1. Introduction

Ferrying is an important part of international freight and passenger transportation as it use eliminates many problems associated with the delivery of oversized (lengthy and large-size) and heavy cargos while reducing the transportation costs and time. The main Ukrainian ferry service provider on the Black Sea market is the shipping company (SC) Ukrferry (Odessa, Ukraine), which has the status of a national carrier of Ukraine [1, 2].

Ferry services are provided by ferry vessels operating in linear shipping. The vessel loading is characterized by diversified cargos, and the booking notes to book space aboard for transportation are served according to the line of various shippers and forwarding agents [2]. Besides:
- loads placed on board a ferry have various linear, volumetric, and mass parameters each of which can specify the cargo type and constitute the key transportation characteristic in the ferry loading [3];
- applications for cargo carriage arrive gradually, during a certain period of time within which they are processed and confirmed for a planned or subsequent shipment [4].

All this complicates the decision-making process on the loading of ferries. At present, the analysis of the possible placement of cargo from each subsequent application and thus the formation of an operational plan for the vessel loading are carried out by the line agents using the standard features of the program MS Excel spreadsheet with the entry of a large number of exogenous parameters. In this regard, the decision-making process on the formation of the ferry loading at the planning stage of the work is characterized as:
- strongly influenced by the “human factor”;
- labor-intensive,
- time-consuming.

- largely iterative, with additional input data depending on the receipt of new applications;
- restricted by the number of possible alternatives;
- prone to underestimate many factors and, as a result, to overlook the best option [5].

In turn, a wrong decision in terms of loading a vessel leads to an irrational use of the vessel’s production capacity and a frequent need to deny the customer the sale of transportation services for a planned shipment. It results in a loss of profits incurred by the shipping company and a high probability of losing its potential customer, who will be forced to use the services of competitors.

The topicality of research in this direction is justified by the need to develop an appropriate methodology of maintaining the processes of developing and making decisions on the loadings of separate vessels, which would be adapted to the peculiarities of operating freight and passenger ferries.

2. Analysis of previous studies and statement of the problem

The present study has primarily considered published research [6–19] about the formalized decision-making process on forming the loadings of vessels of different types. For example, in [6] there is an initial formulation of the problem of formalizing the vessel loading with general cargo; in [7] a method is developed for selecting an optimal general cargo loading that would comply with the dimensional and weight restrictions as well as the seaworthy state of a vessel, which also applies to ferries.

The research findings in [8] determine the procedure of forming the optimal bulker loading, whereas in [9] there is
an attempt to solve the problem of choosing an optimal variant of cargo operations on bulk carriers, and in [10] there is a description of how to optimize the loading of bulk carriers by using the simplex method. However, the object of bulk transportation is very different from the object of ferry traffic, which makes the suggestions presented in [8–10] only slightly applicable to ferries.

In [11] there are suggestions on forming a composite loading of ro-ro vessels; in [12] relevant economic and mathematical models are presented with regard to diverse transported cargos; and in [13] there is a model suggested to optimize the loading of a ro-ro vessel, based on an algorithm of economical progress. However, despite the functional similarity between ro-ro vessels and ferries, the latter extensively transport passengers, which is a condition not considered in [11–13].

In [14, 15] provisions are formulated on forming the loading of ro-ro vessels [14] and ro-ro passenger ships [15], taking into account the stability and security of the vessel by a simulation modeling.

In [16] an economic and mathematical model is suggested for loading a ship that works on the line to determine the optimal composition of the cargos that can maximize the revenue. In [17] an economic and mathematical model is developed to load a container carrier with regard to transportation of ISO containers of various types and sizes. In [18] there is a model of an optimal container handling in the port in order to minimize the movement during its loading. A conceptual model and economic and mathematical models of implementing thereof are suggested in [19] to optimize the consistent work of a container carrier. The ferry, which operates similarly to the container carrier in terms of carrying out small linear shipments, can be suggested the same provisions as in [16–19], particularly concerning the condition of continuous shipments, vessel capacity in terms of the cargo nomenclature, and compliance with the pre-announced schedule.

The aforementioned economic and mathematical models take into account the specifics of the respective vessels. Some of them study the problems of composite loadings of ships [11–15]. However, no solution has been suggested to the problems of a combined loading of a vessel with both cargo and passengers. In this respect, the described models can not be adequately applied to combined freight and passenger ferries. Therefore, it is necessary to undertake additional research in this area.

3. The purpose and objectives of the study

The purpose of the study is to improve the efficiency of the vessel fleet operations through the development of a methodological apparatus that would facilitate an optimal loading of individual ferry vessels.

To achieve the goal, the set objectives are the following:
– to clarify the specific features of a freight and passenger ferry transportation;
– to formulate the task conditions for loading a freight and passenger ferry;
– to develop an economic and mathematical model to optimize the ferry loading with regard for the technical and operational as well as structural characteristics of a freight and passenger ferry, taking into account the specifics of ferry traffic.

4. An economic and mathematical modeling to optimize the loading of a freight and passenger ferry

4. 1. The specifics of ferry traffic

To achieve the goal and the objectives of the study, it is primarily necessary to examine several specific features of ferry traffic.

1. A specific peculiarity of a freight and passenger ferry is the possibility of a composite loading, when various objects to be transported are placed in various transport facilities (spaces) of the vessel [1, 2]:
   – cargo deck (B\textsuperscript{CARGO}), for freight on board in transport units (railway (r/w) wagons, heavy trucks (TIR), and containers) and vehicles as cargo;
   – cabins (B\textsuperscript{PAX}) for drivers of TIR trucks and for other passengers.

The whole diversity of railway transport (I\textsuperscript{R/W}) (r/w wagons, r/w tanks, and locomotives) can be placed only in those areas of the vessel that are equipped with rails. In this regard, many cargo decks (B\textsuperscript{CARGO}) have special compartments that are equipped with railway tracks and intended for placement and transportation of the relevant freight (B\textsuperscript{R/W}), i.e. B\textsuperscript{R/W} \in B\textsuperscript{CARGO}.

As to other types of wheeled vehicles, they do not require specially equipped decks.

In the absence of a turnover of cargo trains (I\textsuperscript{R/W}), the load can be placed and transported on the same decks or parts thereof that are equipped with rails [3].

Thus, on the basis of mathematical logic and set theory, it is possible to present the logical relationship of subordination and intersection of ferry premises used for commercial purposes while operating the vessel (B\textsuperscript{COMM}) as follows (1):

\[
\{B\textsuperscript{COMM} = B\textsuperscript{CARGO} \cup B\textsuperscript{PAX};
\]

\[
B\textsuperscript{R/W} \in B\textsuperscript{CARGO}. \tag{1}
\]

2. Based on the techniques of cargo handling operations, the whole set of freight (I) placed on ferries should be divided into:
   – loads I\textsuperscript{RO-RO}, placed horizontally, and
   – loads I\textsuperscript{LO-LO}, placed vertically, e.g. with the help of cranes. This method of loading is less common since it requires equipping the dock with cargo cranes.

Therefore, it is relevant to note (2):

\[
I = I\textsuperscript{RO-RO} \cup I\textsuperscript{LO-LO}. \tag{2}
\]

3. When the loading is horizontal, the methods that are used are the following [3]:
   – “roll on / roll off”, when the freight (cars and trucks) get on board on their own;
   – “drive on / drive off”, when the cargo (containers, flats and other bulky loads) get on board on special platforms with chassis (roll-trailers);
   – “truck to truck”, when the loads get on board with the help of various forklifts.

4. The limiting factor in loading vessels of the roll-on type, including ferries, is the capacity of the vessel in cargo units restricted by the length of the rails and the size of the deck areas [1, 2].

5. The ferry is assigned to certain ports and is operated on a regular schedule, which is to be strictly observed unless in force majeure cases [3].
6. A peculiar feature of ferries is transportation not only of a wide range of cargos and passengers but also of a wide variety of cargos that belong to the same nomenclature but differ in linear and mass characteristics [3]. This, in turn, complicates the process of developing and making decisions on the ferry loading.

In the SC Ukrferry, for example, the whole diversity of loads is divided into the following main groups: r/w wagons (I\textsubscript{WAG}), TIR trucks (I\textsubscript{TIR}), containers (I\textsubscript{CONT}), cars (I\textsubscript{AUTO}), and deck cargo (I\textsubscript{D/C}).

The most extensive range of cargos consists of a wide variety of rolling stock (3):

\[ I_1^{\text{WAG}}, I_2^{\text{WAG}}, \ldots, I_v^{\text{WAG}}, \; v \in \mathbb{V}, \]  

where \( V \) is the type of the rolling stock (covered wagons, platforms, semi-wagons, refrigerator wagons, or railway tanks).

The TIR type includes various trucks (4):

\[ I_1^{\text{TIR}}, I_2^{\text{TIR}}, \ldots, I_r^{\text{TIR}}, \; r \in \mathbb{R}, \]  

where \( r \) is the type of the truck (trailers, contrailors, and hindcarriages).

The nomenclature "containers" includes containers of various sizes of ISO standards (5):

\[ I_1^{\text{CONT}}, I_2^{\text{CONT}}, \ldots, I_w^{\text{CONT}}, \; w \in \mathbb{W}, \]  

where \( w \) is the standard size of the container (20-feet or 40-feet).

The category of "automobiles" includes motorcycles (I\textsubscript{MOTO}) and various cars (6):

\[ I_1^{\text{CAR}}, I_2^{\text{CAR}}, \ldots, I_a^{\text{CAR}}, \; a \in \mathbb{A}, \]  

where \( a \) is the type of the automobile (e.g., sedan, SUV, minivan, etc.).

The deck cargo (I\textsubscript{D/C}) includes palletized goods (construction materials, equipment) (I\textsubscript{PAL}), loaded on a vessel’s deck in a vertical manner or using forklifts, as well as project cargo (I\textsubscript{PR}) (heavy and oversized loads), rolled on board on chassis or railway platforms, or driven in (excavators): I\textsubscript{D/C} = I\textsubscript{PAL} U I\textsubscript{PR}. Using the tools of set theory, let us represent the logical relationship of subordination and intersection of cargo transported by ferries in (7) and (8):

\[ \begin{align*}
I & = I_{\text{LO-LO}} \cup I_{\text{CAR-LO}}, \\
I_{\text{LO-LO}} & = I_{\text{WAG}} \cup I_{\text{TIR-CONT}} \cup I_{\text{AUTO-CONT}} \cup I_{\text{D/C}}, \\
I_{\text{CAR-LO}} & = I_{\text{PAL-CONT}} \cup I_{\text{PR}}, \\
I_{\text{WAG}} & = I_1^{\text{WAG}} \cup I_2^{\text{WAG}} \cup \ldots \cup I_v^{\text{WAG}}, \; v \in \mathbb{V}, \\
I_{\text{TIR}} & = I_1^{\text{TIR}} \cup I_2^{\text{TIR}} \cup \ldots \cup I_r^{\text{TIR}}, \; r \in \mathbb{R}, \\
I_{\text{CONT}} & = I_1^{\text{CONT}} \cup I_2^{\text{CONT}} \cup \ldots \cup I_w^{\text{CONT}}, \; w \in \mathbb{W}, \\
I_{\text{AUTO}} & = I_1^{\text{MOTO}} \cup I_2^{\text{CAR}} \cup \ldots \cup I_a^{\text{CAR}}, \; a \in \mathbb{A}, \\
I_{\text{D/C}} & = I_{\text{PAL}} \cup I_{\text{PR}}.
\end{align*} \]

7. It is clear that the nomenclature of the transported cargo determines the spatial stowing of the cargo aboard the vessel and, consequently, the amount of the revenue received as a result of transporting [4]. However, the nomenclature of the cargos is not the only characteristic that affects the stowing and the cost of transportation. Considering the example of Ukrferry, the characteristics affecting the stowing and the cost of transportation can also include, among many other factors, linear dimensions of the cargo units, the level of stowing the cargo units, the hazard degree, the recipient country and/or the country of the consignor, and belonging to a particular railway (for railway wagons).

Besides, in the cargo units belonging even to the same nomenclature, several of the aforementioned characteristics can differ. For example, commercial vehicles of the same type can differ in length, weight or the load transported.

8. Freight shipments by ferries are in particular demand among consignors of the railway rolling stock and heavy-duty trucks; passenger traffic is required by truck drivers. Cargo in containers, deck cargo, passenger cars and their drivers form an additional demand for cargo and passenger transportation by ferries.

The loading of a freight and passenger ferry for a voyage is planned on the basis of requests from consignors. At each stage of receiving such applications, the possibility of providing transport services is analyzed for a particular load in the planned shipment. As a rule, a booking agent is given a certain quota for loading wagons and cars, taking into account the priority of the traffic on the line and the technical capabilities of the vessel. However, the allocated quota can be used partially. In this case, the carriage may be offered for another cargo or for a cargo for which the quota has been exhausted. Thus, the vessel loading is supplemented with interchangeable cargos provided that it does not contradict the technical and operational characteristics of the vessel.

Passenger traffic is carried out according to the purchase of tickets. Availability of a seat/berth in the cabins of various types and categories of accommodation is monitored at each stage of ticket sales.

It should be mentioned that applications for carriage of cargos and transportation of passengers accompanying them are submitted simultaneously, and they are not transported separately. It means that while reserving some space on the ferry for a load, it is necessary to take into account the availability of seats/berths in the cabins to accommodate the passengers accompanying the consignment (usually, the passengers are drivers of trucks and/or personal cars) and vice versa.

Thus, the multitude of passengers (J) can be conditionally divided into two categories:

- passengers accompanying cargo (J\textsubscript{CARGO});
- passengers without cargo (J\textsubscript{D/OCARGO}).

Passengers accompanying cargo can also be classified into:
- drivers of TIR vehicles (J\textsuperscript{TIR});
- individuals accompanying the cargo and belonging to any other i-th nomenclature (J\textsubscript{CARGO}).

Using the tools of set theory expressed in symbols (9), the relation of subordination and logical intersection of passengers transported by ferries can be expressed as follows:

\[ \begin{align*}
J & = J_{\text{CARGO}} \cup J_{\text{D/OCARGO}}, \\
J_{\text{CARGO}} & = J_{\text{TIR}} \cup J_{\text{AUTO}} \cup J_{\text{CARGO}}, \quad i \in \mathbb{I}.
\end{align*} \]  

It is obvious that the cost of transporting passengers depends on the type and category of cabins and accommodation.
tion in them, but it also depends on the passengers’ belonging to any privileged category (children, diplomatic staff, officials of the shipping company, and so on). It is also important to note that drivers of TIR can be accommodated in any free cabin at the discretion of the vessel’s administration.

9. Within a single nomenclature, freight can be calculated for transportation on the basis of a loading unit, a meter of its length, or its volume. For example, in the SC Ukrferry, the charge for the transportation of wagons as well as cars, containers and trucks at least 12 m long is made for a cargo unit. For TIR up to 12 m in length, the charge is set for one meter of the vehicle length.

10. The cost of transporting certain categories of cargos can involve various, preliminarily stipulated, discounts, allowances, and surcharges.

For example, for transporting mineral water Borjomi on the ferries of the SC Ukrferry, there are set discounts of the amount of which depends on the direction of the overland wagon movement. On the other hand, the cost of transporting dangerous cargo includes a surcharge factor 2.

11. In addition to freight, the company’s revenues from shipping are supplemented by surcharges for types of cargo.

In the SC Ukrferry, transportation of wagons and containers is subject to surcharges BAF and GRI.

4. 2. Statement of the problem

Taking into account the aforementioned features of operating freight and passenger ferries, the task of optimizing the loading of a ferry can be formulated as follows.

On the line between ports \( P \) (\( P = \{P_1, P_2, \ldots, P_{11}\} \)), there is a set freight and passenger ferry with the following characteristics:

- the estimated value of the duty (load capacity) corresponding to the conditions of the ferry operation – \( D_{n,t} \);
- the allowable load on the deck \( n \rightarrow D_{n} \) (\( n \in N \)), \( t \);
- the allowable load for the nomenclature of cargo \( i \) on the track \( b \) on the deck \( n \rightarrow D_{n,b,i} \) (\( n \in N; b \in B_j; i \in \Gamma^{(w)} \)), \( t \);
- the total length of all ferry rails – \( L \), m;
- the total length of all rails on the deck \( n \rightarrow L_{n} \) (\( n \in N \), m);
- the length of the track on the deck \( n \rightarrow L_{nb} \) (\( n \in N; b \in B_j \), m);
- the useful area of the vessel – \( S \), m²;
- the area of the deck \( n \rightarrow S_{n} \) (\( n \in N \)), m²;
- the area of the track \( b \) on the deck \( n \rightarrow S_{n,b} \) (\( n \in N; b \in B_j \)), m²;
- the passenger seating – \( N^{PAX} \), pax;
- the number of seats/berths in the cabins according to any free cabin at the discretion of the vessel’s administration in the M categories of accommodation – \( N_{m} \) (\( m \in M \)), seats/berths.

In accordance with the requests of consignors, ferry transportation is planned for G consignments (\( g \in G \)). Each consignment \( g \), in its turn, consists of I nomenclatures (\( i \in I \)) of cargo in the amount of \( K \) units (\( k \in K_i \)). Besides, transportation is planned for H groups of passengers (passengers who wish to travel together) (\( h \in H \)). Furthermore, each group consisting of \( J \) passengers (\( j \in J \)) may wish to be accommodated according to the M categories of cabins (\( m \in M \)).

It is necessary to optimize the loading of a freight and passenger ferry.

4. 3. The development of an economic and mathematical model

4. 3. 1. The development of the target function and control parameters

As is known, ferries operate in a linear regime of navigation. Companies providing linear services act as public carriers. It obliges them to undertake the carriage of cargos depending on the sequence of receiving applications from cargo owners. In this regard, the choice of cargo from the point of profitability of its transporting to the linear shipping company as a public carrier is a technically incorrect criterion. Despite this, the mechanisms for attracting highly-chargeable cargos in the practice of regular shipping still work; their implementation is provided at the level of line agents and is reflected in the amount of their remuneration.

In this regard, as the optimality criterion for the ferry operation planning level, we have selected an expected return for voyage \( F \) (10), which is equal to the sum total of transporting cargos \( F^{cargo} \) (11) and passengers \( F^{pass} \) (14).

\[
Z = F = F^{cargo} + F^{pass} \rightarrow \max,
\]

\[
F^{cargo} = \sum_{m=1}^{m} \sum_{n=1}^{n} \sum_{b=1}^{b} \sum_{k=1}^{k} \left( \tau_{nk}^{(i)} + R_{nk}^{(i)} \right) x_{nbi}\]

\[
+ \sum_{n=1}^{n} \sum_{b=1}^{b} \sum_{k=1}^{k} \sum_{l=1}^{l} \tau_{nk}^{(i)} \left( l_{nk} + R_{nk}^{(i)} \right) x_{nblk} +
\]

\[
+ \sum_{n=1}^{n} \sum_{b=1}^{b} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{m=1}^{m} \tau_{nk}^{(i)} \left( v_{nk} + R_{nk}^{(i)} \right) S_{nbgkl} (11)
\]

where \( x_{nbi} \) is a control parameter under the following conditions:

\[
x_{nbi} = \begin{cases} 1, & \text{if the k-th cargo unit of an i-th nomenclature of a g-th consignment is placed on the b-th rails of the n-th deck;} \\
0, & \text{otherwise.}
\end{cases}
\]

\[
\tau_{nk}^{(i)}, \tau_{nbi}^{(i)} \text{ and } \tau_{nbi}^{(i)} \text{ are the tariff rates for carriage of the k-th cargo unit of an i-th nomenclature of a g-th consignment as charged respectively for the cargo unit (CU), the length of the CU, and the volume of the CU.}
\]

Tariff rates quoted by line agents for customers may depend of relevant discounts and surcharges (12):

\[
\begin{align*}
\tau_{nk}^{(i)} &= \tau_{nk}^{(i)} + k_{nbi}^{(i)} Z_{nk}^{(i)} + \tau_{nbi}^{(i)} k_{nbi}^{(i)} Z_{nbi}^{(i)} \\
\tau_{nbi}^{(i)} &= \tau_{nbi}^{(i)} + k_{nbi}^{(i)} Z_{nbi}^{(i)} + \tau_{nbi}^{(i)} k_{nbi}^{(i)} Z_{nbi}^{(i)} \\
\tau_{nbi}^{(i)} &= \tau_{nbi}^{(i)} + k_{nbi}^{(i)} Z_{nbi}^{(i)} + \tau_{nbi}^{(i)} k_{nbi}^{(i)} Z_{nbi}^{(i)} \\
\end{align*}
\]

where \( \tau_{nbi}^{(i)} \) is the base tariff rate for the carriage of a cargo unit of an i-th nomenclature of a g-th consignment; \( k_{nbi}^{(i)} \) is the discount rate on the tariff for transporting a k-th cargo unit of an i-th nomenclature of a g-th consignment; \( k_{nbi}^{(i)} \) is the surcharge rate on the tariff for transporting a k-th cargo unit of an i-th nomenclature of a g-th consignment in the SC Ukrferry, surcharge factor 2 is applied to alcoholic beverages, whereas surcharge factor 3 is used in case of dangerous cargos and animals; \( Z_{nbi}^{(i)} \) is a parameter that is set to 1 if there is a discount on the freight rate for a k-th cargo unit of an i-th nomenclature of a g-th consignment; otherwise it is 0; \( Z_{nbi}^{(i)} \) is a parameter that takes the value of 1 if there is a surcharge rate for transporting a k-th cargo unit of an i-th nomenclature of a g-th consignment; otherwise it is 0; \( L_{ng} \) is the length of a k-th cargo unit of an i-th nomenclature of a g-th consignment that can take the values of \( 1_{nk}^{(i)} Z_{nbi}^{(i)} \) or \( 1_{nk}^{(i)} Z_{nbi}^{(i)} \) for which: \( Z_{nbi}^{(i)} \) is a parameter that takes the value of 1 if the charge for transporting a k-th cargo unit of an i-th nomenclature of a
The 32-th consignment is set for the actual meters of length of the CU; otherwise it is 0; $Z_{32}$ is a parameter that takes the value of 1 if the charge for transporting a k-th cargo unit of an i-th nomenclature of a g-th consignment is set for a linear meter of length of the CU; otherwise it is 0. The "began meter of length" is understood as the length of a k-th cargo unit of an i-th nomenclature of a g-th consignment, which takes the values of $\nu_{gik}$, $Z_{gik}$, or $\nu_{gik}Z_{gik}$, for which: $Z_{gik}$ is a parameter that takes the value of 1 if a discount rate is provided for transporting a k-th cargo unit of an i-th nomenclature of a g-th consignment; otherwise it is 0; $Z_{gik}$ is a parameter that reflects the possibility to accommodate a k-th cargo unit of an i-th nomenclature of a g-th consignment set as a conditional volume of the CU; otherwise it is 0.

In view of the aforementioned, (15) can be specified in the following way (16):

$$F_{\text{carg}} = \sum_{\text{m}, \text{n}, \text{b}, \text{g}, \text{i}, \text{K}, \text{m}, \text{h}, \text{j}, \text{K}} \left( \tau_{\text{m}} \cdot k_{\text{m}} \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} + \tau_{\text{m}} \cdot k_{\text{m}} \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} \right) \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} \rightarrow \text{max},$$

where $\tau_{\text{m}}$ is the base charge rate for accommodating the passenger according to the m-th category; $k_{\text{m}}$ is the discount rate for a preferential passenger (a child, or an ambassador); $k_{\text{m}}$ is the surcharge rate for single occupancy (without sharing the cabin); $Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}}$ is a parameter that takes the value of 1 if a discount rate is provided for transporting a j-th passenger of an h-th group of passengers accommodated in a cabin of an m-th category; otherwise it is 0; $Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}}$ is a parameter that takes the value of 1 if a surcharge is set for transporting a j-th passenger of an h-th group of passengers accommodated in a cabin of an m-th category; otherwise it is 0.

In view of the aforementioned, (15) can be specified in the following way (16):

$$F_{\text{carg}} = \sum_{\text{m}, \text{n}, \text{b}, \text{g}, \text{i}, \text{K}, \text{m}, \text{h}, \text{j}, \text{K}} \left( \tau_{\text{m}} \cdot k_{\text{m}} \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} + \tau_{\text{m}} \cdot k_{\text{m}} \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} \right) \cdot x_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} \rightarrow \text{max},$$

The parameter $Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}}$ depends on the possibility to get the cargo onto the n deck, which is limited by the dimensions and load capacity of the lift.

$$Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}} \rightarrow \text{max},$$

The parameter $Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}}$ depends on the possibility to accommodate a j-th passenger of an h-th group of passengers accommodated in a cabin of an m-th category; otherwise it is 0.

The parameter $Z_{\text{m} \cdot \text{h} \cdot \text{g} \cdot \text{i} \cdot \text{K} \cdot \text{m} \cdot \text{h} \cdot \text{j} \cdot \text{K}}$ depends of the category of accommodation applied for by the passenger when buying/booking.
the ticket, as well as the possibility of accommodating drivers of TIR according to the m-th category.

The ferry load optimization is aimed primarily at developing planned freight traffic with the rational use of the production capacity of the vessel. To solve the problem of forming the vessel’s optimal load, all the requirements should be considered in the economic and mathematical model by forming a system of restrictions that can reflect relevant conditions.

4. 3. 2. The formation of a system of constraints

In addition to maximizing the revenue, the optimal loading of a ferry is aimed at ensuring that the number of the cargos does not exceed the net carrying capacity calculated according to the operating conditions in a given direction. This condition is reflected in the restriction (18).

\[ \sum_{n \in N, b \in B, g \in G, i \in I, k \in K} x_{nhbk} \cdot q_{igk} \cdot Z_{nhbk} \leq D_h, \]  
(18)

where \( q_{igk} \) is the weight of a k-th cargo unit of an i-th nomenclature of a g-th consignment, t;

When placing the cargo on board a vessel, it is necessary to observe the maximum permissible load:

– on each deck (19);
– on one axis of the same rails for the rolling stock (20);
– on one way for the other cargos (21);

\[ \sum_{b \in B, g \in G, i \in I, k \in K} x_{dhbk} \cdot q_{igk} \cdot Z_{dhbk} \leq D_{dh}, \]  
(19)

\[ \sum_{b \in B_i} x_{dhbk} \cdot q_{igk} \cdot Z_{dhbk} \leq D_{shg}, \]  
(20)

where \( q_{igk} \) is the load on the axis by a k-th cargo unit of an i-th nomenclature of a g-th consignment, t;

The limiting factors in loading the ferry are:

– the total useful area of the vessel (25):

\[ \sum_{n \in N, b \in B, g \in G, i \in I, k \in K} x_{nhbk} \cdot s_{igk} \cdot Z_{nhbk} \leq S; \]  
(25)

– the area of each deck (26):

\[ \sum_{b \in B, g \in G, i \in I, k \in K} x_{nhbk} \cdot s_{igk} \cdot Z_{nhbk} \leq S_{nhg}, \]  
(26)

– the area of each track (27):

\[ \sum_{g \in G, i \in I, k \in K} x_{nhbk} \cdot s_{igk} \cdot Z_{nhbk} \leq S_{shg}, \]  
(27)

where \( s_{igk} \) is the area of a k-th cargo unit of an i-th nomenclature of a g-th consignment with account for technological gaps.

Besides, when loading a ferry, it is necessary to take into account the maximum possible loading of the equivalent (EQ) of the cargo unit of an i-th nomenclature of the same size range (28):

\[ \sum_{n \in N, b \in B, g \in G, i \in I} x_{nhbk} \cdot I_{igk} \cdot Z_{nhbk} \leq EQ, \]  
(28)

where \( I_{EQ} \) is the length of a cargo unit of an i-th nomenclature of the same standard size accepted as a base parameter. In the practice of Ukrferry, such a conventional unit, according to historical tradition, is a standard covered wagon 14.7 m in length; in designing the vessels for the shipping company, such wagons are the main intended units for transportation.

Condition (29) restricts the use of the allotted quota for the carriage of cargo units of an i-th nomenclature (\( i \in I^{WAG} \cup I^{TIR} \)), providing the main demand for freight transportation by ferries with regard to the priority of traffic on the line and the technical capabilities of the vessel. The quota is set for units of an i-th nomenclature of the same type and size.

\[ \sum_{n \in N, b \in B, g \in G, i \in I} x_{nhbk} \cdot I_{igk} \cdot Z_{nhbk} \leq Q, \]  
(29)

where \( U_i (i \in I^{WAG}) \) takes the following values:

\[ E_i = \begin{cases} 1, & \text{if the traffic of cargo units of an i-th (} i \in I^{TIR} \text{) nomenclature is sufficient for the allotted quota;} \\ 0, & \text{otherwise.} \end{cases} \]

\( U_i (i \in I^{TIR}) \) takes the following values:

\[ E_i = \begin{cases} 1, & \text{if the traffic of cargo units of an i-th (} i \in I^{WAG} \text{) nomenclature is sufficient for the allotted quota;} \\ 0, & \text{otherwise.} \end{cases} \]

where \( I^Q \) is the length of a cargo unit of an i-th nomenclature of the same type and size for the amount of which the quota is
made available. If a wagon is taken, for example, as a unit of account, a standard covered wagon 14.7 m long is used as an equivalent (EQ) for calculating the load at the SC Ukrferry. For the TIR, this unit of account can be represented by a standard heavy-duty truck 17.0 m in length; Q is the size of the quota set for an i-th cargo nomenclature.

According to an analysis of the commercial ro-pax plans for the ferry “Greifswald”, the stability of the vessel can be ensured by placing a minimum of cargos in the amount of 35 % of the vessel’s bearing capacity on the upper decks and a maximum cargo in the amount of 65 % of the vessel’s bearing weight on the lower decks. Thus, due to the restrictions (30) and (31) introduced to the model, it is possible to ensure adequate stability of the vessel.

\[
\sum_{\text{n} \in \text{N}(\text{N}=\text{N}' \cup \text{N}')} \sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot q_{\text{g}_{\text{i}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq 0.35 \cdot D_{\text{b}} \quad \text{(30)},
\]

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot q_{\text{g}_{\text{i}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq 0.65 \cdot D_{\text{b}}, \quad \text{(31)},
\]

where \( \text{n} \in \text{N}(\text{N}=\text{N}' \cup \text{N}') \) is the unity of all the decks located above (\( \text{n} \in \text{N}' \)) and below (\( \text{n} \in \text{N}' \)) the main deck.

Condition (32) limits the loading of the vessel in the number of cargos of each nomenclature declared for carriage:

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq N_{\text{i}} \quad (i \in \text{I}), \quad \text{(32)}
\]

where \( N_{\text{i}} \) is the number of cargos of an i-th nomenclature declared for carriage:

\[
N_{\text{i}} = \{1,2,...,I\}.
\]

Moreover, the consumer market situation is such that the supply of marine transportation exceeds the demand for it, and the restriction (32) becomes an equation used to ensure the development of all cargo traffic.

Besides, if the cargo owner requests integrity of the consignment, its non-divisibility into separate nomenclatures, the consignment integrity is stipulated by the following constraint (33):

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} = N_{\text{g}_{\text{i}}} \quad (g \in G; i \in I_{\text{g}}), \quad \text{(33)}
\]

where \( N_{\text{g}_{\text{i}}} \) takes the value of \( 0 \) or \( 1 \) \( \{N_{\text{g}_{\text{i}}} = \{0;1\}\} \). Moreover, \( N_{\text{g}_{\text{i}}} = 1 \) means that a k-th cargo unit of an i-th nomenclature of a g-th consignment is accepted for a stipulated shipment. Besides, \( N_{\text{g}_{\text{i}}} = 0 \) means that fulfillment of application liabilities on transporting the i-th nomenclature of the g-th consignment is delayed, as agreed with the cargo owner, until the next shipment. Importantly, in the case where the ferry loading is planned for shipment by implementing the model \( N_{\text{g}_{\text{i}}} = 0 \), the preferable condition is \( N_{\text{g}_{\text{i}}} = 1 \) when decision is made on the next shipment. Otherwise, the carrier is at risk of losing the client if the latter has to wait long for transportation.

In its turn, passenger accommodation aboard is limited to the total passenger capacity of the ferry (35) as well as to the seating capacity in each category of accommodation in cabins (36).

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq N_{\text{PAX}}, \quad \text{(35)}
\]

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq N_{\text{pax}} \quad (m \in M), \quad \text{(36)}
\]

Condition (37) limits the accommodation of passengers by the number of the passengers who have booked/bought the tickets:

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq N_{\text{pax}}, \quad \text{(37)}
\]

where \( N_{\text{pax}} \) is the number of the passengers who have booked/bought the tickets.

Moreover, the consumer market situation is such that the supply of marine transportation exceeds the demand for it, and the restriction (37) becomes an equation used to ensure the development of all cargo traffic.

Furthermore, if passengers request their travelling together (on the same voyage), the integrity of the passenger group is taken into account by the following restriction (38):

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \leq N_{\text{g}_{\text{i}}}, \quad (g \in G; i \in I_{\text{g}}), \quad \text{(38)}
\]

where \( N_{\text{g}_{\text{i}}} \) takes the value of \( 0 \) or \( 1 \) \( \{N_{\text{g}_{\text{i}}} = \{0;1\}\} \) where \( D_{\text{g}_{\text{i}}} \) is the number of passengers belonging to an h-th group. Besides, \( N_{\text{g}_{\text{i}}} = D_{\text{g}_{\text{i}}} \) means that an h-th group of passengers is accepted for transportation in full for the planned voyage. However, \( N_{\text{g}_{\text{i}}} = 0 \) means that fulfillment of application liabilities for transporting the h-th group of passengers is postponed until the next shipment.

Just as in the case with cargos, for an adequate machine solution of the problem, it is necessary to impose a restriction (39) that prevents simultaneous placement of the same passenger in several cabins or for several seats/berths of a cabin.

\[
\sum_{\text{n} \in \text{N}(\text{N})} \sum_{\text{b} \in \text{B}} \sum_{\text{g} \in \text{G}} \sum_{\text{i} \in \text{I}} x_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} \cdot Z_{\text{a}_{\text{b}_{\text{g}_{\text{i}}}}} = N_{\text{h}_{\text{j}}}, \quad (h \in H; j \in J_{\text{h}}), \quad \text{(39)}
\]

where \( N_{\text{h}_{\text{j}}} \) takes the value of \( 0 \) or \( 1 \) \( \{N_{\text{h}_{\text{j}}} = \{0;1\}\} \). Moreover, \( N_{\text{h}_{\text{j}}} = 1 \) means that a j-th passenger of an h-th group is accepted for transporting during the stipulated voyage. Furthermore, \( N_{\text{h}_{\text{j}}} = 0 \) means that fulfillment of application liabilities for transporting the j-th passenger of the h-th group is postponed, upon the passenger’s consent, until the next voyage.
Mathematics and cybernetics – applied aspects

It is obvious that the passengers accompanying cargo units want to travel along on the same voyage; an example can be a family with a car. Moreover, in most cases, the voyage of accompanying passengers is obligatory; for example, when it concerns driver(s) of heavy-duty vehicles. That is, if the cargo is accepted for carriage on a particular voyage, then the carriage of passengers accompanying the cargo on this voyage is a mandatory condition for transporting (unless it conflicts with other factors, such as lack of necessary documents, etc.).

Thus, the expression (40) prevents movement of passengers without the cargo they accompany, and it also prevents transportation of cargo without accompanying passengers (if any):

\[
\sum_{\text{meh}, \text{h}, \text{i}, \text{k}} n_{\text{shik}} \cdot x_{\text{shik}} \cdot Z_{\text{shik}} = \sum_{\text{meh}, \text{h}, \text{i}, \text{k}} n_{\text{shik}} \cdot x_{\text{shik}} \cdot Z_{\text{shik}} \quad (g \in G),
\]

where \( n_{\text{shik}} \) is the number of passengers accompanying a \( k \)-th cargo unit of an \( i \)-th nomenclature of a \( g \)-th consignment;
\( Z_{\text{shik}} \) is a parameter that reflects the transportation of a \( k \)-th cargo unit of an \( i \)-th nomenclature of a \( g \)-th consignment with accompanying passengers:

\[
Z_{\text{shik}} = \begin{cases} 1, & \text{if a } k \text{-th cargo unit of an } i \text{-th nomenclature of a } g \text{-th consignment is transported accompanied;} \\ 0, & \text{otherwise.} \end{cases}
\]

\( Z_{\text{ghij}} \) is a parameter that reflects a \( j \)-th passenger belonging to a \( h \)-th group is placed according to the \( m \)-th category of the \( g \)-th consignment:

\[
Z_{\text{ghij}} = \begin{cases} 1, & \text{if a } j \text{-th passenger belonging to an } h \text{-th group is accommodated according to the } m \text{-th category of } g \text{-th consignment;} \\ 0, & \text{otherwise.} \end{cases}
\]

One of the important conditions of operating vessels on the line is compliance with a preliminarily drawn, pre-announced and systematically specified schedule. In this regard, the time of the loading operations at each port of the line should not exceed the period stipulated by the linear schedule, taking into account the established reserve of the parking time (41).

\[
\max \left\{ 2 \cdot \left( \sum_{\text{meh}, \text{h}, \text{i}, \text{k}} n_{\text{shik}} \cdot t_{\text{shik}} \cdot Z_{\text{shik}} \right); 2 \times \left( \sum_{\text{meh}, \text{h}, \text{i}, \text{k}} n_{\text{shik}} \cdot t_{\text{shik}} \cdot Z_{\text{shik}} \right) \right\} 
\leq T^* \cdot K^*,
\]

where \( t_{\text{shik}} \) is the time of the roll off of a cargo unit of an \( i \)-th nomenclature from the \( n \)-th deck; \( t_{\text{max}} \) is the time during which one passenger goes off the vessel board; \( T^* \) is the time during which the vessel stays in a port according to the schedule; \( K^* \) is the reserve time of staying in a port before the subsequent voyage.

Expressions (42) and (43) represent possible values for the control parameters:

\[
x_{\text{shik}} = \{0;1\}, \quad (n \in N; h \in H; i \in I; k \in K),
\]

\[
x_{\text{ghij}} = \{0;1\}, \quad (m \in M; h \in H; j \in J),
\]

Moreover, given the gradual flow of applications for transportation, the efficiency of decision-making in developing the plan of loading a vessel can be achieved by applying the aforementioned model at each \( t \)-th stage of receiving a new application for the transportation of a \( k \)-th cargo unit of an \( i \)-th nomenclature of a \( g \)-th consignment.

5. The results of developing an economic and mathematical model for optimizing the loading of a freight and passenger ferry

The study has formulated the statement of the problem of optimizing the composite load for a freight and passenger ferry according to which there has been developed and presented a general economic and mathematical model that takes into account the technical, operational, and design features of the freight and passenger ferry as well as the specific conditions of its use. The target function, presented in the model, maximizes the revenue from freight and passenger transportation. A system of restrictions ensures effective and safe use of the technical and operational characteristics of the ferry, including the linear dimensions of the decks and tracks, loads on the decks and tracks, and also the capacity of the ferry cabins. At the same time, it takes into account the limitations of the system parameters of the current freight and passenger traffic, which include the amount of various cargos and the requirements of cargo owners on the continuity of consignments and passengers regarding the accommodation of the latter in different categories of cabins. The control parameters in the model are Boolean variables that can be set to 0 or 1.

6. Discussion of the results of optimizing the loading of a freight and passenger ferry

In the process of optimizing the loading of a freight and passenger ferry, there has been identified such as a distinctive feature of the ferry vessel as a composite loading of the vessel not only with different types of cargo but also with the simultaneous loading of cargos and placement of passengers. This peculiarity produces a number of other distinctive features that differentiate ferries from other vessels whose load optimization is presented in [6–19]. In this respect, the economic and mathematical model presented in this study takes into account limitations not only in placing cargos but also in accommodating passengers in cabins with regard to the number of passengers and the classes of cabins in which such passengers would like to stay.

Furthermore, the model considers the condition of the cargo being transported and accompanied on the same voyage. Another peculiarity of ferries is a wide variety of cargos being transported, whereby some cargo units (even within the same nomenclature) differ in their linear and weight characteristics. In this connection, the economic and mathematical model uses Boolean variables represented by individual cargo units and individual passengers.
The value of 1 means that a cargo unit or a passenger is transported on the current voyage; the value of 0 means otherwise.

The suggested model helps plan the shipping of cargos in such a specific facility of cargo consolidation as a train. The model sets limitations on the length of the railway gauge and the axle load. Moreover, the model stipulates restriction on the continuity of the consignment in accordance with which the cargo is sent in the current shipment, in the full amount of the consignments, or is postponed for transporting in the next shipment. Thus, the process of developing the economic and mathematical model was based of taking into account the specific features of freight and passenger ferries; therefore, the model can be adequately applied while organizing and managing ferry operations.

However, it is important to specify that the developed model is more focused on decision-making while developing the transportation services provided by the Ukrainian ferry operator such as the shipping company Ukrferry. Therefore, when used by other operators, the model can be modified, some limitations can be eliminated, and new restrictions can be added. For example, some cargo ferries do not transport railway stock, which implies that track length restrictions can be added. For example, some cargo ferries do not transport railway stock, which implies that track length restrictions for the decks should be excluded from the models. Some ferries widely use the practice of container loading in a vertical way, which implies a need for introducing additional restrictions.

References

1. Introduction

Poor structure of considerable part of modern applied problems and tasks, the lack of reliable and representative input data and the possibility to consider non-standard and unpredictable factors during their solving underlie the need to apply such an approach as the expert surveys. The problem of determining numerical evaluation of an object plays an important role among the tasks where the expert opinions are largely used to solve them. As a rule, certain complex immeasurable characteristics of the object are assessed, which requires professional knowledge and experience of the experts. The theory of decision making described a number of rules that determine collective evaluation which are based on the processing of both the expert assessments and the values that characterize degrees of expert group competence.

Subjective nature of expert assessments, in its turn, predetermines a possibility to use the apparatus of fuzzy sets prior to collective evaluation. The successful application of models and methods that are based on fuzzy logic allows improving the results of solutions of such problems by a specific criterion.

System analysis of time series forecasting methods detected a class of economical, medical and sociological problems that are needed to consider not only the internal regularity of the time series but the external significant factors too, in the design of efficient methods of forecasting. Taking into account the fact that while solving the practical tasks, it is not always possible to formalize the action of external factors due to their sporadic and unpredictable nature, it would be expedient to involve the experts to evaluate them, which makes it possible to consider additional factors which can have a significant impact on the investigated phenomenon.

Thus, a development of heuristic models and forecasting methods of time series under real conditions, which allow taking into account the experts’ (group of experts) assessments, is an actual and practical important task.