This paper considers the organization of a goods delivery process, which is selected as the study object. It has been established that the main problems that arise in this case can be caused, for example, by the imperfection of infrastructure and transport for delivery. This can be partially solved through the use of unmanned aerial vehicles for the delivery of goods, as well as by solving tasks related to effective control over their movement. However, there is another issue associated with the insufficient efficiency of existing mathematical models of goods delivery systems involving unmanned aerial vehicles since the maximum possible delivery speed is not provided. Therefore, there is a need to find a better solution to this problem.

A model of the goods delivery system that uses unmanned aerial vehicles based on priority has been built. The resulting model takes into account the intensity of requests and provides a shorter waiting time in the queue, and therefore a greater delivery speed.

Models of single-channel and multichannel goods delivery systems with failures and expectations were investigated according to probability. It was found that the devised goods delivery system is on average less loaded per unit of time and makes it possible to serve more orders while incoming orders are in line for less time. The same models have also been investigated according to the waiting time in the queue. It has been established that the devised goods delivery system provides a shorter waiting time in the queue. At the same time, the deviation between the theoretical and experimental values of probabilities and waiting time is 2% and 3%, respectively, which allows us to assert high accuracy of the results and the devised model as a whole.

The results reported here could be used in practice in the absence of an extensive network of logistics and sales and remoteness of recipients.

Keywords: time allocation, delivery priority, goods delivery system, unmanned aerial vehicle

1. Introduction

Logistics, as an economic process and as a management function, involves a single system for the movement of information, material and financial goods and services. In this case, an integral part is transportation, which is inextricably linked with production and trade processes because it allows for the delivery of goods and transportation of people [1].

The main problems that arise when organizing the delivery of goods can be caused by various factors. These are infrastructure imperfections, unsatisfactory condition of transport for maintenance and its deterioration, poor quality of transport routes, congestion, long distances between points of reception and delivery of goods, poor organization of the delivery process, unpredictable costs, etc. [2, 3]. These problems can be solved by using unmanned aerial vehicles (UAVs) as transportation means for the delivery of goods and by solving tasks related to effective control over their movement [4]. To this end, there are mathematical models of goods delivery systems (GDS) that use UAVs [5] but they are not effective enough since they do not provide the maximum possible speed of delivery of goods. Therefore, there is a need to find a better solution to the problem of UAV traffic control. This can be done by increasing the efficiency of existing GDS models by reducing the average waiting time in the queue when delivering one unit of goods.

GDS can be described as a mass service system (MSS) because it includes requirements for service, service stations, and order queues. Certain requirements are put forward to it, namely:

- high probability of implementing the order;
- low waiting time in line.

There are a large number of MSS models for performing a variety of tasks, namely for building territory maps [6, 7], creating a more reliable communication system [8], modeling UAV flight traffic [9–11]. Therefore, there is a need to analyze these models and select such an implementation that can be used for GDS that use UAVs in accordance with the above requirements, in particular the short waiting time in the queue.

A relevant direction in the development of GDS is the use of UAVs as a means for delivering a variety of goods in the context of solving business problems, in particular the delivery of orders from online stores. At the same time, it is also relevant to build an appropriate GDS model that would ensure that the delivery process is as efficient as possible. Important criteria for this are the GDS parameters, namely the average number of orders per unit of time, the number of orders and places in the queue, the consolidated intensity of service and channel loading.
2. Literature review and problem statement

In [6], the UAV model using MSS as a point Poissonian process for constructing 3D maps of the territory in a cylindrical and Cartesian coordinate system is considered. Nevertheless, the use of this model is possible provided that constant communication between UAVs is maintained. In [7], a model of a distributed measuring system based on mixed integer linear programming is proposed, taking into account the possibility of recharging UAVs. This model can be adapted for different types of measurement for which the result can be obtained using numerical methods. However, this model is not adaptive for GDS as there are unresolved difficulties associated with replacing UAVs directly during delivery. In [8], the construction of a more reliable communication system using a UAV network with a decrease in computational complexity for calculating the position of each individual UAV is considered. This model is well suited for building a low-latency communication system for low-power devices. However, unlike information in mobile low-power networks, GDS models are difficult to consider as distributed. The adaptability of models [6–8] to perform specialized applied tasks, taking into account the limitations of the battery life of UAV, is shown. But issues related to modeling systems as a whole remained unresolved. This causes difficulties in assessing the effectiveness of these models under different parameters and the possibility of their application to GDS.

In [9], the results of the modeling of UAV flight traffic are reported. Work [10] also shows the high adaptability of models for performing a variety of applied tasks. But, in works [9, 10], the battery life of the UAV is not taken into account and the issues related to insufficient efficiency of service requests remained unresolved. This causes difficulties in the possibility of application for GDS under conditions of high and medium request intensity. Work [11] reports the results of modeling MSS using a real-time UAV under heavy loads. The high efficiency of the model for emergency response systems is shown. Nevertheless, this system will not be so effective under various loads. This causes difficulties in the possibility of application for GDS under conditions of low and medium intensity of requests, that is, it makes it costly and excessive.

An option to overcome the above difficulties may be to use MSS based on priority. This approach is used in [12, 13], which consider the models of systems that are based on the preliminary determination of priority and without, respectively. Nevertheless, these models do not provide sufficient service speed. Also, a similar principle is implemented in [14], where it is proposed to reduce the time of delivery of goods by reducing the likelihood of UAV downtime. However, depending on the intensity of the requests, a situation is possible when some UAVs are not involved in the work while others are excessively loaded. This leads to an increase in waiting time in the queue and, as a result, a deterioration in the effectiveness of GDS.

Thus, the problem of insufficient efficiency of existing mathematical models of GDS that use UAVs can be stated as follows. Although GDS can be described as MSS, but most existing MSS models are not adaptive for GDS [6–8] or adaptive only partially [9–11]. Adaptive for GDS are only MSS based on priority but they do not provide sufficient service speed [12, 13]. At the same time, existing GDS with the help of UAVs based on priority [14] are not effective enough.

All this gives reason to argue that it is advisable to conduct a study to improve the effectiveness of GDS based on priority, which would reduce the waiting time in the queue.

3. The aim and objectives of the study

The aim of this work is to devise a model of GDS that use UAV based on priority. This will make it possible to build a new model of GDS involving UAV based on priority, which will take into account the intensity of requests and provide a shorter waiting time in the queue.

To accomplish the aim, the following tasks have been set:

- to investigate GDS models according to performance indicators;
- assess the performance of a GDS model using UAV based on priority.

4. The study materials and methods

The object of our study is the process of organizing the delivery of goods using UAVs. The construction of a suitable GDS that use UAVs based on priority was carried out by improving the model given in [14] while hypothesizing that the average waiting time in the queue will be shorter if we take into account the battery life. It has been suggested that the probability distribution of the requirements for the delivery and for servicing these requirements is close to the Poisson distribution and can be described using MSS. To simplify the construction of a GDS model using UAVs based on priority, battery aging was not taken into account, and when modeling the operation of UAVs for GDS based on priority, the limitation of the modeling environment was taken into account.

Checking the effectiveness of the devised model and comparing the data were provided by the Mathcad computer algebra system (USA). This system has a powerful mathematical apparatus that makes it possible to find solutions to algebraic and differential equations, to carry out operations with vectors and matrices, and also has powerful means of graphical representation of information [15]. To evaluate the GDS model based on priority, the WeBots environment (Switzerland) was used, which is a professional software package for modeling the operation of mobile robots, in particular UAVs, and the process of loading and unloading at a virtual station. This makes it possible to create 3D-virtual worlds with physical properties, program individual objects to achieve the desired behavior or provide a mechanism for their coordination, considering the influence of external factors insignificant, that is, controlled [16]. The combination of the Mathcad system and the WeBots environment makes it possible to evaluate the effectiveness of the devised model and test it in practice by simulation in the created virtual world with further analysis of the results.

In GDS involving UAVs based on priority, the number of orders that are being serviced is equal to the number of free transport channels. Since UAVs are used, the number of free transport channels is theoretically unlimited but in practice, this number is limited to the number of UAVs. Thus, the work of GDS involving UAVs based on priority, which determines the states of functioning of the system, can be described using the following system of equations [14]:
where $P_0, P_1, P_2, P_3, \ldots, P_{n-1}, P_n$ are the probabilities of finding the system in states $S_0, S_1, S_2, \ldots, S_{n-1}, S_n$ respectively; $\lambda$ – the intensity of receipt of orders in GDS, that is, the average number of orders in the system per unit of time; $n$ – the number of free UA Vs; $\mu$ – consolidated intensity of service.

The solution to this system is the probability $P$ of UA Vs engagement and, accordingly, of the channel in the GDS that use UA Vs based on priority:

$$P_t = \frac{1}{\sum_{i=0}^{n} \frac{n! \lambda^i t^i}{(n-i)!}},$$

$$P = 1 - P_0,$$

(2)

where $P_0$ is the probability of UAV downtime; $t$ – delivery time.

The GDS model with a UAV based on priority (1), (2) has limitations since it does not take into account the need to replace the UAV battery. It takes some time $t_b$, which takes into account the actual replacement of the battery, the UAV’s flight to the place for replacement, and departure after its completion. If the battery life is short, the delivery requirement will be canceled and the UAV responsible for it will fly to replace the battery. To eliminate this limitation, the GDS priority UAV model was used as the base model for the construction of a new GDS model by taking into account the UAV battery life. Thus, priority $K$ was introduced, which takes into account the delivery time (from the base GDS) $t$, and the battery life of UAV $t_{W_b}$ and is described using a system of equations:

$$K = \begin{cases} 1, & t_f < 0.9 t_{w_b} - t; \\ 0.5, & 0.5 t_f > 0.9 t_{w_b} - t; \\ 0, & t_f > 0.95 t_{w_b}. \end{cases}$$

(3)

where $t_f$ is the flight time since the last battery change.

Thus, taking into account the priority of battery life $t_{W_b}$ will allow $k+1$ to be met as a higher priority and perform battery replacement after completing the application. Taking into account the total operating time $T = W + 1/\mu$ [5], the system on average serves $L_2$ applications during the service time $T_s$. Therefore, during the time of UAV operation, the following number of requirements for delivery can be fulfilled:

$$K = \frac{0.9 t_{w_b} - T_s}{T_s} L_2.$$

(4)

Given the average time spent in the system, fulfilling the last delivery requirement takes time $T_s = 1/\mu$, during which the UAV is used until the battery is replaced, and this requirement is not transferred to another UAV. Therefore, on average, the consolidated intensity of service will increase slightly and become:

$$\mu_s = \mu + \frac{k+1}{k} (1 + \mu \cdot t_f).$$

(5)

Thus, the devised model of GDS that use UA Vs based on priority can be described using a system of equations:

$$\frac{d}{dt} P_{m0} = -n \lambda P_{m0} + \mu^2 P_{m1};$$

$$\frac{d}{dt} P_{m1} = -n \lambda P_{m1} + \mu^2 P_{m2};$$

$$\frac{d}{dt} P_{m2} = -n \lambda P_{m2} + \mu^2 P_{m3} + (n-1) \lambda P_{m1} - \mu^2 P_{m2};$$

$$\ldots$$

$$\frac{d}{dt} P_{mn} = -\lambda P_{m(n-1)} - \mu^2 P_{mn};$$

$$\sum_{i=0}^{n} P_i = 1,$$

(6)

where $P_{m0}, P_{m1}, P_{m2}, P_{m3}, \ldots, P_{m(n-1)}, P_{mn}$ are the probabilities of finding the system in states $S_0, S_1, S_2, \ldots, S_{n-1}, S_n$ respectively.

The solution to this system, and therefore an indicator of efficiency, is the probability $P_{m}$ of UAV engagement and, accordingly, the channel in the devised GDS that use UA Vs based on priority:

$$P_{m0} = \frac{1}{\sum_{i=0}^{n} \frac{n! \lambda^i t^i}{(n-i)!}},$$

$$P_{m} = 1 - P_{m0},$$

(7)

where $P_{m0} = W$ is the probability of UAV downtime.

Another indicator of efficiency is the waiting time $W$ in the queue, which can be determined from the probability distribution of the waiting time in the queue:

$$P(W) = 1 - e^{-W},$$

(8)

where $W$ is the average waiting time in the queue.

Comparison of models shows that solution (6), taking into account the priority, which takes into account the time of delivery and battery life, makes it possible to move from minimizing the probability of UAV downtime to minimizing the average waiting time in the queue. This is illustrated by (7) and (8), which is the basis of the devised model.

Our study was conducted in the WeBots environment by testing the effectiveness of the proposed GDS model using UA Vs based on priority. A virtual world (landscape, trees, and man-made objects) was created, in which the GDS was implemented with the help of three Mavic 2 Pro UA Vs. Delivery of goods in the form of the same type of cargo weighing 150 grams was carried out during the UAV maximum flight time of 30 minutes.

Delivery efficiency is determined by higher probability values with smaller quantities of loaded channels, and smaller values – at larger quantities. Also, the efficiency of delivery is determined by a shorter waiting time in the queue. GDS corresponds to the Poisson flow [5], so the probability distribution of the waiting time in the queue is characterized by an average value for the exponential law of probability distribution. Validation of results is defined as a small deviation of the values of the probability distribution and waiting time of theoretical and experimental GDSs, which indicates the adequacy of the devised model.
5. Results of investigating the devised model of the goods delivery system using unmanned aerial vehicles based on priority

5.1. Investigating models of the delivery system according to performance indicators

A study of GDS models was carried out, in which \( \lambda = 5 \) orders per hour for servicing goods are received at the input while the average intensity of service (system loading) is \( \rho = 0.5 \). The system has \( n = 3 \) channels that can be free or engaged. In the system at the same time, on average, there are \( \mu = 10 \) orders waiting for service. At this intensity, the system on average serves orders faster than orders arrive, so the system is stable and not overloaded.

Single-channel and multi-channel GDSs were considered [17] with failures and expectations for the delivery of goods based on priority and without. Since the systems had three channels, the following states are possible for them:

- \( i = 0 \) – all channels are free;
- \( i = 1 \) – one channel is engaged, the other two are free;
- \( i = 2 \) – two channels are engaged, one channel is free;
- \( i = 3 \) – all three channels are engaged, the service requirement is refused.

The study of single-channel and multichannel GDSs with failures was carried out taking into account five waiting slots in the system.

Verification of the effectiveness of these models is illustrated by plots (Fig. 1–8), the construction of which was carried out in the system of computer algebra Mathcad.

Fig. 1 shows the probability distribution for the model of a single-channel GDS with failures based on priority and for the devised model of a single-channel GDS with failures based on priority.

![Fig. 1. Probability distribution for a single-channel delivery system based on priority \( P \) with failures and for a developed single-channel goods delivery system based on priority \( P_m \) with failures](image1)

Fig. 1 shows that the use of the devised model in single-channel GDSs with failures makes it possible to increase the probability of the state \( P_m \) of the free channel at \( i = 0 \). It will also reduce the probability of the state \( P_m \) of the engaged channel at \( i = 1 \). Thus, in general, the devised GDS with the priority of delivery on average per unit of time is less loaded and makes it possible to serve more orders while incoming orders receive a smaller number of failures. The sum of the probabilities for a single-channel GDS with failures based on priority \( P \) and the developed single-channel GDS with failures based on priority \( P_m \) for all states \( i = 0–1 \) is 1. This shows the correctness of our calculations.

Fig. 2 shows the distribution of the waiting time for the model of a single-channel GDS with failures based on priority and the devised model of a single-channel GDS with failures based on priority. The waiting time distribution plots for a single-channel GDS with failures based on priority and the developed single-channel GDS with failures based on priority are constructed for the average waiting time \( W \) and \( W_m \), respectively.

![Fig. 2. Distribution of waiting time for a single-channel goods delivery system with failures based on priority and the developed single-channel goods delivery system with failures based on priority](image2)

Fig. 2 shows that at a probability of 0.95, the waiting time for a single-channel GDS with failures based on priority is 0.267 \( W_q \), and for the developed single-channel GDS with failures based on priority – 0.2 \( W_q \). Thus, the devised model provides a shorter waiting time in the queue.

Fig. 3 shows the probability distribution for a single-channel GDS model based on priority with waiting and the devised model of single-channel GDS based on priority with waiting.

![Fig. 3. Probability distribution for a single-channel goods delivery system with waiting based on priority and the developed single-channel goods delivery system with waiting based on priority](image3)
Fig. 3 shows that the use of the devised model in single-channel GDSs with waiting makes it possible to increase the probability of the state $P_m$ of the free channel at $i=0$. It will also reduce the likelihood of the state $P_m$ of the engaged channel with an increase in the queue of orders $i$. Thus, in general, the devised GDS with the priority of delivery on average per unit of time is less loaded and makes it possible to serve more orders while incoming orders are in line for less time. The sum of the probabilities for a single-channel GDS with waiting based on priority $P$ and the developed single-channel GDS with waiting based on priority $P_m$ for all states $i=0–3$ is 1. This shows the correctness of our calculations.

Fig. 4 shows the distribution of the waiting time for the model of a single-channel GDS with waiting based on priority and the devised model of a single-channel GDS with waiting based on priority. The plots of waiting time distribution for a single-channel GDS based on priority with waiting and the developed single-channel GDS based on priority with waiting are built for the average waiting time $W$ and $W_m$, respectively.

Fig. 4 shows that at a probability of 0.95, the waiting time for a single-channel GDS with a priority-based waiting is $0.3W_q$, and for the developed single-channel GDS with a priority-based waiting – $0.267W_q$. Thus, the devised model provides a shorter waiting time in the queue.

Fig. 5 shows the probability distribution for the priority-based multichannel GDS model with failures and the devised model of a multichannel GDS with failures based on priority.

Fig. 5 shows that the use of the devised model in multichannel GDS with failures makes it possible to increase the probability of the state $P_m$ of the free channel at $i=0$. It will also reduce the probability of the state $P_m$ of the engaged channel with an increase in the queue of orders $i$. Thus, in general, the devised GDS with the priority of delivery on average per unit of time is less loaded and makes it possible to serve more orders while incoming orders are in line for less time. The sum of the probabilities for a multichannel GDS with failures based on priority $P$ and the developed multichannel GDS with failures based on priority $P_m$ for all states $i=0–3$ is 1. This shows the correctness of our calculations.

Fig. 6 shows the distribution of the waiting time for a model of multichannel GDS with failures based on priority and the devised model of a multichannel GDS with failures based on priority. The waiting time distribution plots for a multichannel GDS with failures based on priority and the developed multichannel GDS with failures according to priority are built for the average waiting time $W$ and $W_m$, respectively.

Fig. 6 shows that at a probability of 0.95, the waiting time for a multichannel GDS with failures based on priority is $0.08W_q$, and for the developed multichannel GDS with failures based on priority – $0.06W_q$. Thus, the devised model provides a shorter waiting time in the queue.

Fig. 7 shows the probability distribution for a multichannel GDS model with waiting based on priority and the devised model of a multichannel GDS with waiting based on priority.

Fig. 7 shows that the use of the devised model in multichannel GDS with waiting makes it possible to increase the probability of the state $P_m$ of the free channel at $i=0$. It will also reduce the likelihood of the state $P_m$ of the engaged channel with an increase in the queue of orders $i$. Thus, in general, the devised GDS with the priority of delivery on average per unit of time is less loaded and makes it possible
Control processes
to serve more orders while incoming orders are in line for
less time. The sum of the probabilities for a multichannel
GDS with waiting based on priority $P$ and the developed
multichannel GDS with waiting according to priority $P_m$ for
all states $i=0–3$ is equal to 1. This shows the correctness of
our calculations.

5.2. Evaluating the operability of the model of a goods
delivery system using unmanned aerial vehicles based
on priority

In practice, the process of delivering goods using UAVs
requires high control accuracy and orientation in the space
of these devices. Therefore, the simulation of UAV operation
for GDS based on priority was carried out in the WeBots
environment by creating a virtual world, the general view of
which is shown in Fig. 9.

Fig. 8 shows the distribution of waiting time for the model
of a multichannel GDS with waiting based on priority and
the devised model of a multichannel GDS with waiting based
on priority. The waiting time distribution plots for the multi-
channel GDS with waiting based on priority and the deve-
loped multichannel GDS with waiting based on priority are
built for the average waiting time $W$ and $W_m$, respectively.

Fig. 9 shows a virtual environment that looks like a city
containing trees, buildings, roads, and other man-made
objects. It also depicts UAVs that are involved in the delivery
of goods. The working areas of the UAV’s territory and trajec-
tory were assigned as a set of coordinates, which were input
data for UAVs, in the corresponding software controller of the
WeBots environment.

The evaluation of the model’s performance was carried out
by performing a series of 10 virtual launches of GDS with three
UAVs, so the number of channels was limited to 3, and the sys-
tem states to 4 ($i=0, 1, 2, 3$). $\lambda=5$ orders per hour for servicing
goods at an average service intensity of $\rho=0.5$ were received
at the input. In the system at the same time there were an
average of $\mu=10$ orders waiting for service. For single-channel
and multi-channel GDS with failures, five waiting slots in the
system were used. The resulting experimental probability va-
lues for GDS with waiting based on priority $P_m$ and the devised
GDS $P_{exm}$ under different states of the system were compared
with the theoretical values of $P$ and $P_m$, respectively. Similarly,
the experimental values of the waiting time in the queue were
compared with the theoretical ones. This is illustrated by
plots (Fig. 10–17), the construction of which was carried out
in the system of computer algebra Mathcad.

Fig. 10 shows probability distribution for the model of
a single-channel GDS with failures based on priority and
the developed model of single-channel GDS with failures
based on priority for experimental and theoretical probabil-
ity values.

Fig. 11 shows the distribution of the waiting time for
the model of a single-channel GDS with failures based
on priority and the devised model of a single-channel GDS
with failures based on priority for experimental and theore-
tical values.
Fig. 11 shows that the difference between theoretical and experimental waiting time values for a single-channel GDS with failures based on priority and the devised GDS differs not significantly, namely by 3%.

![Graph showing the comparison between theoretical and experimental waiting time values for a single-channel GDS.]

Fig. 12 shows the probability distribution for a single-channel GDS model with waiting based on priority and the devised model of a single-channel GDS with waiting based on priority for experimental and theoretical probability values.

Fig. 12 shows that the difference between the theoretical and experimental probability values for a single-channel GDS with waiting based on priority and the devised GDS differs not significantly, namely by 2%.

Fig. 13 shows the distribution of waiting time for the model of a single-channel GDS with waiting based on priority and the devised model of a single-channel GDS with waiting based on priority for experimental and theoretical values.

Fig. 13 shows that the difference between the theoretical and experimental waiting time values for a single-channel GDS with waiting based on priority and the devised GDS differs not significantly, namely by 3%.

![Graph showing the comparison between theoretical and experimental waiting time values for a single-channel GDS.]

Fig. 14 shows the probability distribution for a multichannel GDS model with failures based on priority and the devised model of a multichannel GDS with failures based on priority for experimental and theoretical probability values.

Fig. 14 shows that the difference between the theoretical and experimental probability values for a multichannel GDS with failures based on priority and the devised GDS differs not significantly, namely by 3%.
Fig. 14 shows the probability distribution for the priority-based multichannel GDS model and the devised model of a multichannel GDS with failures based on priority for experimental and theoretical probability values.

Fig. 14 shows that the difference between theoretical and experimental probability values for a multichannel GDS with failures based on priority and the devised GDS differs not significantly, namely by 2%.

Fig. 15 shows the distribution of waiting time for the model of a multichannel GDS with failures based on priority and the devised model of a multichannel GDS with failures based on priority for experimental and theoretical values.

Fig. 15 shows that the difference between theoretical and experimental waiting time values for a multichannel GDS with failures based on priority and the devised GDS differs not significantly, namely by 3%.

Fig. 16 shows the probability distribution for a multichannel GDS model with waiting based on priority and the devised model of a multichannel GDS with waiting based on priority for experimental and theoretical probability values.

Fig. 16 shows that the difference between theoretical and experimental probability values for a multichannel GDS with waiting based on priority and the devised GDS differs not significantly, namely by 2%.

Fig. 17 shows the distribution of waiting time for a multichannel GDS model with waiting based on priority and the devised model of a multichannel GDS with waiting based on priority for experimental and theoretical values.

Fig. 17 shows that the difference between the theoretical and experimental waiting time values for a multichannel GDS with waiting based on priority and the devised GDS differs not significantly, namely by 3%.

The reliability of the devised GDS model was assessed using UAVs based on priority in comparison with other models. These are p-medians taking into account engaged UAVs (p-Median w/CoBU) (USA) and random selection (Random Target) (Switzerland) [18], weighted planning (WBSS) (USA), and shortest travel time or distance (STTD) (Turkey) [19]. These models were chosen as the closest to the devised model due to the similarity of the mathematical apparatus, taking into account the priority used for both MSS and GDS that use UAVs. Calculations were performed in the Mathcad system based on data obtained from the WeBots environment. Averaged results are given in Table 1.

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<td>7.4</td>
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<td>Devised</td>
<td>0.027</td>
<td>–</td>
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</tr>
<tr>
<td><strong>Multi-channel</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>GDS with waiting</td>
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<tr>
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<td>11.8</td>
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<tr>
<td>Random Target</td>
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Table 1 illustrates that the shortest waiting time is demonstrated by the devised GDS model using UAV based on priority. For a single-channel GDS with failures, it is 0.17 hours, for a single-channel GDS with waiting – 0.193 hours, for a multi-channel GDS with failures – 0.027 hours, for a multi-channel GDS with waiting – 0.051 hours. This indicates the high reliability of our model.

6. Discussion of results of investigating the devised model of a goods delivery system using unmanned aerial vehicles based on priority

The results of studies of the devised model of GDS that use UAVs based on priority show slightly less loaded systems with the same intensity of delivery requirements. This is explained by minimizing the probability of UAV downtime (Fig. 1, 3, 5, 7), which leads to minimization of the average waiting time in the queue (Fig. 2, 4, 6, 8). This indicates the effectiveness of the devised GDS model using UAVs based on priority. Fig. 10, 12, 14, 16 show that the difference between the theoretical and experimental probability values for GDS based on priority and the devised GDS based on priority differs not significantly, namely by 2 %. Fig. 11, 13, 15, 17 show that the difference between the theoretical and experimental values of the waiting time for GDS based on priority and the devised GDS based on priority differs not significantly, namely by 3 %. All this indicates the adequacy of the devised GDS model involving UAVs based on priority and high accuracy of our results. Also, Fig. 1, 3, 5, 7 demonstrate that the sum of the probabilities of the devised GDS model using UAVs based on priority for all states is 1. Fig. 2, 4, 6, 8 show that the plot of waiting time distribution for the devised GDS model using UAV based on priority is exponential in nature and asymptotically approaches 1. All this indicates the convergence of this model. The resulting data were confirmed by research in the WeBots environment (Fig. 10–17).

The reliability of the devised GDS model using UAVs based on priority is comparable (Table 1) with other models [18, 19] and is not inferior to them. That is, the waiting time in the queue for the devised GDS model is 1–15 % less. This is achieved by taking into account the battery life of UAV.

The devised model of GDS involving UAVs based on priority takes into account the intensity of requests and provides a shorter waiting time in the queue. Taking into account the delivery time and battery operation makes it possible to increase the efficiency of GDS, by minimizing the likelihood of UAV downtime, and, consequently, minimizing the average waiting time in the queue. This is reflected in the process of theoretical and experimental research, namely the probability and waiting time distributions for single-channel and multichannel GDSs with failures and waiting.

Our results can be used in the implementation of GDS that use UAVs for commercial needs, for example, when delivering orders from online stores. The condition for the application of the devised GDS is the absence of an extensive network of logistics and sales. The potentially expected effect of the implementation of the devised GDS will be to increase profits due to a decrease in the waiting time for goods in the queue. In addition, the devised GDS involving UAVs can be used for emergency services in the delivery of rescue equipment, in particular medicines. The conditions of application, in this case, are the remoteness of the recipients and the efficiency of delivery. Potentially expected effect will be to save human lives, especially in cases of natural and man-made disasters, accidents, etc.

The devised GDS model GDS involving UAVs based on priority was investigated in the WeBots environment, which is not a direct UAV flight simulator. This imposed certain limitations. That is, the influence of external factors (turbulence, wind, cloudiness, precipitation, etc.) on the movement of UAV and its battery was not taken into account. In addition, the state of the battery, namely its aging, was not taken into account. There are also limitations to system parameters. That is, the reduced intensity of service cannot be less than the intensity of receipt of orders, and for GDS with failures, the number of waiting slots cannot be less than the number of UAVs in the system. If the specified restrictions are not met, then the system is inoperative. Thus, when applying our model in practice, as well as in further theoretical studies, it is necessary to take into account all the above limitations.

The disadvantages include the complexity of conducting research in the WeBots environment for virtual worlds with a large number of objects. This disadvantage can be overcome by using more powerful software environments and hardware resources and parallel computing.

The development of the study of this model implies further reducing the waiting time in the queue by taking into account a larger number of GDS parameters. This will require complex mathematical modeling and software simulation.

7. Conclusions

1. Models of single-channel and multichannel GDSs with failures and waiting based on probability have been investigated. It was established that the use of the devised model in the considered GDS makes it possible to increase the probability of the state of the free channel at \( i = 0 \) and reduce the probability of the state of the engaged channel with an increase in the queue of orders \( i \). The resulting values suggest that, in general, the devised GDS is less loaded on average per unit of time and makes it possible to serve more orders while incoming orders are in line for less time. In addition, the sum of the probabilities for the devised GDS involving UAVs based on priority for all states is 1. This shows the correctness of our calculations. Models of single-channel and multichannel GDSs with failures and waiting according to the waiting time in the queue were also investigated. It was found that the waiting time for a single-channel GDS with failures based on priority is 0.267 \( W_q \), and for the developed single-channel GDS with failures based on priority – 0.21 \( W_q \). It was found that the waiting time for a single-channel GDS with waiting based on priority is 0.3 \( W_q \), and for the developed single-channel GDS with waiting based on priority – 0.267 \( W_q \). It was established that the waiting time for a multichannel GDS with failures based on priority is 0.08 \( W_q \), and for the developed multi-
channel GDS with failures based on priority – 0.06 \( W_q \). It was established that the waiting time for a multichannel GDS with waiting based on priority is 0.15 \( W_q \), and for the developed multichannel GDS with waiting based on priority – 0.12 \( W_q \). Thus, the devised model provides a shorter waiting time in the queue.

2. The value of the probability and waiting time estimate in the queue was obtained when assessing the performance of a GDS model using UAVs based on priority. It was found that the difference between theoretical and experimental probability values for single-channel and multichannel GDSs with failures and waiting based on priority and the devised GDS differ by 2 \%. It was found that the difference between theoretical and experimental waiting time values for single-channel and multichannel GDSs with failures and waiting based on priority and the devised GDS differs by 3 \%. The obtained values allow us to assert the high accuracy of the results and the devised model as a whole.

### Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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The study was conducted without financial support.

### Data availability

All data are available in the main text of the manuscript.

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**References**


