1. Introduction

Nowadays, one of the most urgent problems of power electronics is to minimize the size and to optimize power supply for semiconductor converters. As a result of downsizing, electronic components are becoming more sensitive to external influences, current surges, and thermal stresses. Due to an increase in thermal capacity, heat transfer during fast processes becomes complicated.

According to previous research [1], almost 60% of accidents occur as a result of semiconductor converters overheating. In this case, every 10°C increase in temperature doubles the accident rate.

In order to ensure trouble-free operation of a converter, it is necessary to stabilize the thermal conditions. This can be done with the help of methodological and circuit approaches. The methodological approach is a simulation of thermal processes and the selection of the converter component parameters so that, even with a decrease in the maximum permissible parameters of the device components due to the heat during the restart and the occurrence of transients, these parameters are not exceeded. Moreover, magnetic materials and elements of the device can be chosen so that the simultaneous heating of several elements can result in a mutual compensation of the influence of changes in the electrical parameters of the heated components on the output characteristics of the circuit or device. The circuit design approach involves introduction of additional devices or chains in the circuit with a view to respond to the emergency sections of the device.

Creating thermal protection of converting devices is complicated by the fact that using power switches due to the thermal capacity makes it impossible to perform a full removal of heat from the semiconductor device chip. Especially dangerous are the reclosing modes in which the heated appliance with lowered boundary values gets a thermal stress. These processes must be modelled by solving the problem in a methodological way, calculating the electrothermal processes that are combined in the device and selecting the elements of the circuit so as to reduce the amount of thermal stress and soften the character of the transition process when the heated converter is switched on.

2. Analysis of the published studies and formulation of the problem

To create thermal protection, it is necessary to analyse the combined electromagnetic and thermal processes of the
3. The purpose and tasks of the study

The purpose is to calculate the electrothermal processes of a converter so that the models of electromagnetic and thermal processes or the control system during operation could prevent an emergency mode of the device caused by overheating at the design stage.

To achieve this purpose, it is necessary to do the following tasks:
- to develop and solve differential equations on the system state for electromagnetic processes in a matrix form, which are expanded by adding the thermal component;
- to analyse the matrix exponentials that characterize the transition process as well as to determine the parameters by adjusting which it is possible to reduce oscillation during the transition process in the moments that are close to overheating or an accident;
- to select the maximum parameter values of elements in the desired temperature range, to choose the inductor core with the necessary dependencies of the magnetic permeability on the temperature and the magnetic field strength, and to calculate, if necessary, additional chains of protection against overheating.

4. A thermal simulation of pulse converters

4.1. Thermal models of the device components

It is necessary to measure the parameters of the converter components at the required operating frequency range and to determine the analytical dependence of the component parameters on the temperature deviation from the normal value.

The research concerns the circuit of a step-down pulse-width modulator (PWM) of the 2nd order shown in Fig. 1.

The circuit elements:
- VT is a MOSFET transistor IRFP4232;
- VD is a diode D650N;
- L is an inductor with a core 3S11 by Philips [10];
- C is a capacitor Epcos B43415 [11].

When the mechanism is switched on or at intermittent modes of operation, its semiconductor devices are at the forefront. During the fast processes of switching on, the temperature of the semiconductor device crystal can greatly exceed the device body temperature. Fig. 4 shows a thermal transition process when applying a pulse current to the transistor VT at the housing and heat sink temperature amounting to 50 °C [7].

The heat capacity depends on the composition of the semiconductor material between the crystal and the housing as well as their splicing. The heating of the crystal is calculated as:

\[ T_i(t) = T_i(0) + P_i(t) \sum_{i=1}^{n} R_{th,i} \left( 1 - e^{-t/T_{th,i}} \right) \]  

(1)
where $T_b$ is the body temperature, $P$ is the instantaneous power, $R_s$ and $C_{th}$ are thermal resistance and capacitance. The expression under the sum operator is the thermal impedance between the crystal and the case of the semiconductor device $Z_{thc}$. This value is given by the manufacturer in the form of a graphic dependence of a family of characteristics at various duty cycles on the pulse time. Selection of the data values can simplify the calculation of the crystal temperature [8]:

$$T_c(t) = T_b(t) + P(t) \cdot Z_{thc}(t),$$

where $P = U_1 \gamma$ and $\gamma$ is the duty cycle.

To simulate the active parameters of semiconductor devices, it is possible to represent them in the formula form. Thus, the author of [9] considers temperature dependence of the resistance of a drain-source transistor $R_{DS}$, direct resistance of a diode $R_{VD}$, and forward voltage of a diode $U_{V_D}$.

$$R_{DS} = R_{DS0} \left[ 1 + \frac{T - 20}{298} \right],$$  

$$R_{VD} = R_{VD0} \left[ 1 + \frac{T - 20}{298} \right],$$  

$$U_{VD} = U_{V_{D0}} - 2 \cdot 10^{-3} \cdot (T - 20),$$

where all zero values are reference data of the manufacturer.

During transients, the main objective when dealing with a system of differential equations is to analyse the first member of the matrix of coefficients, precisely the value of $t + \Delta t$. The other parameters during fast processes will not have time to heat up enough to significantly alter the electromagnetic parameters. However, by controlling the parameters of passive components, it is possible to change the nature of the transition process, thereby providing thermal protection of the system.

When semiconductor devices are heated, there is a decrease of the maximum current for operating at which the apparatus is designed. Due to the vibrational transition process at re-enabling a heated device, the maximum allowable value of the current can be exceeded, leading to the device failure.

By taking the data of the magnetic permeability of the material at a temperature of 25 °C. The loss in the magnetic circuit is calculated as:

$$P_{loss} = \frac{V^2}{T} \cdot \int dB(t) \cdot |B(t)|^{-\gamma} dt,$$

where $V$ is the volume of the magnetic circuit; $c, x, y$ are the reference properties of the magnetic circuit, and $B$ is the flux density [12].

In the case of the capacitor impedance simulation, the points' arrangement is of an exponential shape. With the exponential component, the characteristic begins to look like the original already at $n=3$:

$$Z = a_l e^{-x} + a_r \theta^x + a_\theta + Z_0.$$

At a frequency of 10 kHz, the response will appear as shown in Fig. 4. With this characteristic, it is possible to determine the dependence of the capacitance on temperature.

The power of the active losses in the capacitors can be found through the work of the reactive power on the loss tangent:

$$P = P_{max} \cdot \tan \delta = 2\pi fU^2 C \cdot \tan \delta.$$

The reactive power of the capacitor must not exceed the permissible values specified by the manufacturer lest
the condenser temperature at a given ambient temperature should exceed the maximum permissible value. The permissible value of the current through the capacitor depends on the permissible temperature of heating and the geometric dimensions of the housing, and it is provided in the technical documentation.

\[ \frac{dX}{dt} = AX + B, \quad (9) \]

where X is a vector of state variables, A is a coefficient matrix, and B is a matrix of external influence.

For the step-down PWM converter that is shown in Fig. 1, the system looks as follows:

\[ \begin{bmatrix} \frac{d_i}{dt} \\ \frac{d_u_i}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & -\frac{1}{r} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_c \\ u_i \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad (10) \]

where \( s \) corresponds to the state of the transistor. When the transistor is open, \( s = 1 \); when the transistor is closed, \( s = 0 \);

\[ r = r_i + r_d, \quad (1-s) + r_i \]

is the sum of the resistances of the transistor, the diode and the inductor based on the transistor state; \( L, C, \) and \( R \) are the coil inductance, the capacitor, and the load resistance, respectively, and \( E \) is the power supply voltage.

Let all the coefficients be equal to the values of the electric components of the parameters under normal conditions, excluding heating.

Since this system can be modified, then (9) takes the following form:

\[ \frac{dX}{dt} = (A_0 + A_\xi)X + B, \quad (11) \]

where \( A_0 \) is a component matrix corresponding to the operation of the device at a temperature of 25 °C, \( A_\xi \) is a matrix of changes in the electrical parameter values of the components due to heating, and \( \xi \) is a thermal coefficient matrix that is characterized by heat removal. At an ideal heat removal, the coefficient values are equal to zero, and the matrix of thermal coefficients stops affecting the system. If \( \xi = 1 \), the heat is not removed; whereas at \( \xi > 1 \) the device is heated up from the outside.

The matrix of variable coefficients for the circuit of a step-down PWM converter should include changing resistance of the transistor, diode and inductor \( \Delta r(\theta_{VT}, \theta_{VD}, \theta_{L}), \) a change in the magnetic permeability of the inductor core, as well as the capacitance change \( \Delta C(\theta_c) \), where \( \theta = T - 25 \) °C is a temperature of a particular component that differs from 25 °C.

The \( L_\xi \) matrix is ill-conditioned, as it contains temperature-dependent components. Consequently, within one matrix, it is necessary to calculate both transients that last microseconds and thermal processes that last several hours. To simplify the calculations, the matrices are separated; in complex circuits of converters where it may be difficult to single out the models, it is possible to use the methods of diakoptics [13].

During steady operation modes, passive components set temperature regimes. In contrast to the intermittent operation mode, the semiconductor crystal temperature barely exceeds the device body temperature, which simplifies its adjusting with the help of the heat sink. Therefore, it is reasonable to consider steady operation modes while examining heating of passive components and considering that heating of semiconductor devices corresponds to a certain operational temperature point or changes periodically.

If the components are in close proximity to one another, they will have the same heating temperature, and the system will be much simpler. For example, at \( \theta_0 = \theta_1 \), the value of the coefficient \( \xi_0 \) is the same for all components of the matrix except the first one. Assuming that for a steady operating mode of the system the semiconductor temperature is stable, without changes in a particular segment of work, this ratio will be a scalar number within \( 0 \leq \xi \leq 1 \) in the absence of external heat sources.

The solution of the system of differential equations on the interval of an open transistor is the following:

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{At} \begin{bmatrix} I(mT) \\ U(mT) \end{bmatrix} + A^{-1} \begin{bmatrix} e^{Bt} - 1 \\ 0 \end{bmatrix} \frac{E}{L}, \quad (12) \]

where \( A = A_0 + A_\xi \) or \( A_\xi \), \( A^{-1} \) is an inverse matrix, and I is an identity matrix.

In the period of a closed transistor, the solution takes the following form:

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{A(t-t_0)} \begin{bmatrix} I(mT + \gamma mT) \\ U(mT + \gamma mT) \end{bmatrix} \quad (13) \]

where \( A = A_0 + A_\xi \) or \( A_\xi \).

The nature of transient and steady processes is determined by the matrix exponential \( e^{At} \), whose specific form is determined, in turn, by the eigenvalues \( \lambda \) of the matrix \( A \). The eigenvalues are found by the expression:

\[ \det(A - \lambda I) = 0. \quad (14) \]

Using the formula of the matrix bisection on the basis of binary vectors [14], we have:

\[ \det(A_0 + \xi A_\xi - \lambda I) = \det(A_0 - \lambda I) + \sum (-1)^{\sigma} \Delta_d(d_{\xi A_\xi}) - \lambda^2 + \det(A_\xi - \lambda I) = 0, \quad (15) \]

where \( \sigma \) is the sum of the column and row values of an element of the matrix, \( \Delta_d(d_{\xi A_\xi}) \) is a minor that corresponds to the \( d_{\xi} \) of the electromagnetic coefficients matrix, \( \Delta_d(d_{\xi A_\xi}) \) is a minor that corresponds to the complement of the binary vector \( d_{\xi} \) of the matrix of heating-based changes in the
electromagnetic coefficients. This total in the unfolded form looks as follows:

\[ a_1 \cdot \xi_{12} a_2(\theta) - a_2 \cdot \xi_{22} a_1(\theta) - a_{12} \cdot \xi_{12} a_1(\theta) \mbox{.} \]

(16)

On simplifying this equation for \( \lambda \), we obtain the following quadratic equation:

\[ \lambda^2 + b \cdot \lambda + c = 0, \]

(17)

where \( b = -a_{11} - a_{22} - \xi_{11} a_{11} - \xi_{22} a_{22} \), and \( c \) is the sum of all available components of the equation.

The roots of the characteristic equation are defined as follows:

\[ \lambda_{1,2} = -\frac{1}{2}(b \pm \sqrt{b^2 - 4c}), \]

(18)

If the roots are real, the transition process is aperiodic. This characteristic does not have current or voltage jumps, which is advantageous when switching on a heated device with reduced maximum allowable values of parameters of the semiconductor components. The temperature increase on the chip of the key elements will not be abrupt either, as shown in Fig. 2.

With complex roots, the quantity of the vibration component of the process depends on the proximity of the roots to the hypochondriac axis (Fig. 5).

![Fig. 5. The placement of the characteristic roots in the complex plane](image)

The oscillation criterion is determined by the angle \( \phi \) in the following way:

\[ \tan(\phi) = \frac{\text{Im}(\lambda)}{\text{Re}(\lambda)}, \]

(19)

where \( \eta \) is the minimum value of the real root that determines the duration of the transition process as a value that is inversely proportional to the control time. The boundary \( \xi \), which is distant from the imaginary axis, is determined by the roots that have very little impact on the transition process.

According to Vieta’s formulas, \( b = \lambda_1 + \lambda_2 \), and \( c = \lambda_1 \lambda_2 \). By introducing a value that is equal to the geometric mean of the roots of the equation \( \Omega = \sqrt{\lambda_1 \lambda_2} \), the characteristic equation can be written as follows:

\[ \lambda^2 + \frac{b}{\Omega} \lambda + \lambda^2 + B \cdot \Omega \cdot \lambda + \Omega^2 = 0. \]

(20)

In this form, the value \( B \) characterizes the curve of the transition process, and \( \Omega \) is the scale of the process time \( \tau = \Omega \cdot t \).

With the equation in this form, it is possible to ensure thermal protection of the device at the methodological level, during transients in the design of the device, or during the realization of the circuit beyond the stationary mode, or while carrying out feedback on the temperature and gradually stabilizing the operation of the converter. For example, in the intermittent mode, during the oscillatory transient process, the current peaks are higher than the limit value of the key parameters. Hereby, it is essential to choose properly the core of the inductor so that it, at a certain frequency and intensity of the magnetic field during the heating, could change the inductance of the throttle as much as to decrease the vibrational component. Thus, the roots of the characteristic equation are moving away from the imaginary axis, the value \( B \) is reduced, but \( \Omega \) on the contrary increases. The nature of the transients is close to aperiodic, and the process duration increases.

### 5. Thermal converter protection management

As a result of the aforementioned calculations, it is possible to create thermal protection of a converter both during transition processes by managing the degree of the process oscillation and during steady-state operation. The nature of the transition process can be directly influenced just by changing the intensity of the magnetic field of the coil, for example, by introducing a parallel DC-controlled coil and thus changing the angle \( \phi \). It is also possible to choose the magnetic material of the core so that the heating will increase the inductance of the coil and the amplitude of the transient will decline to a safe level, which is necessary for heated semiconductor devices with lowered boundary values of the parameters (Fig. 6).

![Fig. 6. A transient within a step-down PWM converter at the following parameters: a — the inductor temperature is 25 °C; b — the inductor temperature is 90 °C](image)

These results show that even by properly choosing the material of the core of the inductor it is possible to protect the converter from overheating. By controlling the intensity of the magnetic field of the inductor, the nature of the transition process can be changed from oscillatory to aperiodic. An aperiodic nature of the process is the safest option in the case of a recloser with heated semiconductor devices.

### 6. The use of electrosimulation for thermal protection

This approach can be used in the theoretical calculations for electric processes and for modelling systems that require high precision. For example, for adjusting the transition process characteristics, it is possible to create a thermal...
protection system that operates like a feedback chain on
temperature (Fig. 7).

![Diagram](image)

**Fig. 7. A thermal protection system for a pulse converter**

C in Fig. 7 is a pulse converter whose switch receives
switch-on and switch-off pulses of a specific frequency that
is set by the generator or a chain of command. The convert-
er circuit contains an LC filter, as in the above described
second order step-down PWM. The converter components
have thermal sensors (1) that transmit the temperature
value of the device elements to the microprocessor (2) with
an incorporated thermal model of all the elements. The next
steps include: recalculation of the electromagnetic values of
the elements’ parameters, analysis of the complexity of the
transition process on determining the temperature by the
temperature sensors, and stabilization of the processes by
changing the input frequency of switching on the converter
switch, thereby affecting the reactance value of the inductors and circuit capacitors. At the same time, adjustment can
be made through the operational amplifier (3) by changing the
DC that powers the inductor (4) that is connected with
the magnetic transducer of the filter coil, thereby changing
the magnetic field intensity and thus the thermal character-
istics of the magnetic permeability of the core.

The dependence of the magnetic permeability on the tem-
perature at low magnetic field intensity means an increase of
the magnetic permeability as the heating increases to the Cu-
rie temperature and then sharply declines after reaching this
temperature. As the magnetic field intensity increases, this
dependence begins to grow at a slower rate; on reaching the
predetermined value of tenseness, this dependence changes
into the opposite direction, which means that the magnetic
permeability decreases while temperature increases. It is pos-
sible to control this feature to stabilize the electromagnetic
processes as to temperatures that are critical for the circuit or
to change the nature of the transition process by increasing its
time and turning the oscillation process into aperiodic.

**7. Discussion of the results of using the electrothermal
modelling to create thermal protection of
pulse converters**

A combined electrothermal modelling is necessary to
ensure thermal protection of devices. There are many tech-
niques and simulation packages of both electromagnetic
transducer processes and separate thermal processes. For
example, when modelling transients of a pulse transmitter
in computer environments (e.g., Simulink or Plecs), it is
possible to connect components that are responsible for heat
transfer and heat capacity. However, these applications will
not take into account that, as a result of changes due to heat
or hot restart of the device, the inductor will receive a modi-
fied current, and hence the intensity of the magnetic field of
the coil will be different from the one specified by the user.
 Consequently, the dependence of the magnetic permeability
on the magnetic core temperature will be different. A similar
problem may arise in relation to other elements. The advan-
tage of this method is that, while analysing the electromag-
etic and thermal processes, these nuances are taken into
account and the calculation will be more accurate, which
will expand the boundaries of safe operation of converters.

This method is useful when calculating devices that work
in conditions bordering on the maximum modes. This method
is also relevant to ensure heat protection of semiconductor
devices during transients when the thermal protection devices
as well as the cooling system can not prevent thermal shock in
the crystal of semiconductor devices due to the brevity of the
cooling process and the inertia of the heat capacity of the ele-
ments. The procedure can be used to create additional thermal
protection systems by choosing materials of the device elements
so that during their heating either the transient oscillations will
diminish or the transition process will change its character.

The present study is a ground for further creation of ther-
mal protection systems, soft-start systems of pulse convert-
ers, and control systems for electric stabilization processes
in pulse converters.

**8. Conclusions**

1. To solve the problem of a combined electrothermal
modelling, the study suggests equations of the state system
with addition of a thermal component and heat transfer
characteristics. Thermal and electrical components of the
equations were singled out due to poor conditioning of the
coefficient matrix; while solving the differential equation,
the integration time for calculating electromagnetic and
thermal processes, these nuances are taken into
account and the calculation will be more accurate, which
will expand the boundaries of safe operation of converters.

2. The example of a step-down PWM converter pro-
vides solutions to the equations of state. The nature of the
transition process in these solutions depends on the matrix
exponentials. The position of the characteristic roots of the
matrix in the complex plane characterizes the nature of the
transition process. The closer are the roots to the real axis,
the smaller is the oscillation in the process. If the roots are
not complex, the transition process is aperiodic, which is the
most secure form that should be aimed at when switching on
a heated device. Vieta’s formulas can help select parameters
of the circuit elements so as to obtain the desired form of
electromagnetic processes at different temperatures.

3. By using this technique for electrothermal process mod-
delling, it is possible to choose the elements of the converter
circuit. For example, in a step-down PWM converter with a
selected magnetic material of the inductor core for high per-
meability dependence on heating, the transient oscillations at high
temperatures will be smoothed, thereby making the reclosing
of the device safer for the key semiconductor elements. An ad-
ditional chain of thermal protection on a microprocessor with
a programmed electrothermal model can be created to serve as
feedback on temperature or to work as a system of soft start-up
when the device is re-enabled.
References