1. Introduction

An efficient use of energy resources is a priority in the energy policies of the EU and the entire world. A significant reduction in the reserves of such main heat transfer agents as oil and natural gas predetermines a purposeful search for new energy and resource saving technologies. Annually, there increases the share of alternative and renewable energy sources, such as biofuels, solar, wind, water and geothermal energy, in the structure of the world power supply. Thus, in 2013, renewable energy provided about 19 % of energy consumption in the world [1]. The tendency is also reflected in the maximum amount of heat energy.

2. Analysing the previous studies and formulating the problem

Cavitation is generated with ultrasonic (US) and hydrodynamic devices [8]. Given numerous shortcomings of ultrasonic cavitation generators, such as uneven processing, considerable specific energy consumption, and high probability of erosion of work items [9], their use is limited to laboratory experiments. Since hydrodynamic cavitators are devoid of such drawbacks, the scope of their application, compared to ultrasound devices, expands – they provide a highly efficient uniform cavitation processing of media and objects in different industries.

The most important characteristics of cavitation are energy-related, in particular: (1) energy consumption for the creation, development and maintenance (at a particular level) of a cavitation zone, (2) specific energy consumption, and (3) the amount of heat energy released during cavitation. Energy characteristics of cavitation processing, as a rule, are determined by cavitation generators’ design elements, in particular: the shape of cavitating parts [10, 11], their size (diameter) [12, 13], number, and spatial location. Thus, a venturi nozzle applied for developing and maintaining the cavitation zone uses up to 43 % of the flow energy, while a truncated diffuser-confuser nozzle – 62 %, and a full-size diffuser-confuser nozzle – 62–89 % [14]. The use of cavi-
tors with venturi nozzles (that have expansion chambers for synthesising methyl esters of fatty acids) made it possible to achieve a 95% output of the reaction product when the value of specific energy consumption is 3.9 MJ/m^3 [15]. The authors of [16] present optimal design parameters of cavitation vortex heat generators that are used to pasteurise brine for dairy products. When the circulation multiplicity factor equals to 1 and the heat output for the liquid phase Q=42.1 MJ/h, the necessary difference of temperatures at the inlet and outlet of the cavitator (Δt=2.3 K) is provided by vortex heat generators with the input nozzle width of 0.063 m, diaphragm diameter of 0.04 m and cylindrical vortex tube length of 0.8 m.

The aforementioned facts prove that high efficiency of cavitation processing of liquid-phase media require a thorough analysing of the effect of design parameters of hydrodynamic cavitation on the energy efficiency of the process, including the amount of heat energy released during cavitation. Most studies focused on the effect of one of the design parameters of cavitating parts (shape, diameter, etc.) on the energy characteristics of cavitation processing. Although comprehensive studies of the joint effect of several parameters are more important since they allow deriving a multifunctional dependence of the value of heat energy (released during cavitation) on the design and technological parameters, which enhances a cavitator capacity to control the efficiency of liquid-phase media processing.

3. The purpose and objectives of the study

The objective of the study was to determine the optimal design parameters of cavitating parts (their number, diameter, and spatial location) that provide the most efficient mode of cavitation processing of aqueous media.

To achieve the above goal, it is necessary to solve the following tasks:

1. To use experimental research in building a 4-factor mathematical model that would link the value of heat energy with technological (inlet pressure) and design (nozzle diameter, the number of nozzles, and the angle of attack jets) parameters;
2. To verify the adequacy of the constructed model and its accuracy;
3. To analyse the graphical interpretation of the model of cavitation processing of aqueous media;
4. To measure the impact of small amounts of air (injected in an aqueous medium) on the intensity of cavitation and the attendant fluctuation effect.

4. Materials and methods of studying cavitation processing of aqueous media

4.1. Materials and methods of studying the effect of technological (inlet pressure) and design (nozzle diameter, the number of nozzles, and the angle of attack jets) parameters on the value of heat energy released during cavitation

Cavitation fields were generated by means of a hydrodynamic jet cavitator with a system of removable nozzles of different diameters, while the angle between them was regulated by means of a hinge mechanism. The pressure in the cavitator inlet was regulated in a range of 0.36–0.60 MPa by means of a bypass. The pump drive capacity of the jet cavitator was 1.1 kW. The amount of released heat energy was measured with a calorimetric method [9], which required experimental measuring water temperature in the cavitator inlet and outlet as well as its flow in the circulation circuit.

The development and progress of cavitation phenomena excited under various experimental conditions were also evaluated with the method sonochemical analysis, which involves a PC obtaining and processing of characteristics of an acoustic signal generated by the cavitation field. The signal was fixed with a spherical hydrophone of 8105 variety that was commutated from the PC. The operating frequency range of such hydrophone is 0.1 Hz–160 kHz and sensitivity in the reception mode is −205 (dB re 1 V)/µPa. The graphical interpretation of the cavitation field development was carried out in Adobe Audition 1.5.

4.2. Mathematical modelling the cavitation processing of aqueous media

The cavitator design was optimised via analysing a 4-factor mathematical model that linked the value of released energy with technological (inlet pressure) and design (nozzle diameter, the number of nozzles, and the angle of attack jets) parameters. Since construction of multi-factor nonlinear regression equations by means of analytical methods in most cases is impossible, we used empirical methods that provide adequate results, in particular, the Brandon method of consistent excluding the effects of independent variables [17, 18], which allows deriving multiplicative models as follows:

\[ y(x_1, x_2, ..., x_n) = b_i \Pi_{i=1}^{N} f_i(x_i), \]  

(1)

where \( b_i \) is the model coefficient, \( f_i(x_i) \) is an arbitrary one-dimensional function, \( N \) is the number of input factors, and \( i = 1, N \).

The accuracy of the model is affected by the order of the independent input variables \( x_i \) and the corresponding \( f_i(x_i) \). To find index numbers of the variables, we built graphs of each \( f_i(x_i) \) as functions of one variable and derived empirical regression lines. Approximation of the multi-factor function was started with the variable, for which the difference between the experimental data and those calculated with the use of the derived regression equation was minimal.

The model coefficient \( b_i \) is equal to the arithmetic mean of the experimental function values of \( N \) sample set:

\[ b_i = \frac{1}{N} \sum_{i=1}^{N} y_i. \]  

(2)

Factor \( f_i(x_i) \) was determined by calculating the sample set of the new fictitious input variable \( y_{ii} \):

\[ y_{ii} = y_i / b_i, \]  

(3)

where \( i = 1, N \).

In MS Excel, there was built a graphical dependence \( y_{ii} = f(x_i) \) and selected the type of regression equation, for which the value of approximation reliability \((R^2)\) was maximum. The equation was part of the mathematical model as factor \( f_i(x_i) \).
Energy-saving technologies and equipment

Factor $f_i(x_i)$ was a basis for calculating the sample set of the next fictitious input variable $y_{ul}$; we built a graphical of the number of nozzles, $n$; and the angle of attack jets. The intervals of varying parameters were as follows: the inlet pressure – 0.12 MPa, the nozzle diameter – 0.6 mm, the number of nozzles – 2, and the angle of attack jets – 72 degrees. The ranges of varying parameters were as follows: the inlet pressure – 0.36–0.6 MPa, the nozzle diameter – 1–2.2 mm, the number of nozzles – 3–7, and the angle of attack jets – 36–180 degrees.

The adequacy of the derived regression equation was verified by the Fisher criterion [19]:

$$F = \frac{S_{res}^2}{S_{rep}^2},$$

where $S_{res}^2$ is residual variance and $S_{rep}^2$ is reproducibility variance.

The residual variance determines scattering of experimental data about the regression equation, whereas the reproducibility variance – the value of random error.

Value $S_{res}^2$ was calculated as follows:

$$S_{res}^2 = \frac{\sum_{i=1}^{N} (y_{exp} - y_{cal})^2}{(N - (m + 1))},$$

where $y_{exp}$ is experimental value of the unknown quantity, $y_{cal}$ is experimental value of the unknown quantity that is calculated by the derived regression equation, $N - (m + 1) = f_i$ denotes the number of degrees of freedom that is calculated as the difference of the number of experimental points $N$ and the number of factors $m$ that are determined on the basis of these points.

The value of reproducibility variance was found at the stage of the preliminary analysis of experimental data with the help of the following dependence:

$$S_{rep}^2 = \frac{\sum_{i=1}^{N} (y_{exp} - y_{AM})^2}{(N - 1)},$$

where $y_{AM}$ is the arithmetic mean of the experimental values of the unknown quantity and $N - 1 = f_2$ is the number of degrees of freedom.

The Fisher criterion value was calculated by formula (5) and compared with the tabular one for the selected significance level $\alpha$ and degrees of freedom $f_1$ and $f_2$. If $F < F_\alpha$, the derived regression equation adequately interprets the experimental results.

The coefficient of determination and the mean relative approximation error are often used as approximation accuracy criteria [20]. The coefficient of determination was calculated as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_{exp} - y_{cal})^2}{\sum_{i=1}^{N} (y_{exp} - y_{AM})^2}.$$

The mean relative approximation error ($e_{MRE, \%}$) was calculated as follows:

$$e_{MRE} = (1 / N) \sum_{i=1}^{N} \frac{|y_{exp} - y_{cal}|}{y_{exp}} \times 100.$$

The closer the coefficient of determination value is to 1 and the mean relative error value to 0 %, the more accurate is the mathematical model.

5. The results of optimising the jet cavitator design

The required mathematical model to optimise the cavitator design has the following general form:

$$E(P, d, n, \beta) = E_{AM} \cdot f_1(P) \cdot f_2(d) \cdot f_3(n) \cdot f_4(\beta),$$

where $E(P, d, n, \beta)$ is the value of heat energy released due to cavitation (depends on 4 factors: $P$, $d$, $n$, $\beta$); $E_{AM}$ is the arithmetic mean of the values of heat energy of $N$ sample set; $P$; $d$ is nozzle diameter, mm; $n$ is the number of nozzles, items; $\beta$ is the angle of attack, degrees; $f_1(P)$, $f_2(d)$, $f_3(n)$, $f_4(\beta)$ are one-dimensional functions that depend respectively on $P$, $d$, $n$, $\beta$.

The use of the Brandon method allowed deriving the following multiplicative model:

$$E(P, d, n, \beta) = 1.753 \cdot (1.562P)^{\frac{5}{3}} \times (4.969d^{1.5} - 32.294d^{0.979} + 76.133d^{1.5} - 76.926d + 29.08) \times (-0.009n^4 + 0.206n^3 - 1.597n^2 + 5.353n - 5.494) \times (-10^{-2} \beta^4 + 3.10^{-2} \beta^3 - 2.10^{-2} \beta^2 + 0.979).$$

The chart of dependence $E=f(P, d, n, \beta)$ calculated by equation (11) on the basis of experimental data is shown in Fig. 1.

Since the parameters that affect the value of heat energy were changed 5 times, the size of $N$ sample set was 5$^4 = 625$ combinations.

The adequacy of the derived regression equation (11) was verified by the Fisher criterion. The residual variance and the reproducibility variance were calculated as follows:

$$S_{res}^2 = 9.554 / (625 - (4 + 1)) = 0.015,$$

$$S_{rep}^2 = 45.924 / (625 - 1) = 0.074.$$

The Fisher criterion value was found as follows:

$$F = 0.015 / 0.074 = 0.203.$$

The accuracy of the mathematical model was assessed with the use of two criteria – the coefficient of determination and the mean relative approximation error:

$$R^2 = 1 - 9.554 / 45.924 = 0.792,$$

$$e_{MRE} = (1 / 625) \cdot 36.589 \times 100 \% = 5.85 \%.$$
The research on the process of cavitation in a heterogeneous system “water – dispersed solid particles” has revealed the effect of particle flotation. Dissolved gases (their small content in a liquid phase) play the role of cavitation germs. Therefore, in order to intensify cavitation and an attendant flotation effect, before cavitation processing, liquid-phase media were injected with small amounts of air – 0.5–3.0 % by the liquid volume. The dependence of heat energy (E, MJ) that is released during cavitation on the content of air injected in the aqueous medium (C, % vol.) is shown in Fig. 2.

![Fig. 2. The dependence of heat energy (E, MJ) on the content of air injected in the aqueous medium (C, % vol.)](image)

The calculated values of the coefficient of determination ($R^2 = 0.792$) and the mean relative approximation error ($\pm \text{MRE} = 5.85 \%$) indicate a fairly high precision of the mathematical model that describes such a complex phenomenon as cavitation.

As Fig. 2 shows, the dependence of heat energy on the content of air in the aqueous medium has an extreme character. When the amount of air was in a range of 0–0.5 % by the volume of water, there was a significant decrease in the value of heat energy – 0.606 MJ (from 2.539 to 1.933 MJ), i. e. 24 %. This probably results from the accumulation of small “survivable” bubbles that do not flatten [21] and, respectively, formation of a “degenerate cavitation” mode. Further increasing the air content in aqueous media (from 0.5 to 2.0 %) reduces the distances between the bubbles and, consequently, makes them coalesce.

Due to pressure fluctuations and the presence of turbulent flows in the cavitator, there increases a probability of dispersion and subsequent flattening of the formed bubbles, which is proved by an increase of 0.31 MJ in the value of heat energy released during cavitation exceeds 2.5 MJ. Given the increasing probability of erosion of cavitating parts, namely nozzles, when the angle of attack jets approaches 180 degrees, it is appropriate to restrict its value range to 144–170 degrees.

The tabular Fisher criterion value of the selected level of significance $\alpha = 0.05$ and that of the number of degrees of freedom $f_1 = 620$ and $f_2 = 624$ is 1.51 [22]; hence, the regression equation (11) adequately reflects the results of the experiment.

The results of the sonochemical analysis of the cavitation processing with and without small amounts of air injected in the aqueous medium are presented in Table 1. The amplitude of sound pressure (dB) was expressed with reference to the baseline magnitude of $V/\mu Pa$.

### Table 1

<table>
<thead>
<tr>
<th>The amount of air injected into the aqueous medium, % vol.</th>
<th>The amplitude of sound pressure, (dB re 1V)/\mu Pa</th>
<th>Normalized values of the acoustic signal intensity, % of the max</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>35</td>
</tr>
</tbody>
</table>

If the cavitator has 5 nozzles with a diameter 1.6 mm, the inlet pressure of 0.57 MPa, and the angle of attack jets of 150 degrees, the injection of air in an amount of 2 % by the liquid volume allows maintaining the amplitude of the sound pressure throughout the processing at a level of –11 dB. Simultaneously, the amplitude of the sound pressure was lower by 18 %, i. e. the cavitation declined, when the processing parameters were the same but air was not pumped into the system.

6. Discussing the Brandon method use in optimising the jet cavitator design

Fig. 1 shows that the optimal conditions for cavitation processing of liquid-phase media are in the plane of the following values: inlet pressure – 0.54–0.60 MPa, nozzle diameter – 1.6 mm, the number of nozzles – 4–5, and the angle of jet attacks – 144–180 degrees. Under such conditions, the value of heat energy released during cavitation exceeds 2.5 MJ. Given the increasing probability of erosion of cavitating parts, namely nozzles, when the angle of attack jets approaches 180 degrees, it is appropriate to restrict its value range to 144–170 degrees.

The research on the process of cavitation in a heterogeneous system “water – dispersed solid particles” has revealed the effect of particle flotation. Dissolved gases (their small content in a liquid phase) play the role of cavitation germs. Therefore, in order to intensify cavitation and an attendant flotation effect, before cavitation processing, liquid-phase media were injected with small amounts of air – 0.5–3.0 % by the liquid volume. The dependence of heat energy (E, MJ) that is released during cavitation on the content of air injected in the aqueous medium (C, % vol.) is shown in Fig. 2.

![Fig. 1. A chart of dependence $E=f(P, d, n, \beta)$ calculated by the Brandon method](image)
of water and reflects the most favourable conditions for a joint implementation of cavitation processing of liquid-phase media and a flotation separation of dispersed solid particles as successive stages of the cavitation and flotation technology of separating heterogeneous media [9]. The sonochemical analysis findings also prove the feasibility of pumping air into aqueous media.

7. Conclusions

The research results are as follows:
(1) we have constructed a 4-factor multiplicative mathematical model of cavitation processing of aqueous media that reflects the dependence of the heat energy value on the technological (inlet pressure in the cavitator) and design (nozzle diameter, the number of nozzles, and the angle of attack jets) parameters – equation (11);
(2) the adequacy of the derived regression equation is proved by the Fisher criterion \( F < F_\alpha = 0.203 < 1.51 \). The accuracy of the model is assessed by the values of the coefficient of determination \( R^2 = 0.792 \) and the mean relative error of approximation \( \varepsilon_{\text{MRE}} = 5.85\% \);
(3) the analysis of the 4-factor multiplicative model allowed finding optimal conditions for cavitation processing of liquid-phase media; they are as follows: inlet pressure \( 0.54–0.6 \) MPa, nozzle diameter \( 1.6 \) mm, the number of nozzles \( 4–5 \), and the angle of attack jets \( 144–170 \) degrees;
(4) it is found that, in comparison with the absence of air, the content of air of \( 2\pm0.25 \) % by the volume of an aqueous medium greatly intensifies the formation of the “flotation” layer (its height, dispersibility of bubbles, and gas saturation).

References

1. Introduction

Due to the opportunity of selling energy at “the green tariff”, the use of the bio fuel as a renewable energy source is a stimulating factor for producing both electricity and heat from one energy source [1, 2]. In the production of the pellet fuel, the costs of timber drying make up to 25 % of the total costs. The moisture content should not exceed 10–12 %, and raw timber, for example, may contain about 50 % of water. In order to support the energy-saving temperature and the aerodynamic regimes, drying must take place in the approved interactions which is possible to obtain with using the cogeneration technologies, which have a primary engine, an electricity generator, a heat utilization system and a system of the control and management. Moreover, the measurement of the temperature and the air humidity as a drying agent in the drying chamber and the humidity of the dried timber, are not always used appropriately to support timber drying due to the complexity of measurements, which makes it impossible to use the measurements in the coherence to prevent the influence on the change in the parameters of drying for ensuring the continuous production of the pellet fuel. All these facts substantiate the relevance of this work.

2. Analysis of scientific literature and the problem statement

The means of improving the timber drying technology is based both on the intensification of the heat exchange