simultaneous events, underlying the methodology of project management, with the specifics of educational environments, processes in which are characterized by a whole set of features: complexity and uncertainty, multicriteria of decisions, multivariate, dynamic nature of the processes, contradictory and difficult to formalize the functioning of the elements of the management system and the informational nature of most activities. In turn, the projects of educational environments are characterized by considerable diversity in structure, functions, results, stakeholders, tasks, funding sources, legislative field, and constant and significant intersection with the operational activities of educational environment organizations.

Resolving contradictions that arise in the planning management strategies of higher education institutions is possible by applying the components of the project-vector methodology. Known methods of planning, administration, and evaluation of scientific research activities in institutions of higher education do not provide an opportunity to fully and objectively reflect the results of activities, which can lead to the adoption of insufficiently effective management decisions. Imperfect tools for solving the main problems of the project-vector methodology in terms of the formalization of the activities of the subjects of educational environments restrain the development of the theoretical and practical base in this direction. In work [1], the technological components of the project-vector methodology were developed. Means for compensating for the impact of unforeseen situations on the implementation of projects of subjects of educational environments are considered. However, the issue of mathematical formalization of the project-vector environment, in general, has not been fully resolved. Therefore, studies devoted to mathematical models of the design vector space of educational environments are of scientific relevance.

2. Literature review and problem statement

A distinctive feature of the university management system is the planning and control over the content of higher education by a central government. The Ministry of Education and Science regulates the directions and levels of training of specialists in various specialties [2]. A list of compulsory (at the choice of university and students) disciplines included in each curriculum is determined [3]. The minimum amount of hours for each discipline is set. The Ministry strictly controls the presence of compulsory disciplines in the curricula of each university and compliance with the established minimum hours [4].

This management practice differs to a large extent from the practice of managing higher education in developed Western countries, where universities are given significant autonomy in determining the content of education [5]. Therefore, improving the management of the higher education system is inextricably linked with changes in the organizational structure and management principles of each university [6]. And this requires the creation and implementation of effective information systems and university management technologies. The existing structure of the university's information technologies provides for a comprehensive automation of the majority basic functions of university management [7].

Such technologies provide management of all types of activities of a higher educational institution, but their disadvantage is a significant dependence on the accuracy of the entire system's design as a whole [8]. This is due to the fact that the need to solve a particular problem leads to the design of all functions and information structures of the system [9]. Recently, significant number of research has been carried out on the issues of determining the features of university management, analyzing the conditions of their functioning, formalizing managerial functions, and etc.

However, some aspects of the implementation of managerial functions in accordance with the Bologna Process, in particular those related to the creation of information technologies oriented to system planning of the educational process, are not sufficiently covered in publications. There is a need to create information systems and technologies that would make it possible to adapt training plans quickly and efficiently to current realities, the conditions of the Bologna process in universities and the level of knowledge of students [10].

The effective implementation of such systems is based on project management methodology. Where each of the following – the implementation of an information system, educational program or infrastructure projects – is a separate project requiring individual management and approaches. There is also the problem of developing methods associated with the definition of such projects (project integration), which will correspond to the maximum expansion of the educational environment «Universal projects». The calculation of the movement trajectory in the project-vector space ensures the achievement of the project goals with minimal time and financial resources [11]. This issue is relevant primarily in connection with the introduction of a credit-module system for organizing the educational process in higher educational institutions and the need for effective planning of all processes for the educational and methodological support development for training specialists in this system, taking into account the requirements of ECTS [12].

The described tasks can be solved based on the project-vector methodology, the fundamental principles developed in [13]. In work [1], applied information technology was developed for planning and administration of the technological component of the methodology of project-vector management of educational environments. Means for compensating for the impact of unforeseen situations on the implementation of projects of subjects of educational environments are considered. One of the components of the technology is the assessment of the productivity of higher education institutions and teachers. These studies are described in works [14–16] and can complement the methodology of project-vector management of educational environments. In order to evaluate the productivity of higher education institutions, a parametric model of educational institution processes is described in [17]. The impact parameters described in [17] are an essential component of project management of educational environments. In works [18, 19], models of interaction of educational projects and problems of implementation of international standards of assessment of competencies of project managers are considered. The work [20] describes the principles of learning based on project management.
Concepts of project management in technical education, which can be part of the project-vector management methodology, are described in [21, 22]. In general, all the described tasks are a search for new ways of solving the task of improving the management efficiency of higher education institutions.

In work [23], it is shown that to solve the project's time and budget problems, particularly in the field of education, it is necessary to form an organizational culture. This organizational and functional culture tends to cooperate and expand opportunities for project implementation. The work [24] considers the ideological concepts of higher education management, which can be included in the project-vector management of educational environments. So, the described problems solve local problems of management of educational environments. A thorough description of formally design-vector environments is required to solve complex management problems, and this is done in this paper.

The proposal of the model and tools of the methodology of project-vector management of educational environments, namely structures, schemes, algorithms, processes and tools will allow in the future to ensure the management of higher educational institutions and create a project management system based on the project management methodology, namely:
– use methods and means of project management in the functional structures of educational institutions;
– adjust the organizational component of these institutions to the functional structure of project management tasks;
– manage the implementation of project activities due to its decomposition by PVS measurements;
– provide project management of educational environments based on the block-modular organization of functions and procedures of project portfolio management systems;
– implement an effective project management technology through modeling the movement of objects that reflect the essence of projects in PVS.

3. The aim and objectives of the study

The aim of the study is to develop a mathematical model of project-vector space (PVS) of educational environments. This will make it possible to increase the effectiveness of the universities management.

To achieve the aim the following tasks have been set:
– determine the proximity of vectors in the project-vector space;
– determine the dimensions of the project-vector space;
– develop a model of goal-setting in the project-vector space.

4. Materials and methods

The object of research is the processes of managing educational processes. The subject of the study is the methodology of project-vector management of educational environments. It can be assumed that implementing mathematical models of the methodology of project-vector management of educational environments in the activities of universities will increase the effectiveness of their management.

Creation of mathematical models of project-vector space of educational environments were based on the use of systems theory, methods of systems analysis for the decomposition and classification of structures, functions and management technologies of educational environments; vector algebra as a tool for creating a mathematical apparatus of operating the PVS attributes; modeling theory, for constructing the PVS model of educational environments; methods of operations research, for optimizing the movement of objects of educational environment in PVS.

The described methods were used for implementation in two higher educational institutions. For this purpose, the personal data of the participants of the educational process was used. Accordingly, a set of such data cannot be publicly available. The article summarizes the results of implementing the project-vector methodology in these universities.

5. Results of research on project management of educational environments in PVS

5.1. Determining the proximity of vectors in the project-vector space

As a result of the decomposition of the subject area, the classification of projects of educational environments is made, and the task of optimal management in the design-vector space is formulated. The research enables to proceed to formalization of educational environment management processes. Let's define the basic definition of this part of the research.

Definition 1. By project-vector management of educational environments, Let’s understand the implementation of functions that ensure the organization, planning, and control of the distribution of available resources between the objects of educational environments and ensure their maximum rapid progress in the project-vector space.

Mathematically, the multisystem of project-vector management of educational environments will reflect the vectors formed in the project-vector space (direction of change of objects), evaluate and adjust them based on the needs of stakeholders and project goals. The project-vector space contains a set of objects and subjects of the projects developing in time. The development of the objects and subjects of the projects corresponds to the movement in the project-vector space. The evaluation of the effectiveness of the multisystem of project-vector management of educational environments will be carried out through the evaluation of the distance between the vectors reflecting the required and development of objects/subjects of projects.

Mathematically, a set of vectors can represent the unit movements of project objects and subjects in the discrete project-vector space. Each vector coordinate represents the place of the object/subject of the project in one of the measurements at the current moment in time. The endpoint of these movements is the end of the project and the collapse of the project-vector space again (for the project that has ended) into a point. Each vector is defined by coordinates that reflect the state at some discrete point in time t of the object/subject Q of the project P in the project vector space Ω:

$$A^{i,j}_{k}(t) = [x_{1}^{i,j}(t), x_{2}^{i,j}(t), ..., x_{N}^{i,j}(t),..., x_{M}^{i,j}(t)]$$

where $x_{k}^{i,j}(t)$ – the coordinate value of the object/subject $Q$ of the project $P$ along the $N_{k}$ axis in the project-vector space at time $t$.

The motion vector determines the priority measurements (priority values of the project) and is defined through the magnitude of the change in the distance along the given directions per quantum of time.
Building an effective project management system is not the «aspirations» of individual objects that matter but how similar or different their movement vectors are in the project-vector space. The same vectors mean that the movements of different project objects are equally predetermined. Therefore, a single system (or subsystem) of management of these projects can be created.

In order to create an effective multisystem of project management, it is necessary to group projects in such a way that the distance between vectors of projects included in one group is minimal.

The coordinates of the vectors in each measurement are as different as the attributes of the objects and subjects of the projects displayed in this measurement. It is necessary to develop a mathematical apparatus for calculating the distance between the directions of development of objects and the subjects of projects. This will make it possible to create optimal functional procedures for the formation of vectors and minimize the time and cost of creating a multisystem of project-vector management of educational environments.

The mathematical apparatus of vector algebra is used to calculate the distances between vectors and determine the optimal set of groups of projects (and, respectively, the subsystems of the multisystem of project management).

Assume that for some time interval \(dt\) the coordinate value of some object/subject \(Q\) of the project \(\Pi_k\) change from the value:

\[
\begin{bmatrix}
x_{1k}^{(t)}(t-dt), x_{2k}^{(t)}(t-dt), \ldots, \ \\
x_{p_k}^{(t)}(t-dt)
\end{bmatrix}
\]

to the value:

\[
\begin{bmatrix}
x_{1k}^{(t)}(t), x_{2k}^{(t)}(t), \ldots, x_{p_k}^{(t)}(t)
\end{bmatrix}
\]

where \(t\) – instant of time; \(dt\) – the time quantum of the discrete project-vector space.

Then the value corresponding to the development of the object/subject \(Q\) of the project \(\Pi_k\), can be determined from the formula:

\[
l_k^{(t)} = \sqrt{\sum_{i=1}^{p_k} (x_{ik}^{(t)}(t) - x_{ik}^{(t)}(t-dt))^2}
\]

where \(l_k^{(t)}\) – the vector estimate of the change of the object/subject \(Q\) of the project \(\Pi_k\) at time \(t\) (instantaneous displacement velocity).

However, there is one peculiarity in the design-vector space. In operations on vectors, the distance between them in some measurement is given by the difference of coordinates in that measurement. This cannot be used to calculate the degree of proximity of different vectors. Firstly, because the direction of the object/subject cannot be evaluated over small discrete time instants, it is necessary to evaluate them over a sufficiently long-time interval.

Secondly, the creation of a single system (subsystem) of management of many projects is based not only on the «unidirectional» development of individual objects and subjects of projects but also on their proximity in the project-vector space. For example, whether the same methods or means of project management are used, the same performers, and whether the same input information is used. The two problems will be solved.

5.1.1. Estimation of the magnitude of the similarity of vectors at significant time intervals

Expression 1 defines the direction of development of the object/subject \(Q\) of the project \(\Pi_k\) in the project-vector space at moment \(t\) (Fig. 1).

For instantaneous changes it can be written:

\[
A_{k}^{(t)}(t) = \frac{dA_{k}^{(t)}}{dt} = \begin{bmatrix}
\frac{dx_{1k}^{(t)}}{dt} & \frac{dx_{2k}^{(t)}}{dt} & \frac{x_{3k}^{(t)}}{dt} & \frac{dx_{4k}^{(t)}}{dt} & \frac{dx_{5k}^{(t)}}{dt}
\end{bmatrix}
\]

It means that at some point in time, the projects develop similarly. However, this does not mean that on a long-time interval the development of projects is similar, and therefore it is possible to apply unified approaches to the management of them. For this reason, another mathematical model will be used to solve the problem of determining the projects that are close in development.
Substitute the minimum time interval for a longer one, which more accurately characterizes the coincidence or non-coincidence of trends in the development of projects. However, not all vectors obtained from such substitution are equivalent. Eventually, changes over a long period more accurately characterize trends in the development of objects and subjects of projects. For a more accurate determination of the similarity of the methodologies of management of various projects, the weighting coefficient of similarity in the development of objects and subjects of the project-vector space will be set.

Let’s obtain:

\[ R \left( \frac{A_j(t)}{\Delta}, \frac{A_i(t)}{\Delta} \right) = R_{ij}^\Delta(t, \Delta t) = \mu(\Delta t) \left| \frac{\Delta A_j(t)}{\Delta t} - \frac{\Delta A_i(t)}{\Delta t} \right| \]

\[ = \mu(\Delta t) \sum \left[ \frac{x_{ji}^{\Delta}(t) - x_{ij}^{\Delta}(t - \Delta t)}{\Delta t} \right] \]

where \( R \left( \frac{A_j(t)}{\Delta}, \frac{A_i(t)}{\Delta} \right) = R_{ij}^\Delta(t, \Delta t) \) – the difference in the direction of movement of the object/subject \( Q \) in the projects \( \Pi_j \) and \( \Pi_i \), at time \( t \); \( \mu(\Delta t) \) – the coefficient specifying the value of the relationship between the value of proximity of vectors obtained for a given time interval and the actual proximity of the object/subject motion vectors \( Q \) in the projects \( \Pi_j \) and \( \Pi_i \) at time \( t \).

Since the longer the time the objects and subjects of the projects develop alike, the more likely it is that they need the same approach to management. Taking the value of the relationship between the value of proximity of vectors obtained for a given time interval and the actual proximity of the object/subject motion vectors \( Q \), the projects \( \Pi_j \) and \( \Pi_i \), at time \( t \).

The proximity of vectors can be calculated by their coordinates, but it is difficult to apply a continuous numerical measure to many measurements. Many project parameters are set in qualitative rather than quantitative terms. Therefore, it is proposed to apply an inverse approach to determining the proximity of vectors. Each vector coordinate is represented by distances to the coordinates of other vectors (in this dimension) – the distance between the objects of the projects. For identical values of objects (even qualitative ones), a distance of 0 is taken, and for different qualitative – 1. In this case, there is no need to translate qualitative concepts into quantitative measures. Then the «reduced» end coordinate of the vector \( A_{ij}^\Delta \) along the dimension \( N_j \) will be given not by a single number but by a tuple which represents the «differences» on this coordinate with vectors of the same object but other projects:

\[ \pi_{ij}^\Delta = \left< \ell_{ij1}^\Delta, \ell_{ij2}^\Delta, ..., \ell_{ijN_j}^\Delta \right>, \]

where \( \ell_{ij}^\Delta \) – the reduced coordinate of the end of the vector \( A_{ij}^\Delta \) by the measurement \( N_j \) of the object/subject \( Q \) of the project \( \Pi_j \); \( \pi_{ij}^\Delta \) – the reduced distance in the coordinates of project vectors \( \Pi_j \) and \( \Pi_i \), by measurement \( N_i \) for object/subject \( Q \), \( K \) – the number of projects.

To determine the similarity of projects, consider pairs of projects. When determining the proximity of each of the two projects in the project-vector space, it is not the entire tuple that is important but only a pair of values characterizing these projects. From this premise, it is possible to estimate the relative speed of movement (development) of objects of two different projects:

\[ \ell_{ij}^\Delta(t) = \sqrt{\sum_{i=1}^{K} \left( \ell_{ij1}^\Delta(t) - \ell_{ij1}^\Delta(t - dt) \right)^2}, \]

where \( \ell_{ij}^\Delta(t) \) – the vector estimate of the change of the object/subject \( Q \) of project \( \Pi_j \) at time \( t \) (instantaneous displacement velocity) relative to the change of the same object/subject in project \( \Pi_i \); \( \pi_{ij}^\Delta(t) \) – the vector estimate of the change of the object/subject \( Q \) of the project \( \Pi_j \) at time \( t \) (instantaneous displacement velocity) relative to the change of the same object/subject in the project \( \Pi_i \).
Therefore, the slower two projects move relative to each other, the more advantageous it is to assign them to one group and manage them based on a single project management system (subsystem).

To determine the degree of proximity of vectors in the project-vector space, it is necessary to obtain the value of distance for each coordinate with vectors of the same object, but other projects.

The distances can be specified as possible values:
1. \( l_{00}^i = l_{00}^n = 1 \) – maximum distance, defines non-intersecting qualitative coordinate values (Fig. 2). For example, one project uses MS Project, another, Primavera, or the projects are managed by different project managers.

\[
\begin{align*}
X_{0i}^0 & = X_{0n}^0 = 1 \\
X_{ki}^0 & = X_{ki}^1
\end{align*}
\]

Fig. 2. Distance between the non-intersecting qualitative coordinate values of vectors of the project-vector space

2. \( 0 < l_{0i}^j = l_{0n}^j < 1 \) – determines a similar filling of objects with qualitative concepts. It is equal to the fact that the vector components in a given measurement intersect (Fig. 3).

\[
0 < l_{0i}^j < 1, 0 < l_{0n}^j < 1
\]

Fig. 3. Distance between the crossed qualitative values of coordinates of vectors of the project-vector space

3. \( l_{0i}^j = 0; 0 < l_{0n}^j < 1 \) – defines the superposition of vectors. Corresponds to the occurrence of components of one vector in another by the chosen dimension (the projection of one vector on the coordinate axis enters the projection of another vector) (Fig. 4).

\[
l_{0i}^j = 0; 0 < l_{0n}^j < 1
\]

Fig. 4. Distance between vectors provided that the qualitative value of the components of one vector is included in the other vector of the project-vector space

4. \( l_{0i}^j = l_{0n}^j = 0 \) – determines the coincidence of the qualitative components of different vectors. It means that the components of one vector coincide with the components of another vector according to the chosen dimension (the projection of one vector on the coordinate axis coincides with the projection of the other vector) (Fig. 5).

\[
l_{0i}^j = l_{0n}^j = 0
\]

Fig. 5. Distance between vectors provided that the qualitative value of components of one vector coincides with the qualitative value of components of another vector of the design-vector space

To determine a numerical measure of the distance between vectors, let’s define a numerical characteristic of qualitative attributes of objects of different projects. Let for the object/subject \( Q_i \) of the project \( \Pi_k \) in the dimension \( i \) it is set by some value \( x_{ki}^j \). To determine the numerical value, the theory of fuzzy sets will be used.

Then
\[
l_{ki}^i = \frac{s\left(x_{ki}^j \cap x_{ki}^j\right)}{s\left(x_{ki}^j\right)},
\]

where \( s\left(x_{ki}^j\right) \) – fuzzy measure on coordinate \( i \) of object/subject \( Q_i \) of project \( \Pi_k \) in the project-vector space; \( s\left(x_{ki}^j \cap x_{ki}^j\right) \) – a fuzzy measure of the intersection of the atomic structure of the object/subject \( Q_i \) in projects \( \Pi_k \) and \( \Pi_l \) by coordinate \( i \) in the project-vector space.

The given model will make it possible to create an educational environment-oriented project management methodology, the components of which will reflect the specifics of the movement of close objects in the project-vector space.

5. 2. The dimensions of the project-vector space

The dimensions of the design space reflect various evaluation categories (project values). Since project implementation corresponds to the movement of objects in the project-vector space, it is necessary to set the dimensions of this space, by which the movement is carried out. If in the real world we do not choose the dimensions of our space, they are formed by the laws of nature, then in the world of projects the dimensions depend on those objects that are to be tracked in the project management system.

Such objects in the project management system of educational environments, can be classified into groups:

1. Projects. Determine the importance of the project in the life of organizations working in educational environments. What it provides for these organizations separately, and for the educational environment as a whole. Formed by stakeholders. Objects of the project-vector space (PVS) are the components that determine the movement of the project itself in the project-vector space. First of all, the goals of the project.

2. Project products. Their list is determined by the project objectives.

3. Project tools. Something used to produce a project product.

4. The subjects of the projects. These include:
   - project performers (project team members);
   - the management of the organization of educational environments (OEE) – project sponsor;
   - project manager;
   - project management team;
   - OEE staff;
   - students and graduate students.
As a result of the research the most significant for the project management system of educational environments objects and subjects of the project-vector space were obtained. All of them are shown in Table 1.

Table 1

Objects and subjects of the project-vector space

<table>
<thead>
<tr>
<th>Objects Dynamic component of the object</th>
<th>Class</th>
<th>Dynamic component of the object</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>Products</td>
<td>Achievement</td>
</tr>
<tr>
<td>Values</td>
<td>Products</td>
<td>Achievement</td>
</tr>
<tr>
<td>Results</td>
<td>Products</td>
<td>Creation</td>
</tr>
<tr>
<td>Product</td>
<td>Products</td>
<td>Creation</td>
</tr>
<tr>
<td>Plan</td>
<td>Projects</td>
<td>Creation</td>
</tr>
<tr>
<td>Organization</td>
<td>Projects</td>
<td>Building</td>
</tr>
<tr>
<td>Regulations</td>
<td>Projects</td>
<td>Creation</td>
</tr>
<tr>
<td>Work</td>
<td>Projects</td>
<td>Implementation</td>
</tr>
<tr>
<td>Interactions</td>
<td>Projects</td>
<td>Implementation</td>
</tr>
<tr>
<td>Control</td>
<td>Projects</td>
<td>Carrying out</td>
</tr>
<tr>
<td>Methods</td>
<td>Tools</td>
<td>Application</td>
</tr>
<tr>
<td>Tools</td>
<td>Tools</td>
<td>Application</td>
</tr>
<tr>
<td>CPS (methodology)</td>
<td>Tools</td>
<td>Development</td>
</tr>
<tr>
<td>CPS (organization)</td>
<td>Tools</td>
<td>Development</td>
</tr>
<tr>
<td>CPS (technology)</td>
<td>Tools</td>
<td>Development</td>
</tr>
<tr>
<td>Senior management</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>Project Manager</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>Project Management Team</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>Project Executives</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>Functional Managers</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>The OEE personnel</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
<tr>
<td>Representatives of the Ministry of Education and Science</td>
<td>Subjects</td>
<td>Satisfaction</td>
</tr>
</tbody>
</table>

Management of these objects is the essence of educational environment management. In the general case, the development of each object/subject is carried out in the space formed only by some dimensions, i.e., directions of development of these objects.

Based on the list given in Table 1, it is possible to define the dimensions of the project-vector space. These dimensions should reflect the directions of changes in the given objects.

Therefore, the dimensions of the project-vector space in the project management system of educational environments are:

1. Projects:
   - degree of goal achievement (project readiness);
   - cost;
   - duration;
   - quality;
   - impact of the project (project goals) on the life of the OEE;
   - methods and means;
   - project structures;
   - the scope of changes in the project;
   - amount of project analysis;
   - information support (awareness) of the project;
   - efficiency of project product formation.

2. Products:
   - project product readiness;
   - volume of research work on the product;
   - project product cost
   - quality of the project product;
   - volume of product changes;
   - information about the project product;
   - the impact of the project product on the life of the OEE;
   - methods and means of project product formation.

3. Tools:
   - tool readiness;
   - amount of research on the creation of the tool;
   - cost of the tool;
   - timeframe for obtaining the tool;
   - quality of the tool;
   - volume of changes;
   - functionality of the tool (its impact on the project product formation process);
   - resources for the tool;
   - efficiency (effectiveness) of the tool;
   - »awareness« (level of information support) of the tool.

4. Subjects:
   - satisfaction.

Summarize these dimensions and form a universal set of dimensions of the project-vector space (Table 2).

In further work, let’s use all dimensions, and only the structures will be combined with methods and means. The dimension «satisfaction» is relevant to all project-vector space objects and subjects.

Table 2

Variable parameter of objects when moving along the dimensions of the project-vector space

<table>
<thead>
<tr>
<th>Measurements</th>
<th>The dynamic component of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Projects</td>
</tr>
<tr>
<td>Ready</td>
<td>Degree of completion</td>
</tr>
<tr>
<td>Cost</td>
<td>Project cost</td>
</tr>
<tr>
<td>Duration</td>
<td>Lead time</td>
</tr>
<tr>
<td>Quality</td>
<td>Project Management</td>
</tr>
<tr>
<td>Functionality</td>
<td>Project</td>
</tr>
<tr>
<td>Methods and tools</td>
<td>Project implementation</td>
</tr>
<tr>
<td>Resources</td>
<td>For the project</td>
</tr>
<tr>
<td>Structures</td>
<td>Project</td>
</tr>
<tr>
<td>Effectiveness</td>
<td>Project</td>
</tr>
<tr>
<td>Awareness</td>
<td>By project</td>
</tr>
<tr>
<td>Changes</td>
<td>Project decisions</td>
</tr>
<tr>
<td>Analysis</td>
<td>Project</td>
</tr>
<tr>
<td>Satisfaction of subjects</td>
<td>From project</td>
</tr>
</tbody>
</table>
This dimension determines the movement of the subjects of the project-vector space, such as senior management, project manager and team members, the OEE employees, and representatives of Ministry of Education (MES). It is the «Satisfaction» dimension that has a decisive value in project value management. Therefore, it will be considered in the spectrum of movement along all other dimensions of the project-vector space.

The question now arises related to the description of the movement of objects and subjects of projects in the project-vector space. For this purpose, it is necessary first to display the static characteristics of this space, or rather, its environment, contributing to or hindering the implementation of the project.

5.3. The model of goal-setting in the project-vector space

The result of any project is evaluated primarily by the degree to which the project objectives are achieved. The objective is not an internal category of the project; it is generated by understanding the project’s place and role in meeting the stakeholders’ needs. It is determined by the project’s external level or its metalevel. The project objective may reflect the readiness of the project product, the desired level of satisfaction of the project subjects, the cost and time of the project, and more.

In the spectrum of the project-vector approach, the criteria for defining the project objectives can be the implemented measurements. In this case, the objective representation in the project-vector space is possible through some abstract point, the achievement of which provides a given level of satisfaction for the project stakeholders. The movement vectors of objects and subjects during project implementation can represent the degree of achievement of the project objective. The aims will be achieved if the actual coordinates of the objects in the project will equal or exceed the target coordinates.

For effective project management based on the project-vector approach, it is necessary to develop appropriate tools for goal setting and goal-achievement in projects of educational environments.

Starting with goal setting. Each objective is defined by the extent to which stakeholders’ needs are met and the costs that the project owner must incur to achieve those goals. Ideally, the objective should satisfy the stakeholders’ needs as much as possible, and its achievement should be of minimal cost to those stakeholders.

In this case, to create tools for «calculating» the optimal goal setting in the project-vector space, by analogy with the physical space, consider that the movement of objects and subjects of the project-vector space to some target point requires a certain «project energy», as which, it is proposed to use financial resources of organizations working in educational environments. The reason for the «costs» is that the educational environment resists the movement of objects and subjects of the project-vector space. Moreover, the resistance is different in different directions of the project-vector space. Unlike the physical environment, the vectors of resistance in the project-vector space are directed along the coordinate axes. This makes it possible to build a model of resistance to motion along individual coordinate axes and to calculate the total resistance to motion as the sum of these «private» resistances.

Set the numerical values of the resistance to motion. The measurements, the speed of movement of objects will be determined from the formula:

\[ v^h = x^{(h)}_n(t + dt) - x^{(h)}_n(t), \]

were \( v^h \) – velocity of the object/subject PVS \( Q \) of the project \( \Pi_k \) in the direction \( N_i \).

The force of resistance to motion:

\[ F^h = \gamma^h \cdot \left( v^h \right)^2, \]

where \( \gamma^h \) – coefficient of resistance to movement of object/subject PVS \( Q \) of project \( \Pi_k \) in direction \( N_i \); \( F^h \) – resistance to movement of the object/subject PVS \( Q \) of the project \( \Pi_k \) in the direction \( N_i \).

Hence, the energy demand (costs) for the movement along measurement \( i \) is equal to:

\[ e^i_s = F^h_s \cdot s = \int_0^T \left( x^{(h)}_n(t) \right) \left( x^{(h)}_n(t) - x^{(h)}_n(t - dt) \right) dt = \]

\[ = \gamma^h \int_0^T \left( v^h(t) \right)^2 \cdot \left( x^{(h)}_n(t) - x^{(h)}_n(t - dt) \right) dt = \]

\[ = \gamma^h \int_0^T \left( v^h(t) \right)^2 \cdot \left( v^h(t) \right) \cdot dt \cdot dt = \gamma^h \int_0^T \left( v^h(t) \right)^2 \cdot dt, \quad (5) \]

where \( s \) – the Path traversed by the object PVS for the time \( T; e^h_s \) – energy (costs) necessary to move the object/the PVS object \( Q \) of the project \( \Pi_k \) in the direction \( N_i \) by distance \( s \) for time \( T; T \) – the project development time.

In fact, the time in the project is discrete, since the project management system considers moments characterized by important events. Therefore, in expression (5) it can pass from the integral, to the sum of:

\[ e^i_s = \gamma^h \sum_{n=0}^T \left( v^h_n \cdot \left( n \cdot dt \right) \right)^2, \quad (6) \]

where \( n \) – the number of the time quantum.

Alternatively, passing to the coordinates, let’s obtain:

\[ e^i_s = \gamma^h \sum_{n=0}^T \left( x^{(h)}_n((n+1) \cdot dt) - x^{(h)}_n(n \cdot dt) \right)^2, \]

where \( x^{(h)}_n((n+1) \cdot dt) \) – the coordinate value of the object/subject PVS \( n+1 \) of the project \( \Pi_k \) in the direction \( N_i \) in the quantum of time \( n+1; x^{(h)}_n(n \cdot dt) \) – the coordinate value of the object/subject PVS \( Q \) of the project \( \Pi_k \) along the \( N_i \) axis in the quantum of time \( n \).

Movement is always carried out along all the dimensions of PVS. The optimal goal of the project (such a point of space), the achievement of which corresponds to the «maximum expansion of PVS» in those directions that are most important for educational environments, and at the same time, the limited financial resources will be used.

Let’s consider the certainty of movement with the existence of several dimensions. In this case, let’s set the value of the financial resources allocated to the project. Let’s obtain:

\[ n^i_k : E^k = \sum_{N_i} \gamma^h \cdot \sum_{n=0}^T \left( x^{(h)}_n((n+1) \cdot dt) - x^{(h)}_n(n \cdot dt) \right)^2, \]

where \( E^k \) – financial resource (energy) of the project \( \Pi_k \).
Given the impossibility of moving to the origin of coordinates (which would mean the destruction of some of the results of the project, which is generally impossible in real projects) let's obtain:

\[ \forall t, dt: x_i(t+dt) \geq x_i(t) \implies \forall \Pi_i: E_i = \sum_n k_i \left( x_{ij}(T)^2 \right) \]

where \( x_{ij}(T) \) – the final value of the coordinate of the object/subject of the PVS \( Q \) of the project \( \Pi_i \) along the axis \( N_j \).

Then the search for the optimal target for an individual project can be represented through the problem of determining such finite coordinates of objects and subjects of the PVS \( \Pi_i \):

\[ \forall Q_j: x_{ij}^1(T), x_{ij}^2(T), ..., x_{ij}^N(T), \] for which:

\[ \forall \Pi_i: \sum_j \left( k_{ij} \sum_p \left( \sigma_{ij} \cdot x_{ij}^p(T) \right) \right) \rightarrow \text{max}, \quad (7) \]

with constraints:

1. Unforced resistance to motion:
2. \( \forall \Pi_i: E_i \geq \sum_n k_i \left( x_{ij}(T)^2 \right) \] where \( \lambda_{ij} \) – the priority of movement in the direction of \( N_j \) (how important it is that the goal reflects movement in this direction); \( \sigma_{ij} \) – the priority of the object/subject \( Q \) PVS.

So, mathematical models of the project-vector space were developed in work. They are tools of the methodology of project-vector management of educational environments to increase the efficiency of university management. These tools are implemented in applied information technology for the administration of higher education institutions.

The described models and corresponding methods were implemented in the activities of Kyiv National University of Construction and Architecture and Astana IT University.

The mathematical model of PVS of educational environments is based on the personal data of the participants of projects, the development of a mathematical model of educational environment project management. A classification of projects in research environments is also given and it is distinctive from traditional ones as of that reflect the PSV measurements and allows structuring the components of multi-system project management in organizations in the educational sphere of developing countries (Fig. 1, (1), (2)).

**6. Discussion of the development of a mathematical model of PVS of educational environments**

The article has a fundamental focus, but the developed models are essential in project and program management theory, particularly in education. The implementation of PVS is based on the personal data of the participants of the educational process, so they cannot be publicly distributed. Moreover, the implementation of the methodology is characterized by a significant length of time. Since many external and existing internal factors influence the educational environment, it is not easy to immediately assess the effectiveness of implementing the methodology. Preliminary evaluations of the implementation of work results in the activities of Kyiv National University of Construction and Architecture and Astana IT University had a positive effect.

The article formalizes PSV measurements and objects that reflect various evaluative categories (values of educational environments) and characterize the movement of objects in this space, which made possible to develop the mathematical model of educational environment project management. A classification of projects in research environments is also given and it is distinctive from traditional ones as of that reflect the PSV measurements and allows structuring the components of multi-system project management in organizations in the educational sphere of developing countries (Fig. 1, (1), (2)).

Mathematical models have been developed for estimating the magnitude of the similarity of vectors per magnitude by the duration of time intervals, as well as for estimating the magnitude of the vectors’ proximity. For estimating latter vectors are set by qualitative categories, which is based on a multidimensional reverse scheme for estimating the coordinates of these categories, which allows developing a method for grouping PSV objects according to the components of a multi-project management system. The method of grouping PSV objects by vectors’ proximity differs from other methods by high accuracy of distances between quality categories of projects in PSV, which allows optimal structuring of the project management system of educational facilities.

The mathematical model of goal-setting in PSV has been improved. These improvements are based on the expansion model of the «Universe of Projects» and allow to calculate such trajectories in PSV to achieve the objectives of project goals with minimal time and financial resources (5)–(7).

It is shown that to build an effective project management system; it is important to reflect not so much the individual vectors of movement of objects and subjects of the project-vector space as whether these vectors are the same or different. The creation of separate components of multisystem management of many projects is based not only on «unidirectional» development of individual objects and subjects of the projects, but also on their proximity in the project-vector space. For example, the same methods or means of project management are used, the same performers are used, and the same input information is used. If the vectors are the same, the movements of objects in these projects are equally defined. Consequently, a universal component of an educational environment project management multisystem can be created that applies to these projects (Fig. 2–5).
The task of grouping the objects of PVS projects so that the distance between the vectors of objects included in one group was minimal was formulated to create an effective multisystem of managing projects of educational environments. The mathematical apparatus of vector algebra was used to calculate the distances between vectors and determine the optimal set of groups of projects (and, respectively, subsystems of the multisystem of project management). This created a mathematical model of an integral multisystem of project management of educational environments.

Mathematical models are developed to assess the magnitude of the similarity of vectors at significant time intervals and the magnitude of vectors' proximity by qualitative categories. These models show that the slower the shift of objects of different projects relative to each other, the more profitable it is to classify them into one group and manage them based on a single component of the multisystem of project management.

It is shown that the measurements of the project-vector space reflect different evaluation categories (project values), which characterize the movement of the objects of PVS. As a result of this research, the list of the most significant for the multisystem of project management of educational environments of the objects and subjects of the project-vector space was obtained. In order to define the measurements of the project-vector space along which movements are carried out, all objects/subjects of the PVS are divided into groups: projects, project products; project tools; project subjects.

The variables of objects moving along the measurements of the project-vector space are specified and the measurements of the PVS are classified.

The notion of objects' potential movement in the PVS is introduced. It is the characteristic of objects that determines their own ability to change coordinates in the project-vector space. The number of financial resources allocated for developing this object is taken as the universal potential of the object/subject of the project-vector space.

The definition of the unforced resistance of the PVS is given. These are patterns in the impact of a significant number of objects and subjects of the project-vector space on each other that persist in most projects of educational environments and hinder their implementation.

It is shown that in the spectrum of the project-vector approach, the criteria for the certainty of the project goals are the introduced measurements. The goal in the project-vector space is represented through some abstract coordinates. For the end of the vector will be specified not by a single number, but by a tuple that displays the «differences» in this coordinate with the vectors of the same object, but of other projects. This allows to compare the development vectors of projects with each other and identify how close the objectives are to each other, which in turn will allow to apply the same tools and project management approaches to the same projects. The structure of the project management system for educational environments has been developed. To create it, the measurement of grouping objects in the project-vector space was formulated and solved in such a way that the distance between the vectors of objects included in the same class was minimal. The mathematical apparatus of vector algebra was used to calculate the distances between vectors and determine the optimal set of project groups (and, accordingly, subsystems of the project management system).

1. Determining the proximity of vectors made it possible to solve the problem of clustering projects and determining the structure of the project management system for educational environments. The article proposes the calculation of the proximity of vectors by their coordinates, but it is difficult to apply a continuous numerical measure to many measurements. After all, many project parameters are set by qualitative rather than quantitative parameters. Therefore, it is proposed to apply the inverse approach to determining the proximity of vectors. Each vector coordinate is represented by distances to the coordinates of other vectors (in this dimension) – the distance between project objects. For the same values of objects (even qualitative ones), the distance equal to 0 is taken. For different qualitative values – 1. In this case, there is no need to translate qualitative concepts into quantitative measures. Then the «reduced» coordinate of the end of the vector will be specified not by a single number, but by a tuple that displays the «differences» in this coordinate with the vectors of the same object, but of other projects. This allows to compare the development vectors of projects with each other and identify how close the objectives are to each other, which in turn will allow to apply the same tools and project management approaches to the same projects.

2. The article formalizes the dimensions and objects of PVS, which reflect various evaluative categories (values of educational environments) and characterize the movement of objects in this space, which made it possible to develop a mathematical model for managing projects of educational environments. The dimensions of the project-vector space reflect various appraisal categories (project values). Since the execution of the project corresponds to the movement of objects in the project-vector space, it is necessary to set the dimensions of this space, along which the movement is carried out. If in the real world we do not choose the dimensions of our space, they are formed by the laws of nature, then in the world of projects the dimensions depend on those objects that are subject to tracking in the project management system. It is shown that the dimensions of the project-vector space are characterized by the fact that the greater the value of the coordinate, the more developed is the object or subject, the projection of which on this axis has the given value of the coordinate.

3. A distinctive feature of the developed mathematical model of goal setting in PVS is that it is based on the expansion model of the «Universe of Projects» and allows to calculate such trajectories in PVS that will ensure the achievement of project goals with minimal time and financial resources. For effective management of project goals on the basis of the project-vector approach, a mathematical model of goal-setting in the project-vector space has been developed. It is shown that for effective project management based on the project-vector approach, it is necessary to develop appropriate goal-setting and goal-achieving tools in educational environment projects. The article shows that each goal is...
characterized by the degree of satisfaction of the needs of stakeholders and the costs that the project owner must incur in order to achieve these goals. Ideally, the goal should be such that it satisfies the needs of stakeholders as much as possible and its achievement should be minimally costly for these parties.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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