1. Introduction

The process of accumulation of railcars and formation of trains plays a key role. Duration of the accumulation of one train depends on the capacity of railcar traffic of the given purpose. Basic indicators in the process of accumulation of trains are the total railcar-hour accumulation for the given direction per 24 hours, average downtime of one railcar during accumulation, mean time of train accumulation.

To accelerate the process of accumulation of trains, the priority of dispatch of trains are used, which include the final groups of railcars. They also provide for arrival of large groups of railcars by the end of the process of train accumulation. They also form trains with increased weight. In addi-
tion, when developing operational plans, other measures are employed that are aimed at speeding up the working rhythm of railroad station [1]. However, even at rationally organized working process of station, railcars idle period with processing comprises almost half of the total time the railcars spend at the station.

The largest opportunities for reducing daily railcar-hour downtime are in rationalizing the organization of railcar traffic. That is why the development of efficient plan for the formation of trains (PFT) is the most effective measure to improve daily accumulation of railcar-hours. The calculation of PFT is, however, a complex combinatorial problem. Existing classical methods for calculating PFT may be effectively applied only at short polygons. In addition, they do not allow taking into account the limitations that are inherent to the real objects of railway infrastructure. Due to these reasons, at present, building a network PFT at the Ukrainian Railways is actually carried out by expert method. Creating a high-quality procedure for calculating PFT will make it possible to build an automated system for the calculation of PFT. Automated system will allow reassessment of PFT not only once a year but when need be. Such a need is predetermined by the high variability of railcar traffic under conditions of market economy.

2. Literature review and problem statement

PFT implies organization of railcar traffic on sorting, sectional and freight stations and shall ensure:
- improving railcar efficiency and speeding up freight delivery by reducing the time spent at technical stations during accumulation and processing;
- decreasing the cost of transportation by reducing the time and cost of processing railcars and concentration of shunting operations predominantly at well-technically-equipped marshalling yards;
- rational allocation of marshalling operations among stations in accordance with their technical equipment.

Paper [1] outlined the main challenges that stand in the way of building an effective system of organization of railcar traffic under modern conditions. Article [2] presented principles for setting the problem on calculating PFT as a mathematical programming problem. Study [3] proposed a system of techniques for the organization of railcar traffic, which, by means of organizational methods, is able to adapt to the rapid variability of railcar traffic under modern conditions. Paper [4] presented a mathematical model for the calculation of PFT, the construction of which had used the concept of theory of sets. Article [5] proposed a technique for operational correction of PFT, which takes into account the settings, parameters, and condition of the rolling stock, special conditions for transportation, and requirements of the operators and owners of rolling stock.

On the North American continent, a key task in the field of organization of railcar traffic is the task on arranging railcars in sections and assigning them to trains. In addition, at the same time, a task on planning the operations of locomotives and the locomotive squads is solved. Article [6] proposes a specially designed meta-heuristic method, but small dimensionality of the problem, which is given as an example of its implementation, testifies to its imperfection. Paper [7] proposed to state a problem on routing railcar traffic as the problem of determining the graph structure. Railway stations represent vertices of the graph, and the blocks of railcars – arcs. To solve the problem, an algorithm is proposed based on the branch and bound algorithm. The algorithm generates a route for each block, solving the problem of finding the shortest path. As a drawback of this technique, we note a failure to comply with the regulations on the number of railcars in trains. In addition, for real polygons, the use of this method is difficult. The reason is that during simultaneous operations with railcars and blocks all over the range, railcar traffic routing problem passes to the class of problems of large and very large dimensionality.

The article by Swedish researchers [8], which was published in the magazine of the European Research Consortium for Informatics and Mathematics, proposes a method for solving a problem on the disbandment-formation of trains at a marshalling yard. The essence of this method is in the application of the multi-stage formation of “temporary” trains whose number is governed by the model depending on the availability of free tracks. Problem on the formation of trains is stated as a multiprocess flow problem, solving which is proposed with the help of computer program-solver that employs algorithm based on the combination of several methods of mathematical programming with constraints. Article [9] proposed mathematical models based on the use of neuro-fuzzy networks to determine the feasibility of forming and the route of group trains. The article also proposed an original method for operational correction of PFT by using evolutionary selection. Paper [10] proposed an original approach to the solution of problem on the rational allocation of shunting operations among technical stations of a railway network. The method proposed takes into account requests of consignors and provides for a guarantee of timely delivery of cargos in time agreed by the customer.

Although the basic principle to reduce accumulation railcar-hours remains unchanged, the examined methods have almost lost relevance in terms of practical application. They were developed for manual calculations, their use is only possible for polygons that include no more than 10–12 technical stations and 1–2 branches.

3. The aim and tasks of the study

The aim of this study is to develop a method for calculating a network plan for the formation of single-group freight trains. The method should provide for the accuracy of calculations and the possibility of taking into account constraints in the throughput and processing capabilities of stations and the throughput capacity of sections. This method is a key component in constructing a modern multi-level automated system for managing railcar traffic.

To achieve the set aim, the following tasks are to be solved:
- based on the analysis of existing methods, to select a modern mathematical apparatus that utilizes capabilities of modern computational technology;
- to represent a format of solving the problem in the form that is applicable for the chosen method of optimization;
4. Materials and methods for examining the process of constructing PFT

The problem on calculating PFT is a complex task of the combinatorial type. Solving the problems of this type is associated with the operation in large solution spaces. Variables in such problems are discrete, that is why objective functions are not smooth and differentiated. One of the few modern mathematical apparatuses for which such properties of combinatorial problems is not a problem, are genetic algorithms (GA). They do not belong in the class of gradient methods, as are representatives of the class of stochastic optimization methods. A fundamental difference between genetic algorithms and, for example, a method of random search or the method of stochastic approximation is that the genetic algorithm works with a whole population of solutions-candidates. Other algorithms work only with one solution, moving towards optimum and improving this solution only.

To solve a problem using GA, its decision must be represented in the form of a chromosome. Chromosome denotes a sequence of variables. Each variable is represented by a gene. Each gene is assigned with a place in the chromosome, which is called locus. Genetic algorithm simulates biological mechanisms of cross breeding, mutation, crossover and selection among a population of individuals. An objective function is used to evaluate in points the degree of individual’s adaptability and is called the function of adaptability or a fitness function.

One can represent a solution as follows: each section between technical stations is presented by a region of chromosome. The number of genes that are included in such region is the number of flows of railcar traffic that pass this section of polygon. If the motion in the section occurs both in the odd and even directions, then it is matched with 2 regions of the chromosome, separate for each direction. Each gene within its region can take integer values, which can be in the range from 1 to \( N \), where \( N \) is the number of flows that pass through this section. If the genes within a single region take the same values, it is interpreted as the unification of flows, which are represented by these genes, within this section. Thus we simulate partition of a set of flows that pass through this section into non-empty subsets as shown in Fig. 1. Within the first region AB, genes that correspond to flows AE and AF can take the same values, such as value (5). This will mean that flows AE and AF will pass through section AB together, as part of one destination. In region BC, the genes that correspond to flows AE and AF also took the same values (3); this means that along section BC, flows AE and AF will pass together. Thus, when executing the interpretation of an objective function or a fitness function, it is necessary to perform a procedure of logic control. This procedure will analyze subsets of flows of adjacent regions and combine those subsets that contain the same compositions of flows to one destination, that is, to collect destinations bit by bit. Next, each destination receives points that in the problem on calculating PFT may correspond to, for example, railcar-hours of downtime. These railcar-hours match the cost of the destination accumulation. These costs are calculated as the product of the norm of the number of railcars in a freight train in the section and the parameter of accumulation at the Dispatch station to destination. At the last destination station, the points are calculated, or railcar-hours for additional processing of transit railcars of those flows, for which the station of final destination is not their last point of transportation. Fig. 1 shows a scheme of encoding the PFT solution for solving by using GA.

Such representation of grouping the flows in each section corresponds to the set-theoretic representation on splitting the sets into subsets, then the number of combinations for each station is equal to the appropriate Bell number.

Construction of PFT is a complex combinatorial problem, which implies a considerable computational complexity. However, by using modern computing technology and contemporary mathematical apparatus, it is possible to overcome computational complexity. An important parameter that is directly related to the magnitude of time spent on railcars’ downtime during the formation of trains is a parameter of accumulation. The possibilities to influence the magnitude of the accumulation parameter are few, within 5–10% only. However, taking that influence into account when calculating a plan of the train formation will make it possible to improve parameters of the calculated plan. For this purpose, it is necessary to introduce to the model’s objective function the parameters of train accumulation by destinations at the marshalling stations as stochastic variables. Thus, the problem on finding optimal PFT in such statement relates to the problems of stochastic combinatorial optimization. Objective function can be written down as follows:
where $w$ is the number of all possible destinations; $m_i$ is the norm of number of railcars in the train to the i-th destination; $\delta$ is the current parameter of accumulation to the i-th destination; $\sigma$ is the root mean square deviation in the parameter of accumulation to the i-th destination; $\gamma$ is the processing time of train to the i-th destination at the station of disbandment; $\gamma_{dis}$ is the time of disbandment of train to the i-th destination at the station of disbandment; $n_i$ is the number of railcars of the i-th flow; $k$ is the number of flows of railcar traffic; $q_i$ is the number of technical stations to the i-th destination; $z_{i1}$ is the number between technical stations of the i-th destination; $t_{i1}$ is the time of processing transit train without processing at the h-th section to the i-th destination; $W_{i2}$ is the sectional speed of freight trains in the r-th section to the i-th destination; $x_{ij}$ is the variable that takes value 1 if the i-th section belongs to the j-th flow, otherwise it takes value 0; $c_{ih}$ is the cost of railcar-hour; $Sgn$ is the sign function; $erfc$ is the Laplace complementary error function, which is used to calculate the probability of exceeding the parameter of accumulation its mean value.

First term in square brackets represents the cost of railcar-hours for the accumulation to the i-th destination. Second term represents excess cost of railcar-hours for the accumulation to the i-th destination associated with the risk of exceeding the current value of accumulation parameter. These costs model risk. When the current value of accumulation parameter takes values that are less than the mathematical expectation of this magnitude, then there is a risk to exceed it. A Laplace complementary error function is a non-elementary function that represents the Laplace probability integral.

One of the main goals when solving a problem on calculating PFT is the optimal allocation of work between technical stations. The key parameter of technical station is the magnitude of its processing capacity. The processing capacity of station is defined by its rail track development, throughput capacity of bottlenecks and processing capacity of shunting devices. It should be taken into account in the form of limitation:

\[
\sum_{i=1}^{w} \sum_{j=1}^{k} n_{ij} \cdot \gamma_{ij} / m_i \leq N_{pr}^s, \quad s = 1...S, \tag{2}
\]

where $n_{ij}$ takes value 1 if the s-th station is the disbandment station to the i-th destination, takes value 0 otherwise; $N_{pr}^s$ is the processing capacity for the s-th station; $S$ is the number of technical stations on the polygon.

Simultaneously with processing freight trains, technical station processes and passes transit freight trains and passenger trains. Therefore, in addition to the processing capacity, the capabilities of each technical station are limited by its throughput. Station throughput capacity is defined as the number of freight trains (with and without processing) and the assigned number of passenger trains, which can pass the station per day in all directions. It is determined under the operating conditions of station to ensure full utilization of available means, based on the equipment available [11].

Station throughput capacity is determined by the lowest value of throughput by its receiving-dispatching tracks and necks. Limitations on throughput capacity of technical stations can be written down as follows:

\[
\sum_{i=1}^{w} \sum_{j=1}^{k} n_{ij} \cdot \omega_{ij} / m_i \leq N_{pr}^d - N_{p}^d, \quad s = 1...S, \tag{3}
\]

where $\omega_{ij}$ takes value 1 if the s-th station is part of the route to the i-th destination (is a station of formation, disbandment or transit), takes value 0 otherwise; $N_{pr}^d$ is the throughput capacity of the s-th station; $N_{p}^d$ is the number of passenger trains which pass the station in 24 hours.

When creating a mathematical model for making up a plan of train formation, one should also take into account the fact that not only technical stations, but the railway sections that connect them, also have limitations on the throughput. Throughput capacity of a railroad line is the largest quantity of trains or pairs of trains with defined weight, which can pass in 24 hours. It is determined depending on the available technical equipment, type and capacity of rolling stock and train traffic schedule type. The available technical means include a number of tracks, the kind of blocking in section tracks (automated, semi-automatic), power of traction substations, etc. Limitations on the throughput of the sections can be written down as follows:

\[
\sum_{i=1}^{w} \sum_{j=1}^{k} n_{ij} \cdot \eta_{ij} / m_i \leq N_{pr}^d - N_{p}^d, \quad d = 1...D, \tag{4}
\]

where $\eta_{ij}$ takes value 1 if the d-th section belongs in the route of the i-th destination, takes value 0 otherwise; $N_{pr}^d$ is the throughput for the d-th section; $N_{p}^d$ is the number of passenger trains that pass the d-th section over 24 hours; $D$ is the number of sections on the polygon.

5. Results of examining the process of calculating PFT

For the purpose of comparing the existing methods with the one proposed and to verify its adequacy, it is appropriate to solve, using the method proposed, the problem, which was solved by using existing method. Article [12] gives an example of solving a problem by applying a combined analytical comparison method. Fig. 2 shows initial data for the problem. Fig. 2. a shows a schematic of the polygon, railcar-hours of accumulation and the magnitudes of time saving when a transit railcar passes without processing. Fig. 2. b shows a diagram of railcar traffic.

Fig. 3 shows solution presented in [12], which was obtained by a combined analytical comparison method. The solution contains 8 destinations, the calculation of costs according to the optimal plan is given in Table 1. General railcar-hours comprised the magnitude of 6528. Out of which, 1328 railcar-hours — additional costs for processing transit railcars, 5200 railcar-hours — cost for the accumulation of destinations.
Fig. 2. Initial data for calculating a train formation plan:
a—plan of polygon, b—diagram of railcar traffic

Fig. 3. Optimal variant of PFT, calculated by using classical methods

The same initial data were used in [13] to demonstrate adequacy of the new proposed method for calculating a plan of train formation, which is called is a method of sequential growth in the railcar traffic flows [13]. The optimal plan, obtained in [13] by the method of sequential growth in the railcar traffic flows, coincides with the result that was received by means of the combined analytical comparison method, given in [12].

Based on the proposed method, we created software in the language of Matlab. For simulation, we used initial data given in [12, 13]. Fig. 4 shows the optimal PFT and convergence dynamics of objective function. The magnitude of the time spent amounted to about 2 min. As can be seen from Fig. 4, b, the simulation we conducted demonstrated rapid algorithm convergence.

Table 1 gives calculations of the cost of railcar-hours according to the calculated optimal plan.

Table 2 gives calculations of the cost of railcar-hours according to the optimal plan, obtained by using a program created on the basis of the proposed model.

According to data in Table 2, the costs comprise 1728 railcar-hours that are additionally spent on processing transit railcars and 4600 railcar-hours that are spent on the accumulation of destinations; the total cost according the plan is 6328 railcar-hours. Additional costs for processing are 400 railcar-hours larger than in the variant of the plan received in [12]. The optimal plan, however, contains 7 destinations, not 8, as the plan, obtained by applying classical methods. This reduces the cost for the accumulation of
6. Discussion of results of examining the process of calculating PFT

The main shortcoming of the proposed method is the probability when a solution space does not include the solution that matches global minimum of costs. This probability is, however, controlled, that is, one can reduce it by increasing the volume of population of solutions, although it will require additional time or additional computing capacities. The proposed method has demonstrated a larger accuracy than the classical methods. In this case, time costs were absolutely acceptable in terms of practical application. To solve the problem on calculating PFT of large dimensionality, one might need much more time if employing a standard computer. But at present there are possibilities for creating computational clusters. The introduction of the concept of cloud computing will provide the user with access to computing resources of large capacity with the help of the Internet. That is why this is not a big problem now.

The developed method is flexible and allows considering any required limitations.

The method devised has prospects of application as the basis for constructing a new automated system for managing railcar traffic in the network of railroads in Ukraine.

The possibility to further improve the method involves the use of new types of genetic algorithms, which are under development now. Such types of algorithms include, for example, linkage learning genetic algorithms (LLGA), which in the future should greatly increase efficiency of applying mathematical apparatus of GA in comparison with the existing types.

7. Conclusions

1. As optimization method, we chose a method of genetic algorithms considering its advantages over other methods of optimization.

2. An original method is developed for encoding solutions to the problem on calculating PFT in the form of chromosomes for use with the fitness function of GA. The method employs the concept of splitting sets into non-empty subsets for representing combinations of railcar traffic flows that pass the sections.

3. We created by a mathematical model, which uses accumulation parameters as stochastic variables for obtaining the possibilities of finding the most rational variant of PFT. The model also takes into account the limitations on the throughput and processing capacity of objects in the railroad infrastructure.

4. A simulation was performed for a polygon with 6 technical stations, which demonstrated rapid algorithm convergence that indicates adequacy of the model. The variant of the developed plan proved to be 200 railcar-hours more efficient than the variants that were calculated using classical methods. It demonstrates practical value, that is, efficiency of the method devised.

References

