1. Introduction

It is known that in the process of interaction between the rolling stock and tracks, certain part of energy for the traction of rolling stock is used to overcome the forces of dry and viscous friction in the contact «wheel-rail». Energy consumption, in this case, is mostly for the accumulation of residual deformations in the track, wheel and rail, which leads to the formation of point defects and generation of heat. In addition, energy is spent on the accumulation of residual deformations in the rail-sleeper grid, on the radiation in the form of acoustic waves, etc. [1].

If one does not consider these energy losses when calculating the forces of interaction between track and rolling stock, one may receive results that will be by 15–25% different from the actual, those confirmed experimentally. This is particularly important for the sections of track in the curves of small radii (350 m and less), which differ from other sections by significant level of horizontal transverse forces. It should be noted, however, that theoretical calculations of spatial interaction between rolling stock and track were used mainly with the help of the flat calculation schemes.
using subsequently the principle of superposition. That is, they used the hypothesis of linearity of elastic characteristics of the track and its independence from the dynamics of rolling stock in the vertical and horizontal plane. The track in these calculations was considered in the form of beams based on continuous elastic foundation whose modulus of elasticity is a constant magnitude.

But it was demonstrated later in [6] that under certain operating conditions, especially in the curves of small radii, these assumptions differ significantly from the real processes of dynamics of rolling stock and track, yielding large errors. Therefore, such a theoretical base cannot help to achieve practical results, first of all, to enhance safety of transport.

A lot of attention is paid to the question of interaction between rolling stock and the upper structure of the track using computational technology in modern scientific and technical literature. Thus, [7] conducted theoretical and experimental research into elastic qualities of the system wheel-rail under conditions of action of negative factors. As an example, authors conducted computer simulation of the interaction of a freight wagon with a railway line, one of the rails of which has defect in the quality of the corrugated surface. This topic is also considered in [8] – examining dynamic characteristics of the system wheel-rail at the curved sections of railway. A mechanism and a method of computer calculations of these characteristics are presented, which depend on the influence of parameters of the rolling stock, as well as track parameters.

Paper [9] presented a mathematical model for the evaluation of actual operating conditions of railway rolling stock, taking into account a case when a wheel loses contact with the rail. Mathematical modeling demonstrated that amplitude characteristics of the fluctuations depend on the functions of roughness and speed of the wheel motion. When calculating dynamic processes, the contact between a wheel and a rail should be considered non-stable. With increasing speed, the effect of this instability is growing. Therefore, special attention is paid in the research to the issue of improving security for the lines of high-speed motion [10].

If we approach these issues from general positions – from the point of research into dissipative processes, then all of the above considered sources [6–10], each in its aspect, investigate a general phenomenon – the dissipation of energy of the object «wheel-rail». As for the given article, it also explores dissipative processes, which take place in a rail track, at its interaction with the rolling stock. A special feature of this paper, compared with others [4–6], is that we solve a problem of research not into flat but spatial interaction between rolling stock and track using those methods that take into account a case when a wheel loses contact with the rail. Mathematical modeling demonstrated that amplitude characteristics of the fluctuations depend on the functions of roughness and speed of the wheel motion. When calculating dynamic processes, the contact between a wheel and a rail should be considered non-stable. With increasing speed, the effect of this instability is growing. Therefore, special attention is paid in the research to the issue of improving security for the lines of high-speed motion [10].

This task is particularly important for those railway lines that contain curves of small radii, in particular, subways. It should be noted that up to now, in the subways of Ukraine, as well as everywhere in the former Soviet Union territory, intermediate rail fastenings of the «Metro» type and wooden sleepers, in-situ cast by concrete, have been widely used for a long time. Therefore, as noted above, in the Introduction, such sections first of all require special attention in order to ensure safety.

3. The aim and tasks of the study

The aim of the work is to identify indicators of dissipative processes in a railway track at its interaction with the rolling stock, which makes it possible to evaluate current state of the upper structure of the track on the wooden sleepers and to detect the need for repair work.

To achieve the set goal, the following tasks should be solved:

– to develop theoretical positions for determining inelastic resistances of rail supports during vertical and horizontal transverse deformations of rails;
– to identify evaluation characteristics for dissipative processes in the interaction of the system «wheel-rail»;
– to receive numeric values of these characteristics for the track on the wooden sleepers.

4. Materials and methods of research

In order to solve these tasks, we have developed mathematical models of the spatial dynamic system «vehicle – track», in which track is considered as a spatial structure in the form of beams – rails, which are based on elastic-dissipative supports with non-linear characteristics [6].

To determine dissipative characteristics of rail supports, in particular for the determination of equivalent coefficient of dissipation, we used theoretical positions, which consider work of a track under the action of dynamic force $R_{dyn}$ (vertical or horizontal) on it. In this case, the track has elastic properties (rigidity C) and viscous resistance with coefficient of this resistance $\beta$, which is proportional to the velocity of deformation $\dot{\delta}$ (that is, the first derivative from $\delta$).

It is known [6] that dynamic force $R_{dyn}$ is associated with coefficient $\beta$ and deformation by the following ratio:

$$R_{dyn} = C \cdot \dot{\delta} + \beta \cdot \dot{\delta},$$

where $\dot{\delta}$ is the velocity of deformation.

Thus, expression (1) has two components: elastic force $R_{el} = C \cdot \dot{\delta}$ and non-elastic (dissipative) force $R_{dyn} = \beta \cdot \dot{\delta}$.

To consider a more general case, we will assume that the track at first (prior to the impact of dynamic force) is in static equilibrium (point «0»), then all of the above considered sources [6–10], each in its aspect, investigate a general phenomenon – the dissipation of energy of the object «wheel-rail». As for the given article, it also explores dissipative processes, which take place in a rail track, at its interaction with the rolling stock. A special feature of this paper, compared with others [4–6], is that we solve a problem of research not into flat but spatial interaction between rolling stock and track using those methods that take into account a case when a wheel loses contact with the rail. Mathematical modeling demonstrated that amplitude characteristics of the fluctuations depend on the functions of roughness and speed of the wheel motion. When calculating dynamic processes, the contact between a wheel and a rail should be considered non-stable. With increasing speed, the effect of this instability is growing. Therefore, special attention is paid in the research to the issue of improving security for the lines of high-speed motion [10].

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tude $\delta_{\text{dyn}}$ and will continue to be deformed by the periodic law of attenuation, that is, in such a way so that after a certain period of time, variables $\delta_{\text{dyn}}$ and $\delta_{\text{dy}}$ take a zero value as a starting point.

That is, one may write down:

$$\delta_{\text{dyn}} = \delta_0 \sin \omega t, \quad (2)$$

and derive:

$$\dot{\delta}_{\text{dyn}} = \delta_0 \omega \cos \omega t, \quad (3)$$

where $\omega$ is the circular load frequency ($c^{-1}$).

If we divide (2) by $\delta_{\text{dy}}$, and then square both parts of the expressions and sum up received expression, then we obtain:

$$\frac{\delta_{\text{dy}}^2}{\delta_0^2} + \frac{\dot{\delta}_{\text{dy}}^2}{\delta_0^2} = 1. \quad (4)$$

We receive from expression (4):

$$\dot{\delta}_{\text{dy}} = \pm \delta_0 \sqrt{1 - \frac{\delta_{\text{dy}}^2}{\delta_0^2}}, \quad (5)$$

With regard to expression (5), dependence of force $R_{\text{dy}}$ on $\delta_{\text{dy}}$ and $\delta_{\text{dy}}$ (1) may be written down in the following way:

$$R_{\text{dy}} = C \delta_{\text{dy}} + \beta_p \delta_0 \omega \sqrt{1 - \frac{\delta_{\text{dy}}^2}{\delta_0^2}} \cdot \text{sign} \delta_{\text{dy}} , \quad (6)$$

where $\text{sign} \delta_{\text{dy}}$ is the sign of velocity, that is, if $(\delta_{\text{dy}} > 0)$ then $\text{sign} \delta_{\text{dy}} = +1$, and if $(\delta_{\text{dy}} < 0)$ then $\text{sign} \delta_{\text{dy}} = -1$.

Graph of dependence of force $R_{\text{dy}}$ on $\delta_{\text{dy}}$ and $\delta_{\text{dy}}$ was represented in Fig. 1. It follows from the graph and expression (6) that this dependence is curvilinear. At $\delta_{\text{dy}} > 0$, this dependence is represented by the upper curve, and at $\delta_{\text{dy}} < 0$ - by the lower one. The resulting shape is a closed loop of hysteresis.

The work performed by external force $R_{\text{dy}}$ in one period, as well as the external force itself, includes two components:

$$A_1 = \int_0^{T} R_{\text{dy}} (\delta_{\text{dy}} + \dot{\delta}_{\text{dy}}) d\delta_{\text{dy}} =$$

$$= \int_0^{T} C \delta_{\text{dy}} + \beta_p \dot{\delta}_{\text{dy}} d\delta_{\text{dy}} + \int_0^{T} \beta_p \delta_{\text{dy}} d\delta_{\text{dy}}. \quad (7)$$

With regard to $d\delta_{\text{dy}} = \delta_{\text{dy}} dt$, we receive

$$A_1 = \frac{1}{2} C \delta_0^2 + \frac{\beta_p}{\omega} \delta_0 \omega^2 t +$$

$$\int_0^{T} \beta_p \dot{\delta}_{\text{dy}} dt. \quad (8)$$

The first component is the work of external force to overcome elastic resistance. It is obviously that over the entire period (loading – unloading) it will equal 0 because the force of elastic resistance will resume its potential after removal of external loading force.

The second component is the work of external forces to overcome non-elastic (dissipative) resistance. It is equal to:

$$\int_0^{T} \beta_p \delta_{\text{dy}} dt = \frac{1}{2} \beta_p \delta_0^2 \omega^2 t = \pi \omega \beta_p \delta_0^2. \quad (9)$$

From (8), taking into account the fact that the first component is 0, it follows that:

$$\Lambda_1 = \pi \omega \beta_p \delta_0^2. \quad (10)$$

From expression (10), coefficient of viscous resistance of the track to external loads $R_{\text{dy}}$ will be equal to:

$$\beta_{\text{dyn}} = \frac{\Lambda_1}{\pi \cdot \omega \cdot \delta_0^2}. \quad (11)$$

### 5. Results of research into equivalent coefficient of dissipation in vertical and horizontal transversal plane

In order to determine vertical and horizontal transversal equivalent coefficients of dissipation at wooden sleepers, we used data arrays of experimental studies that were performed at 12 examined sections with different operational conditions of railways in Ukraine over 2009–2011 [11].

Given that at each section they tested from 12 to 20 sleepers, the total number of received values of equivalent coefficients of dissipation for wooden sleepers is several thousand. These data have been processed by conventional statistical methods (Table 1). Results of the processing in the form of graphs are given in Fig. 2, 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sleeper type</th>
<th>Axial load (kN)</th>
<th>Dependence $\beta_{\text{axial}}$ (kN·sec/m)</th>
<th>Mean approximation error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical coefficient of dissipation of rail support</td>
<td>Reinforced concrete Sh–1</td>
<td>&lt;265</td>
<td>$\beta_{\text{axial}} = 26.05 + 0.313 \cdot T^{0.462}$</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>265–294</td>
<td>$\beta_{\text{axial}} = 31.3 + 0.329 \cdot T^{0.36}$</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>294–450</td>
<td>$\beta_{\text{axial}} = 34.5 + 0.336 \cdot T^{0.72}$</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>Wooden, type I</td>
<td>&lt;265</td>
<td>$\beta_{\text{axial}} = 16 + 0.205 \cdot T^{0.43}$</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>265–294</td>
<td>$\beta_{\text{axial}} = 19.2 + 0.249 \cdot T^{0.4}$</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>294–450</td>
<td>$\beta_{\text{axial}} = 20.8 + 0.260 \cdot T^{0.77}$</td>
<td>8.3</td>
</tr>
<tr>
<td>Horizontal equivalent coefficient of dissipation of rail support</td>
<td>Reinforced concrete Sh–1</td>
<td>&lt;265</td>
<td>$\beta_{\text{axial}} = 18 + 0.292 \cdot T^{0.4}$</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>265–294</td>
<td>$\beta_{\text{axial}} = 23.4 + 0.288 \cdot T^{0.4}$</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>294–450</td>
<td>$\beta_{\text{axial}} = 25.2 + 0.188 \cdot T^{0.427}$</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>Wooden, type I</td>
<td>&lt;265</td>
<td>$\beta_{\text{axial}} = 11.0 + 0.036 \cdot T^{0.4}$</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>265–294</td>
<td>$\beta_{\text{axial}} = 13.9 + 0.059 \cdot T^{0.4}$</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>294–450</td>
<td>$\beta_{\text{axial}} = 15.5 + 0.078 \cdot T^{0.42}$</td>
<td>7.4</td>
</tr>
</tbody>
</table>
Results of the studies indicate that with an increase in tonnage from 0 to 300 million tons, values of coefficients of dissipation in the vertical plane increases by 1.5–1.9 times (Fig. 2), and in the horizontal plane – a little less, by about 1.3 times (Fig. 3).

6. Discussion of results of research into dissipative characteristics of track on wooden sleepers

Materials presented in this work are a continuation and development of the research, the results of which are represented in [12]. However, the results [12] were obtained for the conditions of the track of not a general usage that differ from the main lines in both design characteristics of the railway tracks and conditions of operation of such tracks, as well as high axial loads, low motion speeds and existence of the curves with a radius less than 150 m.

It should, first of all, be noted that the foundation of research is formed by a fundamentally new calculation scheme of the interaction between track and rolling stock, which is a spatial structure in the form of beams, which are based on the set of elastic–dissipative supports. This scheme allows us to significantly improve the accuracy of calculations without using the principle of superposition when determining the spatial forces of interactions between track and rolling stock.

The results obtained are meant to be applied when calculating such forces of interaction. They make it possible to predict changes in these forces in the course of track operation, to predict occurrence of critical situations, at which a violation of conditions of safety of trains is possible.

Data about vertical dissipative characteristics were obtained for fundamentally different calculation technique; we determined their changes during operation; the assessment of equivalent coefficients of dissipation under winter conditions is given. In future it is planned to continue the studies for the purpose of establishing the impact of motion speeds on the parameters of dissipation.

In this article we present results that confirm the results of study [12], as a result of which we concluded that the resulting diagrams significantly differ from the diagrams of systems that are subject to the law of dry friction (Fig. 4).

In addition, in the paper we assumed that the power of internal friction in the track depends not only on the magnitude, proportional to dry friction, but on the first and second derivatives of the loading velocity, changes in the acceleration of track elements, and perhaps on other dynamic parameters [12].

In [6] they drew conclusions on the fact that the dissipative forces are conveniently considered using equivalent coefficient of viscous friction, reduced to the point of contact between a wheels and a rail (11). In this case, numerical methods of solving a system of differential equations are used that describe the interaction of track and rolling stock. But this is true under condition that the calculation scheme of the track is used in the form of beams that rest on elastic supports.

7. Conclusions

1. Based on analysis of [2–5, 11] and the research in this work, we proposed to detect forces of non-elastic resistances of rail supports by using equivalent coefficient of dissipation, which takes into account the work of all internal friction forces.
2. A formula for mathematical determination of equivalent coefficient of dissipation of rail supports in a track is substantiated. This coefficient is proportional to the work of external force to overcome non-elastic (dissipative) resistance and inversely proportional to angle frequency of oscillations of elastic deformation and the square of magnitude of elastic deformation.

3. We established dependence of equivalent coefficients of dissipation of rail supports in the vertical and horizontal plane on the magnitude of tonnage that passed the section of the track. Results of the experiments indicate that with an increase in tonnage from 0 to 300 million tons, values of coefficient of dissipation in the vertical plane increase by 1.5–1.9 times, and in the horizontal plane – by about 1.3 times.

4. Based on the previous conclusions, there appears an important possibility for the practical application of equivalent coefficient of dissipation to define the current state of track and the need to conduct repair work.

References