1. Introduction

The optimum control problems solution is connected with the use of optimization criterion as a guideline for the choice of such operational process parameters which as much as possible correspond to the owner’s idea (as a supersystem) about the best-case scenario.

It is generally accepted that the pointer that provides the best congruence between the supersystem’s purpose and executive system procession activity parameters is the efficiency.

The result of efficiency definition is used for any processes assessment of all types of operating activities. In this regard, a large number of indicators that are used or proposed to be used for the operational processes efficiency measurement have been developed and continue to be developed.

The relevance of resource efficiency indicator development or search is explained by the fact that its use, for example, as an optimization criterion, allows increasing sharply the economic structure development rates and, therefore, its competitiveness as well.

Thus, an indicator’s search or development problem that really solves the task of operational processes efficiency assessment with the maximum accuracy arises.

The task is complicated because the test for adequacy of an indicator, “declared” as an efficiency measure can’t be carried out by the direct methods. Axiomatically, the efficiency index by itself is an evaluation standard in operations research. Therefore, to carry out the efficiency measure use possibility verification of the developed indicator, the assessment indirect methods development is necessary.
One of such methods has been developed for simple global operations efficiency assessment indicators verification [1]. The method’s idea consists in reference operations classes set creation. Within the class, using the developed absolute prognostic indicator, the rating efficiency assessment of each operation is defined among two alternatives.

Testing of the developed indicator on classes of reference operations allows getting criteria with the use of which the consistent results are received.

At the same time, testing positive results don’t guarantee that the indicator, which has passed these tests, really is efficiency criterion. In other words, tests passing success with the use of reference operations classes is a necessary, but not sufficient condition for making a final decision. The reference operations classes’ number should be expanded.

However, the method, developed in the work [1], can mainly be used for testing of the indicators, developed for an assessment of simple global operations models.

Besides, already developed indicators quantity, designed for the efficiency assessment function performance, is so high that the necessity in such indicators formal signs definition for initially inappropriate alternatives preliminary exception appears.

At the same time, there are operational processes large number that have a considerable duration for assessment of which it is necessary to consider the distributed in time binding and release character, respectively, of input and output operation production. In such cases, simplification of the operation model will cause an essential error in efficiency assessment.

Thus, the task of testing the efficiency assessment indicators should be solved step by step.

At the first step, it is necessary to define formal signs to which the estimated indicator has to correspond as much as possible for the preliminary selection and subsequent verification process maximum formalization.

The second step is to estimate the possibility of reference operations classes’ consistent evaluation.

In the third step, the alternative indicators should pass the stage of mathematical modeling and experimental studies.

Therefore, the development of indicators testing method that are planned to be used as an efficiency measure for operations with the distributed parameters is an important scientific task.

2. Literature review and problem statement

The issue of operational processes efficiency assessment indicator verification is difficult for two reasons. The first, in fact, the technical reason, is connected with the fact that a large number of indicators which researchers associate with the efficiency measure indicators have been developed and continue to be developed.

Because of the efficiency measure indicator has to be used as an optimization criterion, the internal structure of the developed indicator often displays technological process specific nature, for the efficiency assessment of which it is designed.

Widespread approach is the use of technical indicators for making a conclusion about the operational process efficiency. So, in the work [2] the attempt to relate the efficiency to the value of “critical load” is made. In the research [3] the authors try to solve an issue of operations efficiency assessment by the use of a “reliability” indicator, in [4] it is done by means of “space-filling criterion” development, in [5] “error selection energy” is used, in [6] the minimum deviation size from the set trajectory is applied.

In recent years, a large number of works devoted to the issue of energy efficiency have appeared [7]. Traditionally, there are a lot of works in which the efficiency is associated with cost minimization [8] or with the aggregated criterion that is received by important base indicators scaling (Pareto principle [9]).

In the work [10] the use of generalized indicator defined as “efficiency criterion” that is formed by results of integrated indicators extrema values processing is offered.

The use of such approach also excludes the received indicator verification possibility as the applied indicators are quantitatively not comparable among themselves.

The similar problem is observed in the work [10], in which the estimated indicator passes not verification stage, but coefficients scaling stage for the purpose of function extremum coordination with the developers’ idea about the studied process efficiency.

It is obvious that for preliminary indicators selection there is no need to use the verification difficult methods.

It is enough to develop the generalized mathematical converting process model and to define formal signs, the use of which will significantly allow reducing the search problem complexity.

3. The purpose and objectives of the research

The purpose of the work is definition of the formal signs system allowing dividing a set of estimated indicators into two classes: the class of the indicators that aren’t appropriate for use as an operations efficiency measure, and the indicators that possibly can be used to solve this task.

For achievement of the research goal, the following objectives were solved:

– to identify the quantitative characteristics inherent in any operational process, which aren’t dependent on its internal structure and defining its efficiency;
– taking into account the distributed in time operation production flow nature, to establish the sign that has sensitivity to the time character change of these flows.

4. Determination of formal signs of an estimated indicator for operation efficiency evaluation

Any converting process of input production to output production with the use of the mechanism of their converting can be presented in the form of production cybernetic model (Fig. 1).

However, the mechanism, by itself so processes converting products, as the converting products act on the mechanism, causing its wear. Therefore, from the cybernetic point of view, the system mechanism, in relation to the converting operation, is the transformation product by itself. On this basis, the cybernetic operation production model will look as follows (Fig. 2).
Definition 1. We will determine a time-limited process of interaction between production flows, predetermined concerning to their set, qualitative and quantitative parameters, and also rules of their interaction by the concept of "operation":

\[ \forall \tau \in M \{T(\tau) \wedge B(\tau) \wedge C(\tau) \wedge P(\tau) \wedge \ldots \} \wedge P(\tau) \wedge K(\tau) \wedge Q(\tau) \rightarrow \Lambda(\tau) \] 

here M – set of system processes; \( \bar{X} \) – system process; \( t \) – input production flow; p – output production flow; A(\( \bar{X} \)) – process \( \bar{X} \) is an operation, T(\( \bar{X} \)) – time-limited process; B(\( \bar{X} \)) – ordered interactive process; C(\( \bar{X} \)) – finished process; K(r) – product \( r \) is predetermined qualitatively; R(\( \bar{X} \), r) – product \( r \) is input product of \( \bar{X} \) process; Q(r) – product \( r \) is predetermined quantitatively.

Any operation is carried out for the purpose of output products set value increase in relation to the value of input operation products set. As the output product with the set quality can be received at various quantitative proportions of input products, there is an issue of the best operation choice among a set of possible.

For operations with the distributed in time parameters, it is possible to define a quantitative estimating flow for important input production \( r_q(t) \) and for output production \( p_q(t) \).

Corresponding to each important input production \( r_q(t) \) and output production \( p_q(t) \) quantitative estimating flows let their expert (cost) estimates be \( r_s \) and \( p_s \), then the set of comparable input and output functions can be presented by input \( re(t) \) and output \( pe(t) \) global functions (Fig. 3). As a consequence, \( re(t) = r_s r_q(t) \) and \( pe(t) = p_s p_q(t) \).

Because a change of any input product qualitative and/or quantitative parameter results in a change both of input function and output function, the parameters of these functions are the basis for global estimated indicators development and particularly for an efficiency indicator.

Therefore, the researches, connected with the efficiency assessment reliability have to be focused on the class of indicators the structure of which is based on the global \( re(t) \) and \( pe(t) \) functions.

Definition 2. Operation model is the form of operation data presentation that displays the results of system products and system mechanisms procedural interaction, the time of which is started with the registration of the beginning moment of the first input technological production flow, and the end time – the registration of the completion moment of the output technological production last flow delivery.

The example of a global model of operation with the distributed parameters is represented in Fig. 4.
where \( t_1 \) – start time of input production supply; \( t_f \) – end moment of output production delivery.

However, for example, such indicator doesn’t give an estimation possibility for operations with the distributed parameters as it doesn’t have the time change sensitivity of operation products binding or release.

Let’s consider operations models for which the start time and end time of input production supply and the start time and end time of output production delivery are the same (Fig. 5).

Let’s define global operations models for which

\[
\begin{align*}
    t_{r11} = t_{s12} = t_{r12} = ... = t_{r1j};
    t_{p11} = t_{p12} = t_{p1j} = ... = t_{p2j};
\end{align*}
\]

as the synchronized in time operations or as synchronized operations.

So, for example, a) operation and b) operation (Fig. 5) are synchronized operations. At the same time, a) operation is a priori more effective than b) operation. Here

\[
\begin{align*}
    \text{RE}_a = \text{RE}_b, \quad \text{PE}_a = \text{PE}_b, \quad \text{and} \quad \text{ipe}(t)_a = \text{ipe}(t)_b.
\end{align*}
\]

That is, they have identical output functions, but the second operation demands binding of more valuable resources at the initial stage of the operation while the free up volume of valuable resources of the first operation can be used in parallel operation.

Then the current level of reserves (\( \text{ze}(t) \)) in equivalent expression can be defined as follows:

\[
\text{ze}(t) = \text{ZE} - \frac{1}{\alpha} \int_0^t \text{ire}(t) \, dt = \text{ZE} - \text{ire}(t),
\]

where \( \text{ZE} \) – level of reserves in comparable units on initial time point.

For difference definition of the available values current level of reserves (\( \text{dif}(t) \)), the studied operations of input production consumption in the form of input functions of the first (\( \text{re}(t) \)) and the second operation (\( \text{re}(t) \)), it is possible to write down:

\[
\text{dif}(t) = \left( \text{ZE} - \frac{1}{\alpha} \int_0^t \text{re}(t) \, dt \right) - \left( \text{ZE} - \frac{1}{\alpha} \int_0^t \text{re}(t) \, dt \right) = \text{ire}_a(t) - \text{ire}_b(t).
\]

The results of the available reserves difference current level calculation for the operations presented by input functions \( \text{re}_1(t) = (10, 9, 8, \ldots 1) \), \( \text{re}_2(t) = (1, 2, 3, \ldots 10) \) and the initial level of available reserves \( \text{ZE} = 55 \) un. are presented in Table 1.

### Table 1

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<th>( \text{dif}(t) )</th>
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Thus, it is visible (Fig. 6) that in the course of carrying out the second operation additional (in relation to the first operation) reserves of valuable resources remain at the disposal of a supersystem which can be used for carrying out additional operations with the purpose of obtaining the additional added value on the operation time interval.

The second integrated \( \text{vre}(t) \) function from the input function \( \text{re}(t) \) (Fig. 7) has sensitivity to the time of valuable resources certain volume taking:

\[
\text{vre}(t) = \int_0^t \int_0^t \text{re}(t) \, dt \, dt.
\]

That is, for two global models of synchronized operations in the form of input and output functions (\( \text{re}_1(t), \text{pe}_1(t) \)) and (\( \text{re}_2(t), \text{pe}_2(t) \)) which have \( \text{RE}_1 = \text{RE}_2, \text{pe}_1(t) = \text{pe}_2(t) \), the IRE parameter is the index of a more effective operation.

The numerical value of the integrated expression is defined from the input integrated function \( \text{ire}(t) \):
IRE = \int_0^t \int_0^t ir(t) dt = \int_0^t ir(t) dt.

By the same way, it is possible to show that the integrated expression numerical value (IPE) from the integrated output function ipe(t) for operations models, which have PE1=PE2, re1(t)=re2(t), is the index of a more effective operation:

\[ \text{IPE} = \int_0^t \int_0^t \int_0^t \int_0^t \text{ipe}(t) dt = \int_0^t \int_0^t \int_0^t \int_0^t \text{ipe}(t) dt. \]

Therefore, only those estimated indicators will have sensitivity to a change of the dynamics of the input function and output function which connect input and output parameters of the studied operation by using double integration on time of input and output functions.

Thus, the formal signs of the indicator, designed for efficiency estimation of system operations, are the use of global functions of input and output, and also the use of the procedure of double integration on time.

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\[ \text{IPE} = \int_0^t \int_0^t \int_0^t \int_0^t \text{ipe}(t) dt. \]

Thus, for example, indicators RL [11] and EL [12] correspond to such external signs.

\[ RL = \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \text{re}(t) dt \text{pe}(t) dt dt dt dt, \]  
\[ EL = \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \text{re}(t) dt \text{pe}(t) dt dt dt dt. \]

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\[ EL = \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \int_0^{t_0} \text{re}(t) dt \text{pe}(t) dt dt dt dt. \]

Thus, the mathematical relation of two values can be considered as an additional formal sign of an indicator for operations efficiency definition.

5. Discussion of the research results connected with definition of formal signs of indicators designed for assessment of system operations efficiency

The approach to definition of formal signs of the indicators designed for assessment of system operations results efficiency developed in the work is a development continuation of the verification method offered in the work [1] and allows to reduce significantly resource intensity of such
indicators selection process, among the set offered by the developers.

Besides, justification of a sign of such indicator based on definition of the second integrated function from input function and output function, in fact, offers to developers a reference point for research in this direction.

Integration of \( q(t) \) function gives an idea of some quantitative characteristics (XQ) of the object:

\[
XQ = \int_0^t q(t) \, dt,
\]

where \( X \) – the product object with distributed parameters. The second integral characteristic of the time:

\[
IXQ = \left( \int_0^t \left( \int_0^t q(t) \, dt \right) \, dt \right)
\]

displays the physical and cybernetic parameter IXQ of the object X in time, which has so far not received the definition of categories, like weight, volume, speed, and acceleration. In [12] this category is defined as a second integral value of resource use (relative to the input operation) or the impact of resources (relative to output operation).

If to consider the symmetric mathematical operation, in relation to the differentiation operation – integration, then the first integrated function will display, naturally, the quantitative characteristic of the considered object. At the same time, the second integrated function displays the characteristic of the process of “binding” in time of the quantitative characteristic of an object.

The IXQ parameter is determined as physico-cybernetic as the process of binding the object body, as a value, describes the process that is important for biological organisms. It is connected with the fact that the lifetime of biological organisms, as the supersystems, is limited.

Their aspiration to the preservation of the species, competition for access to resources in the form of values and other incentives lead to the formation of motivational tendencies, defined by the concept of “purpose”.

The process of achievement of goals, in the conditions of limitation of valuable resources, results in the need of their rational use for operational processes by means of the index of more favorable operations in the form of efficiency criterion.

It is also possible to note that in relation to the indicators which have undergone selection on formal grounds, it is impossible to use formally the entire apparatus of testing developed for global models of simple operations [1].

For example, the indicator (1) successfully takes a test for relative indicators, though it is an absolute measure. Therefore, for testing of the indicators recommended for assessment of global models of operations with the distributed parameters, it is possible to use not all classes of reference operations which were developed for testing of models of simple operations.

6. Conclusions

1. The definition of the cybernetic production operation model displays in this work a possibility of creation of a global operation model in the form of functions of an input and output \((re(t), pe(t))\). As any change of the input operation production giving leads to a change of operations parameters, the existence of input and output functions is the first formal sign of the indicators designed for efficiency measurement of operations with the distributed parameters.

2. It is established that the second integrated characteristic of the input and output functions has sensitivity to a change of their nature in time.

Thus, the existence of the second integrated characteristic from functions of an input and output is the second formal sign of an estimated indicator.

On the example of assessment of global models of the equally effective simple operations, it is shown that adequate assessment is provided by a dimensionless relative indicator that gives reason to consider existence of functional connections in the form of the mathematical relation in the studied expression as an additional formal sign.

References


1. Introduction

One may define recent years as a period of rapid development of technical means and information technologies with high performance efficiency that led to the creation and implementation of more effective methods of processing and analysis of data and new methods of solving complex applied problems. In this regard, there is a surge of theoretical and practical techniques in the field of neuro-computers and there is increased interest in neuro-like structures, which are widely applied in various areas of human activity – pattern recognition, forecasting, business, medicine, engineering.

Solving applied problems in neuro-basis would be possible if practically applicable methods of the synthesis of neural elements and the synthesis of logical circuits from them are developed.

Significant resources that are invested in creating software and hardware implementation of artificial neural networks, as well as widespread use of neuro-like structures, indicate that the problem of synthesis of neural elements with different activation functions and the construction of logical circuits from them is relevant and practically significant.

In practice, when recognizing discrete images, at the compression and transmission of discrete signals, it is necessary to be able to synthesize neural elements for the realization of discrete functions.