1. Introduction

Currently, the direction finding of radio electronic devices is carried out in a complex electromagnetic environment (EME), which is characterized by multipath propagation of radio emissions and by frequency coverage of the desired signal and the intrusive noises, of a priori uncertainty relative to the parameters of radiations. The promising direction for the implementation of the direction finding for these conditions is the use of digital broadband correlative-interferometric finders with the antenna array (AA) and the digital synthesis of its radiation pattern (RP).

Correlation-interferometric direction finding (DF) techniques provide a wide frequency range, the resistance to interference caused by multipath reception, the high sensitivity and accuracy. However, the most plausible unbiased assessment of estimation of directions to the sources of radio radiation (SRR) is based on the search of a sequential correlational analysis and the review of the space. This fact greatly limits their performance and requires a lot of hardware expenditure of data processing systems, reducing the effectiveness of their application to the dynamic conditions of EME [1]. The disadvantage of this method is also a low exactness of direction finding of SRR, which spectra are completely overlapped by frequency.

Therefore, for the use in the automated radio monitoring systems the development and the research of fast-acting digital methods of correlation and interferometric direction finding is an urgent task.

2. Literature review and problem statement

In [2, 3] the searching correlative-interferometric methods of estimation of directions towards the SRR using the AA, which are effectively implemented in a digital form was studied. However, these methods use the search compensatory method of direction finding, which makes a low speed broadband direction finding of sources of radio emission.

In [4] a method of digital broadband integrated spectral correlative direction finding with the use of linear AA and digital synthesis of its RP is suggested, which provides the increase of the performance of direction finding in a difficult EME. However, in this paper, we proposed to synthesize two RP, the second RP has two times wider beams than the first one, that reduces the potential accuracy and the anti-jamming of the direction finder and reduces its performance. This happens as a result of the hit of intrusive noises to the broadened beams of the second multilobe RP, and also because of the increase of the noise pass band.

In [5] the direct digital method of spectral correlative-interferometric direction finding with the reconstruction of the spatial analytical signal, which has a high accuracy and noise immunity due to the use of preliminary spatial selection and the dispersive-correlative evaluation of direc-
tions to the SRR was proposed. However, this method of direction finding uses the estimation of spatial frequency of the signal on two reconstructed references of the analytical signal. This leaves the possibility of its further improvement to minimize computing costs and increase the performance of direction finding.

In [6–8] the nonlinear spectral methods of direction finding that provide high spatial resolution of the received radiation were studied. However, they have a number of significant drawbacks for radio monitoring systems, such as:

– the high computational complexity (signal processing time), several times higher than the complexity of the searching correlative method of direction finding;
– the need for a priori information about the exact amount of radiation, received in the mixture;
– the bias of estimates of directions, which worsens the accuracy of direction finding;
– the loss of working stability at low (about 10dB) signal/noise ratio.

As a result, the efficiency of spectral methods of direction finding, which is determined, in the first place, by the ratio of speed/accuracy of direction finding is insufficient in radio monitoring systems. This is confirmed by the absence of their use in modern digital direction finders of radio monitoring systems [1, 9].

Thus, the unsolved part of the general problem of the development of means of direction finding of sources of broadband radio signals in real time is to work out a new direct digital method of the correlative-interferometric direction finding with the parallel spatial selection of signals, which will have a minimal computational cost.

3. The aim and tasks of the study

The aim is to develop and to study the high-speed digital direction finder with a spatial selection and two-dimensional correlation processing of signals.

To achieve this goal the following tasks were solved:

– the development of the direct digital correlation-interferometric method of direction finding with the parallel spatial selection of signals and with two-dimensional correlation processing, which has the lowest computational cost;
– the study of accuracy and the width of the working sector of direction finding by simulation.

4. Analytical research and development of the direction finding method

To enable direction finding of a few sources of wideband emissions in real-time in the complex EE conditions we will develop a digital method of correlative-interferometric direction finding with reconstructing of spatial analytic signal and use of a linear AA, which will have a minimal computational cost.

Let the additive mixture \( U(t) \) of \( L \) useful signals \( S(l) \) be adopted spaced by the apart \( Z \) identical direction finding channel of the linear AA subject to the availability of its own additive noises \( n_z(t) \), uncorrelated with each other and which have the same level. The conditions of DF are the following:

\[
U_z(t) = \sum_{l=0}^{L-1} S_z(t - \tau_z(t)) + n_z(t),
\]

where \( U_z(t) \) is a mixture, taken by the \( z \)-th DF channel, \( z=0,1,...,(Z-1) \); \( S_z(t - \tau_z(t)) \) – the \( l \)-th useful signal received by the \( z \)-th DF channel; \( \tau_z(t) \) – the delay time of reception of the \( l \)-th desired signal by the \( z \)-th DF channel relative to the reference channel; \( n_z(t) \) – the own additive Gaussian noise with the even distribution of the power density \( N(o) \) by the frequency \( o \) within the band of the simultaneous analysis of the \( z \)-th channel.

Let the possible values of directions \( o \) to SRR relative to the antenna base of the direction finder be the random variables distributed with equal probability within the sector \([0N, 0V]\) of direction finding. Also, the temporary Fourier spectra \( S_z(l(\omega S.l)) \) of the useful signals, corresponding to the radio emissions of the equation (1), are located within the bandwidth \([oN, oV]\) of the simultaneous reception corresponding to the bandwidth of direction finding channels.

The useful signals \( S(l) \) are casual noise-like emissions of point SRR, located in the far field. For all that the values of the average frequencies \( \omega S.l \) of signal spectra, the width \( \Delta \omega S.l \) of signal spectra, the power capacities \( PS.l \) of signals are random variables with a uniform distribution of the probability density in the respective ranges.

For these conditions of the complex EME the noise immunity of DF of the operating sources is ensured by using a multi-stage previous selection of station and industrial noises within the frequency band \([oN, oV]\) of a simultaneous reception. It is known [10], the high efficiency of these kinds of selection is enabled by the use of the frequency domain with digital processing. Therefore, it is advisable for the use of direction finding to take advantage of frequency domain obtained from the processing, based, for example, on the fast Fourier transform (FFT) algorithm of the time complex frequency spectra of the received implementation of the mixture \( U_z(t) \) on the intermediate frequency \( oIF.k \) [11] by the direction finding channels. The time spectra of mixtures \( \{U_z(t)\}_{z=0,1,...,Z-1} \) received by Z radio channels and defined at the intermediate frequency \( oIF.k \) according to the FFT algorithm will look like:

\[
U_z(joIF.k) = \sum_{l=0}^{L-1} S_z(joIF.k) + N_z(joIF.k),
\]

where \( oIF.k \) – the frequency of the \( k \)-th component in the spectrum \( U(z\omega IF.k) \) of the received mixture \( U_z(t) \) into an intermediate frequency band; \( k=0,1,...,(NS-1) \) – a number of the component in the spectrum \( U(z\omega IF.k) \); \( NS \) – an amount of reading of implementation of the adopted mixture; \( N_z(joIF.k) \) – a complex spectrum of noise implementation in the \( z \)-th channel.

We’ll assume that within the frequency bandwidth \([oN, oV]\) of simultaneous reception of each \( z \)-th DF channel of AA by processing of known methods of the complex spectrum \( U_z(joIF.k) \) of the mixture received at the intermediate frequency \( oIF.k \), the separation of the \( l \)-th integrated time spectra \( S_z(joIF.k) \) of the useful signals, non-overlapping by frequency is made. After that the process of the direction finding of the selected radio emissions can be performed in parallel and independently. The separation of spectra of useful signals and intrusive noises is usually performed by using the amplitude, frequency and spatial selection [10, 12]. As a result, the simultaneous reception frequency band \([oN, oV]\) is divided into a plurality of the bands.
of simultaneous analysis corresponding to the frequency bands of the spectra of the useful signals with the corresponding width:

$$\Delta \omega = \Delta \omega_{ab},$$ (3)

where $\Delta \omega_{ab}$ – the b-th band of frequencies of simultaneous analysis, which may exceed the width of the spectra $S_{z,l}(j\omega_{IF,k})$ of the useful signals.

In the future, we’ll consider the direction finding procedure within just one b-th band $\Delta \omega_{ab}$ of the simultaneous analysis, since the processing of the received compounds in the isolated B bands is performed equally.

It is also advisable to perform a removal of error of compensation by restoring the operating frequency $\omega_{Sk}$ of spectrum components $U_{z,l}(j\omega_{Sk,k})$.

To implement spatial selection with minimal time it is advisable to use parallel spatial selective reception based on FFT and separation of a mixture of radiation $U_{z,l}(j\omega_{Sk,k})$ [13]. To do this, you must implement the processing of the received radio emissions, equivalent to the action of the multi-antenna system with the multilobe RP, which overlaps the given sector of the direction finding. Taking into account the significant own additive Gaussian noise $n_z(t)$ of the DF channels and the interference effects, the treatment should be implemented optimally, delivering the maximum plausibility functional [14]. These requirements should be implemented by the procedure of a digital synthesis of the multilobe RP using the FFT algorithm:

$$U_{z,l}(j\Omega_{z,l}) = \sum_{p=0}^{Z-1} \text{Re}\left[ U_{z,l}(j\omega_{Sk,l}) \right] \exp(-j\Delta \Omega_{z,l} \cdot z) \cdot W(z),$$ (4)

where $U_{z,l}(j\omega_{Sk,l})$ – a complex spatial spectrum for the k-th component of the time spectrum $U_{z,l}(j\omega_{IF,k})$ of the acceptable additive mixture $U(t)$; $\Delta \Omega_{z,l}$ – the value of the spatial frequency, which determines the direction of the p-th leaf of multilobe RP, $p=0,1,...,(Z-1)$; $d$ – the distance between the elements of the AA; $W(z)$ – the weighting function of the spatial digital beam forming.

In view of (4) and the linearity of the operation of the digital beam forming based on FFT the response $U_{z,l}(j\Omega_{z,l})$ of the digital AA with multilobe RP should be presented as an additive mixture of its response to the action of the useful signal $S_{z,l}(j\Omega_{z,l})$ and the inherent noise $N_{z,l}(j\Omega_{z,l})$ of the receiving channel:

$$U_{z,l}(j\Omega_{z,l}) = S_{z,l}(j\Omega_{z,l}) + N_{z,l}(j\Omega_{z,l}).$$ (5)

The synthesis of the multilobe RP allows parallel receiving, separation and spatial selection of counts (5) of the useful signals $S_{z,l}(j\Omega_{z,l})$.

The formed multilobe RP should be presented as a set $Z$ of highly directional parallel partial complex RP:

$$K_{z,p}(j\Omega_{z,l}) = \sum_{p=Ls}^{Hs} K_{z,p}(j\Omega_{z,l} - \Delta \Omega_{z,l}),$$ (6)

where $K_{z,p}(j\Omega_{z,l})$ – the multilobe complex RP; $K_{p}(j\Omega_{z,l})$ – the p-th partial complex RP.

The analysis of the equation (6) shows that the partial complex RPs $K_{p}(j\Omega_{z,l})$ are formed identical in shape and parameters and overlap the sector of direction finding which is appropriate to the range $[0; 2\pi(Z-1)/dZ]$ of spatial frequencies $\Omega_{p}$. They differ only in the direction of the main lobe, corresponding to a particular spatial frequency $\Omega_{p}$. The parameters of the partial complex RP $K_{p}(j\Omega_{z,l})$, such as its shape, the width of the main lobe, the levels of the main and side lobes are determined by the type of weight function of making spatial RP and the number of the Z direction finding channels of AA [15].

Typically, with a decrease of the level $K_{p}.B$ of the side lobes of the partial complex RP the width $\Delta \Omega_{G}$ of the main lobe significantly increases. This, in turn, leads to a significant overlapping of the main lobes of the partial complex RP. The response of the AA with the multilobe RP to the effect of narrow-band or quasi-harmonic radiations is formed as the corresponding sub array of responses of the related partial complex RP. This sub array in future will be defined as a signal group, Fig. 1.

Fig. 1 shows a frequency RP $K_{p}(\Omega_{z,l})$, formed by the FFT algorithm, and the spatial amplitude spectrum $A_{k,l}(\Omega_{z,l})$ of the k-th spectral component of the l-th signal $S_{l}(t)$. In the spatial spectrum $A_{k,l}(\Omega_{z,l})$ at frequencies $\Omega_{z,l}$ with the numbers $p \in [p_{Ls}; p_{Hs}]$ the signal group is highlighted in bold.

The required noise immunity of direction finding of SRR is provided at first by selecting the type of weighting function $W(z)$ of the spatial digital radiation pattern with the appropriate level $K_{p}.B$ of the side lobes of the complex partial RP.

Thus, as a result of processing an array

$$\left[ S_{l,j}(j\omega_{Sk,l}) \right]_{j=0,...,Z-1}$$

of the Z complex time spectral components of the l-th signal at the AA output with the multilobe RP the corresponding signal group

$$\left\{ U_{l,j}(j\Omega_{z,l}) \right\}_{j=0,...,Z-1}$$

of responses of the related partial complex RP will be formed:

$$\left\{ U_{l,j}(j\Omega_{z,l}) \right\}_{j=0,...,Z-1}$$

where $p_{Ls}, p_{Hs}$ – the numbers of the lower and upper frequencies of the signal group

$$\left\{ U_{l,j}(j\Omega_{z,l}) \right\}_{j=0,...,Z-1}$$
For these conditions we will get the most possible direct assessment $\Omega_1$ of the spatial frequency of the signal component $S_z.l(jz)$ by processing a realization of the corresponding signal group $\left\{ U_{Ak.l}(j\Omega) \right\}_{p[n_p, p_n]}$ (7) at the AA aperture with the multilobe RP. To solve this problem, it is advisable to perform a reconstruction within the AA aperture of spatial analytical signal $U_{Ak.l}(jz)$ [11] in such a way:

$$U_{Ak.l}(jz) = \frac{1}{\pi} \sum_{p=1}^{n_p} U_{Ak.l}(j\Omega) \exp(j\Omega p \cdot z) = U_{Ak.l}(jz) + jU_{Ak.l}(jz) = U_{Ak.l}(jz) \exp(j\Omega_{Ak.l}(jz)).$$

where $U_{Ak.l}(jz)$, $U_{Ak.l}(jz)$ – the real and imaginary components of spatial analytical signal $U_{Ak.l}(jz)$; $U_{Ak.l}(jz)$, $\Omega_{Ak.l}(jz)$ – the module and the argument of the spatial analytical signal.

In view of (7) the spatial complex analytical signal $U_{Ak.l}(jz)$ should be presented as an additive mixture of signal $S_z.l(jz)$ and noise $N_{Ak.l}(jz)$ components:

$$U_{Ak.l}(jz) = S_{Ak.l}(jz) + N_{Ak.l}(jz).$$

The model of distribution of counts of the signal component $S_z.l(jz)$ within the AA aperture with the multilobe RP considering (7) and (8) should be presented as follows:

$$S_{Ak.l}(jz) = W(z) \cdot A_{Ak.l} \cdot \exp(i(\Omega_{Ak.l} z + \Psi_{Ak.l})),$$

(10)

where $A_{Ak.l} = \max[S_{Ak.l}(jz)]; \ A_{Ak.l}, \Omega_{Ak.l}, \Psi_{Ak.l}$ – the amplitude, the angular frequency and the initial phase of the signal component $S_z.l(jz)$ of the spatial analytical signal.

The analysis of the equation (10) shows that the counting of the signal component $S_z.l(jz)$ within the AA aperture with the multilobe RP are distributed harmonically with an equal weighting function $W(z)$ of digital radiation pattern (RP). This is due to the use of the FFT algorithm for the synthesis of the multilobe RP and processing of an array of spectral counts $S_z.l(joSoS.k)$ within the AA aperture.

The informative, a priori unknown parameter of the signal component $S_z.l(jz)$, which depends on the direction to the corresponding SSR, is its signal frequency $\Omega_1$, which, in turn, determines the shift $\Delta \Omega$ of the signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}$$

within the AA aperture with the multilobe RP. At the same time, the random parameters are the amplitude $A_{Ak.l}$, the circular frequency $\Omega_{Ak.l}$ and the initial phase $\Psi_{Ak.l}$ of the signal component $S_z.l(jz)$ of the spatial analytical signal. The additive noise component $N_{Ak.l}(jz)$ of the spatial analytical signal $U_{Ak.l}(jz)$, in turn, is a Gaussian noise with a uniform distribution of spatial spectral power density $N^z_{Ak.l}(j\Omega)$ within a limited band of spatial frequencies $[\Omega_{z_-} \cdot \Omega_{z_+}]$, that is within the signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}.$$

For the conditions (8)–(10) let’s define the most plausible direct assessment $\Omega_1$ of the shift of the signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}$$

by single-cycle processing of one implementation of the corresponding reconstructed spatial analytical signal $U_{Ak.l}(jz)$. The optimal criterion of the assessment $\Omega_1$ of the shift of a signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}$$

will be a maximum of a posteriori probability. As a result, we’ll form the equation of the probability as follows:

$$F(z, \Omega_1) = \sum_{k=0}^{Z_1} U_{Ak}(jz) U_{Ak}(j\Omega_1, z) = \max \quad \Omega_1 = \Omega_1,$$

(11)

where $F(z, \Omega_1) – the likelihood functional; U_{Ak}(j\Omega_1, z) – the base waited signal; (,) – the operation of formation of the complex value.

Directly the equation (11) in the general case has no direct solutions [12, 14]. Therefore, to be able to determine the direct assessment $\Omega_1$ of the shift of the signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}$$

with (10) we take as a base signal $U_{Ak}(j\Omega_1, z)$ the received realization of the reconstructed spatial analytical signal $U_{Ak}(jz)$, but removed by the amount of the shift $\Delta z$ within the AA aperture:

$$U_{Ak}(jz) = U_{Ak}(j(z - \Delta z)).$$

(12)

Taking into account (12), the equation (11) takes the form:

$$F(z, \Omega_1) = \sum_{k=0}^{Z_1} U_{Ak}(jz) U_{Ak}(j\Omega_1, z) = \max.$$ (13)

The signal likelihood function $q_{Sk.l}$ of the equation (13) with (10) is equal to [12, 14]:

$$q_{Sk.l} = \sum_{z=0}^{Z_1} \left[ W(z) \cdot A_{Ak.l} \cdot \exp(i(\Omega_{Ak.l} z + \Psi_{Ak.l})) \right] \times \left[ W(z - \Delta z) \cdot A_{Ak.l} \cdot \exp(-i(\Omega_{Ak.l} (z - \Delta z) + \Psi_{Ak.l})) \right] = A_{Ak.l}^2 \cdot \exp(i\Delta \Psi_{Ak.l}) \sum_{z=0}^{Z_1} W(z) W(z - \Delta z).$$

(14)

where $\Delta \Psi_{Ak.l} = \Omega_{Ak.l} \Delta z$ – the argument of the signal function $q_{Sk.l}$. The analysis of the equation (14) shows that an estimate $\Omega_1$ of the shift in the signal group

$$\left\{ U_{Ak}(j\Omega) \right\}_{p[n_p, p_n]}$$

and, therefore, the spatial circular frequency $\Omega_{Ak.l}$ of the signal component of the spatial analytical signal may be used as the argument $\Delta \Psi_{Ak.l}$ of the signal function $q_{Sk.l}$. The argument $\Delta \Psi_{Ak.l}$ of the signal function $q_{Sk.l}$ is proportional to the frequency $\Omega_{Ak.l}$ of the signal and is equal to the difference between the arguments, which are mutually shifted within the AA aperture at the interval $\Delta z$. As a result, taking into account (14) we obtain a direct solution of the likelihood equation (13):
\[ \Delta \Psi_{akl}(\Delta z, z) = \max \left\{ \arg \left[ \sum_{z=1}^{2} U_{akl}(jz) U_{akl}^*(j(z-\Delta z)) \right] \right\} = \arctg \left( \frac{\sum_{z=1}^{2} U_{akl}(z) U_{akl}^*(z-\Delta z) \cdot \sin(\Delta \xi_{akl}(z))}{\sum_{z=1}^{2} U_{akl}(z) U_{akl}^*(z-\Delta z) \cdot \cos(\Delta \xi_{akl}(z))} \right) \] 

where \( \Delta \xi_{akl}(z) = \Delta \xi_{akl}(z) - \Delta \xi_{akl}(z-\Delta z) \) – the difference of the arguments relative to the shifted implementations \( U_{akl}(jz) \) and \( U_{akl}^*(j(z-\Delta z)) \) at the z point of the AA aperture with the multilobe RP.

The analysis of the equations (12)–(15) shows that the evaluation \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group is received on the basis of the spatial correlation analysis of the reconstructed spatial analytical signal \( U_{akl}(jz) \) within the AA aperture with the multilobe RP. The receiving estimate \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group is the most likely and straight, providing its direct determination by the single-channel single-cycle processing.

Also, the equations (12)–(15) can be represented as an equivalent model of the set \( z \) of the parallel operating dual-channel correlation receivers, spaced apart in pairs on the base \( \Delta z \). At the same time the results of the receiving of the equivalent two-channel correlated receivers are coherently added to ensure the efficient processing of all available information on the radiation \( U_{z,l}(j\Omega) \) received across the entire AA aperture with the multilobe RP.

Also, the analysis of the equation (15) shows that this estimation \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group has a continuous distribution of values within \([-\pi; \pi]\) radians. In this case the distribution of values of counts of the spatial analytic signal \( U_{akl}(jz) \) and its corresponding signal group

\[ \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \]

is discrete. It provides the obtaining of the estimate \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift signal group with a minimum amount of the receiving channels of AA and the relevant hardware expenses, as well as the absence of errors of discreteness.

The disadvantage of the algorithm (15) of the determination of the estimation \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group are sufficiently large computational costs \( \text{MOz} \), measured in the number of complex multiplications that are defined as follows:

\[ M_{oz} = (\pi + Z \cdot \log_2 Z) \cdot T_{M}, \]

where \( T_{M} \) – the duration of the implementation of the complex multiplication operation.

Let’s perform the analysis of ways of minimization of the computational costs of estimation \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group with its unchanged effectiveness. To do this, we define the possibility of estimation \( \Delta \Psi_{akl}(\Delta z, z) \) of the shift of the signal group by processing directly the signal group

\[ \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \]

that is, as a result of receiving of the AA with the multilobe RP. We should note that the counts of the signal group \( \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \)

are identified in the spatial-frequency domain and are associated with the counts of the corresponding spatial analytical signal \( U_{akl}(jz) \) through a discrete Fourier transformation [15]:

\[ U_{k,l}(j\Omega_p) = \sum_{z=1}^{2} U_{akl}(jz) \cdot \exp(-j\Omega_p \cdot z). \] (17)

In view of (17) we obtain the relationship of the implementation of the spatial analytic signal \( U_{akl}(j(z-\Delta z)) \), shifted to \( \Delta z \) within the AA aperture and its corresponding signal group \( \{ U_{k,l}(j\Omega_p, \Delta z) \}_{p[n_p, n_{p-1}]} \):

\[ U_{k,l}(j\Omega_p, \Delta z) = \sum_{z=1}^{2} U_{k,l}(j(z-\Delta z)) \cdot \exp(-j\Omega_p \cdot \Delta z) = \]

\[ = U_{k,l}(j\Omega_p) \cdot \exp(-j\Omega_p \cdot \Delta z). \] (18)

The comparative analysis of the equations (17) and (18) shows that the modules of the frequency components of the signal groups

\[ \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \]

and \( \{ U_{k,l}(j\Omega_p, \Delta z) \}_{p[n_p, n_{p-1}]} \)

are the same, and the arguments differ by the component \( -\Omega_p \cdot \Delta z \), which linearly depends on the value of the circular spatial frequency \( \Omega_p \).

Using the Parseval equality (the generalized Rayleigh formula) and taking into account (13), (17) and (18) let’s form the likelihood equation for estimating \( \Delta \Psi_{akl}(\Delta z, \Omega_p) \) the shift of the signal group in the frequency-spatial domain:

\[ F(z, \Omega_p) = \left| \sum_{p[n_p, n_{p-1}]} U_{akl}(j\Omega_p) \cdot U_{akl}^*(j(z-\Delta z)) \right|^2 = \]

\[ = \max \left[ \sum_{p[n_p, n_{p-1}]} U_{akl}(j\Omega_p) \cdot U_{akl}^*(j(z-\Delta z)) \cdot \exp(-j\Omega_p \cdot \Delta z) \right] = \]

\[ = \sum_{p[n_p, n_{p-1}]} U_{akl}(j\Omega_p) \cdot \exp(-j\Omega_p \cdot \Delta z) \] (19).

The direct solution of the likelihood equation (19) is the estimation \( \Delta \Psi_{akl}(\Delta z, \Omega_p) \) of the shift of the signal group

\[ \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \]

according to the equation:

\[ \Delta \Psi_{akl}(\Delta z, \Omega_p) = \arctg \left( \frac{\sum_{p[n_p, n_{p-1}]} U_{k,l}(j\Omega_p) \cdot \sin(\Omega_p \cdot \Delta z)}{\sum_{p[n_p, n_{p-1}]} U_{k,l}(j\Omega_p) \cdot \cos(\Omega_p \cdot \Delta z)} \right). \] (20)

Thus, the equation (20) gives the opportunity to obtain a direct estimation \( \Delta \Psi_{akl}(\Delta z, \Omega_p) \) of the shift of the signal group

\[ \{ U_{k,l}(j\Omega_p) \}_{p[n_p, n_{p-1}]} \]

in a frequency-spatial domain without information loss in contrast to the estimation \( \Delta \Psi_{akl}(\Delta z) \) of the equation (15), defined in the spatial domain.
Let’s define, taking into account (20), the computational costs for obtaining the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ of the shift of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$:

$$M_{sa_k} = (p_{th} - p_n) \cdot T_M. \tag{21}$$

The comparative analysis of the equations (16) and (21) shows that the computational costs $M_{sa_k}$ of obtaining the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ with the same information content of these estimations:

$$K_o = M_{sa_k} / M_{sa} = (Z + Z \cdot \log_2 Z) / (p_{th} - p_n). \tag{22}$$

The gain of $K_0$ of the computational costs in obtaining the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ of the shift of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$ are significantly, $K_0$ times, less than the similar computational costs $M_{sa_k}$ for obtaining the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, z)$ with the same information content of these estimations:

$$K_o = M_{sa_k} / M_{sa} = (Z + Z \cdot \log_2 Z) / (p_{th} - p_n). \tag{23}$$

Thus, in general, the algorithm (20) of determination of the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ of the shift of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$ with the use of the frequency-spatial domain is more effective in terms of performance than the algorithm (15).

Further, in view of (20) we define the estimation $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ of the shift of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$ for all K time spectral components $S_k(l)(j\Omega_p)$ within each $l$-th selected band $\Omega_{ao,l}$ of the simultaneous analysis and form the corresponding complex mutual spectrum $U_{vk,l}(j\Omega_p)$:

$$U_{vk,l}(j\Omega_p) = U_{hk,l}(j\Omega_p) \cdot \exp(j \Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)), \tag{24}$$

where $k_{th}$, $k_{th}$ – the numbers the lower and upper frequency of the spectrum $U_{vk,l}(j\Omega_p)$ of the received mixture within the $l$-th band of the simultaneous analysis accordingly.

The final estimations $\hat{\theta}_l$ of the directions on the $l$-th SRR are determined taking into account (24) as follows:

$$\hat{\theta}_l = \arccos[\Omega_l \cdot c / \omega_{ao,l}], \tag{25}$$

where $c$ – the propagation speed of electromagnetic radiation in free space.

The analysis of the relations (24) and (25) shows that the estimations $\hat{\theta}_l$ of directions on the $l$-th SRR are obtained from the two-dimensional spatial-spectral direct correlative analysis which is performed in two stages. At the first stage during the assessment of the shift $\Delta \Psi_{Ak,l}(\Delta \Omega, \Omega_p)$ of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$ the spatial correlative analysis within the AA aperture with the multilobe RP is carried out. At the second stage of the processing in determining the estimation $\hat{\theta}_l$ of the shift of the signal group $\{U_{k,l}(j\Omega_p)\}_{p|\omega_p,n_l}$ the spectral-time correlative analysis within the band $\Delta \omega_{ao,l}$ of the simultaneous signal analysis is used. Thus, the studied problem is solved.

5. The results of simulation of the direction finders work

We conducted the software simulation of the direction finder according to the developed direct algorithm (24), (25) of the correlative-interferometric direction finding via the developed software model in MathCad environment.

Initial conditions of simulation:

- the signal type – continuous with the linear frequency modulation: $S(t)=A\sin(2\pi f_S t + b t^2)$;
- the spectrum width of the signal $\Delta fS \geq 0.6$ MHz;
- the frequency band of the analysis of the DF radio channel $\Delta fK \geq 10$ MHz;
- the frequency of the signal carrier $f_s = 2$ GHz;
- the frequency of sampling $\Delta f_\text{d} = 2\Delta fK = 20$ MHz;
- the analyzed numbers of time counting of the signal $N_s = 2048$;
- the duration of the review process $T_\text{a} = 0.1$ ms;
- the linear AA with the numbers of the direction finding reception channels $Z = 64$;
- the spatial shift $\Delta \theta = 1$.

As a result of modeling the dependence of the error $\Delta \theta$ of estimation of direction from the direction to the SRR without considering the actions of noises (Fig. 2, series 1) and the dependence of the mean square deviation (MSD) of estimation of direction from the direction to the SRR for a given signal/noise ratio 0 dB (Fig. 2, series 2) were obtained.
The analysis of Fig. 3 shows that the accuracy of the direction finding increases abruptly during the spatial selection with the possible signal resolution. As it can be seen from Fig. 3, the resolution in the direction substantially depends on the ratio signal 1/signal 2 and worsens from 8° to 11° with a decrease of the signal 1/signal 2 relationship with a value of 0 dB to −20 dB.

The resolution can be improved by increasing the number of Z AA elements or by choosing a window of the spectral analysis with a smaller width of the main lobe of the partial RP [15].

6. Discussion of the results of the research of development of a new method of direction finding and of modeling of the work of the direction finder

The results of the research have confirmed the possibility of the parallel spatial selection and the direction finding of radio emission in a complex electromagnetic environment. The advantage of the proposed method of direction finding is the high performance that is achieved by using the multilobe and direct correlation estimation of directions of the SRR. Thus, these results are due to the development of a new direct method of correlation evaluation of the parameter and using it with a linear AA.

The results should be used in the implementation of equipment of radio monitoring and radio navigation systems, which operate in a complex dynamic EME.

The restrictions of the use of the results are the large dimensions of AA at frequencies below 500 MHz, as well as a linear proportional relationship of accuracy and resolution of the direction finder on the signal frequency. The dependence on frequency leads to the need for multiple sets of AA for the direction finder in a wide range of operating frequencies.

These studies are a continuation of the work on the development of digital correlative-interferometric direction finders. In the future, it is necessary to conduct the detailed studies of the accuracy and the noise immunity of this method, its parametric optimization and performance analysis.

In general, the results of simulation of the direction finder confirm the correctness of the theoretical developments.

7. Conclusions

1. A new direct digital method and an algorithm of the correlation-interferometric direction finding with a two-dimensional correlation processing of the reconstructed complex signal space was worked out. It enables the efficient spatial selection of the radio emission in real time due to the formation of the multilobe radiation pattern of the antenna array.

2. The developed method of direction finding has a high precision of the estimation of directions on the radiation source through the use of two-dimensional correlation signal processing. The versions of the algorithm of direction finding are carried out to minimize the computational cost that provides a high performance of the direction finder in general. The simulation results showed that the developed method provides an estimate of the direction finding directions on the radio emission sources that can fully overlap in frequency, with standard deviation of the direction estimation is less than 0.04 degrees.

Analysis of Fig. 2 shows that the examined method provides a higher accuracy of direction finding in comparison with the known method [2] in a wide sector of directions, but it has lower computation costs and provides a significant speed increase of direction finding.

The working sectors of the direction finding with the error $\Delta \theta < 0.005^\circ$ is the sector $(2;84)^\circ$, $(96;178)^\circ$, $(182;264)^\circ$ and $(276;358)^\circ$ relative to the base line of the AA, and according to the signal/noise ratio 0 dB with MSD $\sigma < 0.04^\circ$ – $(25;80)^\circ$, $(100;155)^\circ$, $(205;260)^\circ$ and $(280;335)^\circ$. Thus, for a direction finding in the sector $(0;360)^\circ$ with MSD $\sigma < 0.04^\circ$ it is sufficient to use two AA, arranged at an angle $50^\circ$ in space.

The family of dependencies of errors $\Delta \theta$ of estimation of the direction finder on the shift to the direction of the sources of the two signals, completely overlapped in frequency at different relations signal 1/signal 2 (Fig. 3), is obtained. The set direction to the source of the first signal is $\theta = 60^\circ$, and the direction of the source of the second signal (the interference signal relative to the first one) is changed within $\theta = [40;59]^\circ$.

We used a Blackman window with the side-lobe level of $-58$ dB, which provides a high noise immunity of the direction finder. This level of selectivity at 38 dB is higher than that of the known correlative-interferometric direction finders with the ring AA [1, 9].

Fig. 3 shows: series 1 – for the ratio signal 1/signal 2 0 dB; series 2 – for the ratio signal 1/signal 2 −20 dB.
at a given signal/noise ratio of 0 dB. We showed the possibility of ensuring the resolution of 8 degrees with respect to the signal/noise ratio of 0 dB, which is an important advantage in a complex electromagnetic environment.

The resulting resolution of the direction finder can be improved by increasing the number of Z AA elements or choosing a window of the spectral analysis with a smaller width of the main lobe of the partial RP.

References