1. Introduction

A self-propelled tethered remotely operated underwater vehicle (ROV) is a solid body with six degrees of freedom. No less than three degrees of freedom are typically controlled. Automated control of spatial motion of ROV is a known scientific problem because, as the object of control, it is essentially nonlinear. Basic nonlinearities are caused by the forces and moments of hydrodynamic resistance of the ROV hull, nonlinear dependence of screw propeller thrust on its rotation velocity, and the force action of an umbilical cable [1].

The degrees of ROV mobility as a solid body are shown in Fig. 1.

![Fig. 1. Coordinate systems and degrees of freedom of remotely operated underwater vehicle](image)

The degrees of ROV freedom are typically represented relative to the ROV coordinate system (RCS) $O_{ax}x_ayaz_a$, which moves and rotates along with ROV. The motion of ROV in general is considered in the basic coordinate system (BCS) $O_{bx}xb_yyb_zb$, which is considered stationary relative to the Earth. The assumption on the BCS inertia is also adopted.

Operational control of ROV is essentially a multidimensional control problem. However, it is known that the methodology of developing multidimensional systems of automated control (SAC) permits the synthesis of regulators separately for each degree of freedom [2]. That is why, to operate underwater vehicles, the strategies of decentralized control are mainly used.

In this approach, a multidimensional SAC is obtained by the known methods of synthesis, similar to the single-dimensional problems. In this case, ROV must have a separate moving device to manage each particular degree of mobility. In this case, the influence of the moving device on the other degrees of mobility should be minimal.

It should be noted that, as control objects, ROV have common features with autonomous underwater vehicles (AUV). At a similar architecture of ROV and AUV, the process of synthesis SAC for their motion may also be the same, if we do not take into account the impact of umbilical cable on the former.

Typically, the architecture of ROV does not imply using a separate moving device for each controlled degree of freedom. Instead, the propulsion devices are arranged in a way that some of them may influence both the translational and rotating motion of ROV. Thus, for example, ROV, built by the classical four-propeller pattern (Fig. 1) has separate pro-
pulsion devices for vertical and lateral motion. The marching propulsion devices provide marching motion and rotating movement around vertical axis [3]. That is why these degrees of freedom for control are combined. In addition, even when managing only the translational motion, ROV may change its angular orientation due to the interaction with the flow of water. In this regard, the ROV motion by every translational degree of freedom can depend on all the propulsion devices simultaneously.

In such cases, SAC should not only implement a control law, but also deal with the problem to allocate control functions. Its essence consists in the formation of controlling influences for each propulsion device in accordance with the control law and the configuration of propulsion devices [4]. For this purpose, coupling matrices are employed that bind the configuration of propulsion devices, their forces separately, and their resulting forces and torques.

Remotely operated underwater vehicles, as well as other technical objects, are characterized by nonlinearity of the "restrictions" type. The presence of restrictions for controlling influences can lead to loss of control over certain degrees of freedom. Thus, for example, under marching motion of ROV at utmost speed, SAC enters the saturation mode, and control over its course is not executed. In this regard, relevant is the problem on coordinating the operation of ROV propulsion devices so that the SAC saturation would have a minimal effect on managing its degrees of freedom.

2. Literature review and problem statement

In the theory of automated control, there is a wide range of methods for the synthesis of SAC for complex non-linear objects. They are successfully employed to manage ROV, including treating them as multidimensional objects.

Article [5] proposes to control spatial motion of ROV based on the use of inverse model of its hull. To compensate for the dynamics of each propulsion device, a fuzzy compensator is developed, which operates under sliding mode. Results of the proposed SAC performance are shown for the one-dimensional motion in order to simplify analysis of the effect of propulsion device dynamics on the overall behavior of the system.

Sliding mode control method is applied in [6] for the synthesis of SAC for spatial motion of ROV. The effectiveness of SAC is confirmed by simulating the ROV with four controlled degrees of freedom. A coherence of operation of the propulsion devices is provided by the choice of such parameters of reference model, at which ROV is capable of moving along the reference trajectory. In case ROV deviates from the reference trajectory, the desired values of controlling influences can go beyond permissible limits.

Article [7] proposes adaptive neural network regulator to control the trajectory motion of AUV. Results of SAC modeling are presented for such disturbing influences and desired trajectory, at which SAC does not enter the saturation mode.

Adaptive SAC of AUV with four degrees of mobility is proposed in [8]. Proposed adaptive control algorithm calculates vectors of the desired forces and moments for the motion of AUV by reference trajectory. However, results of the SAC operation are given for the case when there are no restrictions for controlling forces and moments.

A method for the allocation of controlling forces and moments between controlling mechanisms of moving object is developed in [9]. There are results of the SAC operation under assumption on negligibly low inertia of propulsion devices in comparison with the inertia of a moving object. The operation of SAC under the mode of saturation is not considered.

For ROV of inspection class with four controlled degrees of freedom, SAC for trajectory motion is developed in [10]. Control law consists of three levels; controlling signal contains the equivalent component, adaptive PID component and robust component. The focus of the work is control under conditions of incomplete parametric information on the control object.

Paper [11] develops a ROV automated control system based on multidimensional PID regulator whose operation is corrected by reference model, and an observer with a high coefficient of amplification. Article [12] synthesizes, to manage ROV, an adaptive neural network regulator based on the observer. However, there is no information on the behavior of regulators under the modes of saturation.

A multi-dimensional SAC based on PID-regulators to manage ROV with four degrees of mobility is developed in [13]. Each regulator is designed to control one degree of mobility of ROV. In this case, motion at the utmost marching speed will result in the loss of controllability over rotating degree of mobility.

An analysis of scientific publications reveals that SAC are synthesized mainly for ROV with four controlled degrees of freedom. The proposed approaches can also be applied to manage ROV by all six degrees of freedom. But studies on effectiveness of the developed SAC are conducted at such desired spatial trajectories and such external perturbations at which controlling influences practically do not exceed permissible limits. However, the author's experience in the practical operation of ROV, particularly under extreme conditions, demonstrates the following. Disturbing influences may take on magnitudes that will force SAC to operate under saturation mode. The desired trajectories of motion may also be evolving in a way that the contours of SAC can enter the saturation mode for a certain period. Control over such modes in the ROV operation has not been described in the scientific literature up to now.

The existence of restrictions in controlling influences under certain parameters of control object and SAC as a whole leads to the emergence of "strong" and "weak" degrees of freedom. This means that the propulsion devices in the first place will provide control by a strong degree of freedom with the loss of manageability by the adjacent low degree of freedom. Recovery of manageability will occur in the process of exiting saturation mode of the SAC contour that is in charge of the strong degree of freedom.

In practice, when controlling ROV, this may lead in general to the unstable work of SAC. The known methods of SAC synthesis did not take into account this peculiarity. That is why, in order to avoid the saturation mode, the SAC performance speed is reduced, or the dynamics of the desired ROV trajectory is slowed down.

3. The aim and tasks of the study

The aim of present study is to develop a method for maintaining manageability at multi-dimensional automated con-
control over tethered underwater vehicle. The method should provide for the work of automated control system contours on the verge of saturation. This will make it possible to manage controlled automated motion of underwater vehicle by all controlled degrees of freedom.

To achieve the set aim, the following tasks are solved:
- a mathematical model is developed for the spatial motion of ROV with six controlled degrees of freedom;
- employing the method of inverse dynamics, a regulator of spatial motion of ROV by six degrees of freedom is synthesized;
- the operation of ROV regulator under the modes of saturation is modeled;
- the fundamentals for the method of maintaining SAC on the verge of saturation are formulated;
- SAC for the ROV spatial motion with maintaining the manageability by its degrees of freedom is designed;
- using a method of computer simulation, the operation of SAC is examined with the unit that maintains its contours on the verge of saturation.

4. Materials and methods for devising a method for maintaining SAC on the verge of saturation

4. 1. Development of a mathematical model for the ROV spatial motion

A mathematical model for the ROV motion is obtained based on the laws of amount of motion and the moment of amount of motion of a solid body:

\[
\begin{aligned}
\frac{d}{dt} \mathbf{P} &= \mathbf{F}, \\
\frac{d}{dt} \mathbf{L} &= \mathbf{M},
\end{aligned}
\]

where \(\mathbf{P}\) is the amount of motion (the momentum) of body, \(\mathbf{F}\) is the main vector of external forces, \(\mathbf{L}\) is the moment of amount of motion (angular momentum) of body relative to its pole, \(\mathbf{M}\) is the main moment of external forces relative to the body’s pole.

Index B in the differential sign means that the derivative of vector is calculated relative to basis B, that is, the basis of BCS. A pole of the body is usually selected in the center of its masses.

When using a non-inertial reference system, that is, RCS, one should consider the following. Geometric sense of derivative from vector by time comes down to its representation of velocity of motion of the end of the vector from which the derivative is taken. In this regard, the laws of amount of motion and the moment of the amount of motion of a body take the following form [14]:

\[
\begin{aligned}
\frac{d}{dt} \mathbf{P} + \mathbf{\omega} \times \mathbf{P} &= \mathbf{F}, \\
\frac{d}{dt} \mathbf{L} + \mathbf{\omega} \times \mathbf{L} + \mathbf{v} \times \mathbf{P} &= \mathbf{M},
\end{aligned}
\]

where \(\mathbf{v}\) is the translational speed of ROV motion, \(\mathbf{\omega}\) is the rotating speed of motion of the body.

Index A in the differential sign means that the derivative of vector is calculated relative to A basis, that is, the basis of RCS.

In order to obtain kinematic parameters of ROV, it is necessary to use an equation of solid body motion with vectors \(\mathbf{v}\) and \(\mathbf{\omega}\) instead of vectors \(\mathbf{P}\) and \(\mathbf{L}\). For this purpose, we separate the components from vectors \(\mathbf{P}\) and \(\mathbf{L}\) that correspond to the amount of motion and the moment of the amount of motion of the added masses. Next, we accept the assumption about symmetry of ROV that allows you to write down vector equation of motion in the form of six scalar differential equations [15]. Such approach is suitable to study the motion of symmetrical bodies. If a control object is asymmetrical, then additional transformations of system of equations will be required. The purpose of such transformations is to obtain a dependence of derivatives from speed kinematic parameters through the rest of equation members.

A more convenient to use is the matrix method for recording basic laws of motion of a rigid body. It allows obtaining the derivatives of speed kinematic parameters by applying arithmetic operations over matrices. In the practice of modeling marine movable objects, the following matrix equation of motion is applied [4]:

\[
\mathbf{M} \frac{d}{dt} \mathbf{v} + \mathbf{C} \mathbf{v} = \mathbf{T},
\]

where \(\mathbf{M}\) is the matrix of own masses and moments of ROV inertia and the added masses of water with dimension 6×6; \(\mathbf{V}\) is the matrix of ROV speed kinematic parameters of dimension 6×1; \(\mathbf{C}\) is the matrix to express coriolis and centripetal forces of ROV and added masses of water of dimension 6×6; \(\mathbf{T}\) is the matrix of resultant forces and moments that act on ROV, of dimension 6×1.

The elements of matrix \(\mathbf{C}\) are formed based on the elements of matrix \(\mathbf{V}\), as well as on their own and added masses and moments of inertia of ROV. Using such a form of record is also not very convenient because the inertial and speed parameters of ROV are duplicated in matrix \(\mathbf{C}\).

In present work, we shall obtain a matrix equation of the ROV motion based on the laws of amount of motion and the moment of the amount of motion of body (1). For this purpose, we shall introduce designations for the components of vectors \(\mathbf{P}, \mathbf{L}, \mathbf{v}, \mathbf{\omega}, \mathbf{F}\) and \(\mathbf{M}\):

\[
\begin{aligned}
\mathbf{P} &= \{P_x, P_y, P_z\}, \\
\mathbf{L} &= \{L_x, L_y, L_z\}, \\
\mathbf{v} &= \{v_x, v_y, v_z\}, \\
\mathbf{\omega} &= \{\omega_x, \omega_y, \omega_z\}, \\
\mathbf{F} &= \{F_x, F_y, F_z\}, \\
\mathbf{M} &= \{M_x, M_y, M_z\}.
\end{aligned}
\]

Index \(A\) indicates that the vector coordinates represent their mapping on the RCS axes.

We shall also introduce designations for the matrix of amount of ROV motion \(\mathbf{Q}\), matrix of speed kinematic parameters \(\mathbf{V}\) and the matrix of forces and moments \(\mathbf{T}\):

\[
\begin{aligned}
\mathbf{Q} &= \begin{bmatrix} P_x & P_y & P_z \\ L_x & L_y & L_z \end{bmatrix}, \\
\mathbf{V} &= \begin{bmatrix} v_x & v_y & v_z \\ \omega_x & \omega_y & \omega_z \end{bmatrix}, \\
\mathbf{T} &= \begin{bmatrix} F_x & F_y & F_z \end{bmatrix}.
\end{aligned}
\]
\[ T = \left[ F^T M \right] = \left[ F_x^T \ F_y^T \ F_z^T \ M_x \ M_y \ M_z \right]^T. \]

Superscript index \( T \) denotes a matrix transpose operation. Let us form matrix \( K \) that considers movable motion when taking a derivative relative to RCS. Multiplication of matrix \( K \) by some matrix \( X = \left[ \dot{x} \ \dot{y} \right]^T \) should form a matrix of product vectors:

\[ \dot{K}X = \left[ \dot{\omega} \times \dot{x} \ \dot{\omega} \times \dot{x} + \dot{v} \times \dot{y} \right]^T. \]

Therefore, the elements of matrix \( K \) will be formed based on the components of vectors \( \dot{v} \) and \( \dot{\omega} \):

\[
K = \begin{bmatrix}
0 & -\omega_z & \omega_y & 0 & 0 & 0 \\
\omega_z & 0 & -\omega_x & 0 & 0 & 0 \\
-\omega_y & \omega_x & 0 & 0 & 0 & 0 \\
0 & -v_z & v_y & 0 & -\omega_z & \omega_x \\
v_z & 0 & -v_x & \omega_z & 0 & -\omega_x \\
-\omega_y & v_x & 0 & -\omega_x & \omega_y & 0
\end{bmatrix}.
\]

The application of matrix \( K \) allows us to write down the laws of motion of body (1) in the matrix form:

\[
\frac{dQ}{dt} + KQ = T. \tag{2}
\]

In hydromechanics, the forces acting on a marine movable object (MMO) as a result of its interaction with the environment (fluid), are typically divided into inertial forces and forces of viscosity. Inertial forces are accounted for by assuming that MMO moves in an unconstrained incompressible perfect fluid.

Let us consider the effect of inertial forces of fluid as the amount of fluid motion as the left part of the law of motion of body (2) by the principle of superpositions:

\[
\frac{d}{dt}(Q + Q_i) + K(Q + Q_i) = T.
\]

In this equation, the forces and moments in the right side do not contain fluid inertial component since it was considered in the left side.

Let us replace the matrices of amount of motion with the matrices of speed kinematic parameters and masses of ROV:

\[
\frac{d}{dt}(MV + AV) + K(MV + AV) = T;
\]

\[
M = \text{diag} \{ m, m, m, i_x, i_y, i_z \}.
\]

Let us form matrix \( K \) that considers movable motion to BCS. The elements of matrix \( K \) will be formed based on the components of vectors \( \dot{v} \) and \( \dot{\omega} \):

\[
K = \begin{bmatrix}
\hat{i}_1 & \cdots & \hat{i}_{16} \\
\vdots & \ddots & \vdots \\
\hat{l}_{61} & \cdots & \hat{l}_{66}
\end{bmatrix}
\]

where \( M \) is the matrix of masses and moments of ROV inertia; \( m \) is the mass of ROV; \( i_x, i_y, i_z \) are the moments of ROV inertia, calculated along the respective RCS axes; \( \Lambda \) is the matrix of added masses; \( \lambda_{ij} \) are the added masses of fluid, \( i=1, 2, \ldots, 6 \), \( j=1, 2, \ldots, 6 \).

Matrix \( \Lambda \) is symmetric, that is, \( \lambda_{ij} = \lambda_{ji} \). The elements of matrix \( \Lambda \) depend solely on the geometry of the ROV outer surface.

It should be noted that the speed kinematic parameters of ROV (elements of matrix \( V \) ) are defined relative to water. Under conditions of current, to determine the ROV velocity relative to BCS, it is necessary to consider its speed relative to RCS.

Let us introduce matrix of own and added masses \( M \) of ROV positional kinematic parameters \( R \), and matrix of correction by current \( V \). We shall write down the basic law of ROV dynamics in the normal Cauchy form:

\[
\frac{dQ}{dt} + M\dot{\Lambda} = \Lambda Q; \quad \dot{\Lambda} = \{ \theta \ \psi \ \phi \}^T;
\]

\[
\frac{d\dot{q}}{dt} = \left[ \dot{\theta} \ \dot{\psi} \ \dot{\phi} \right]^T;
\]

\[
\frac{dR}{dt} = K_r \{ V + V_s \}.
\]

where \( \Lambda \) and \( \Lambda \) are the vectors of translational and rotational coordinates of MMO, respectively; vector \( \Lambda \) connects the start of BCS and the center of MMO mass, the elements of vector \( \dot{q} \) are the Euler angles; \( \dot{V} \) is the vector of current velocity relative to BCS whose elements are assigned in RCS; \( K_r \) is the kinematic matrix of coupling between positional and speed kinematic parameters of ROV with dimension 6×6.

The elements of matrix \( K_r \) are formed based on equations of coupling between translational and rotating motions of ROV:

\[
K_r = \begin{bmatrix}
K_r & 0_{6 \times 3} \\
0_{3 \times 6} & K_o
\end{bmatrix}
\]

where \( K_r \) is the matrix of coupling between projections of vector \( \dot{\Lambda} \) on the axes RCS and BCS, \( K_o \) is the matrix of coupling between the projections of vector \( \dot{\omega} \) onto the RCS axes and speeds of change in the Euler angles.

The elements of matrix \( K_r \) are defined either based on the Euler angles or based on basis vectors RCS \( \hat{i}_1, \hat{i}_2 \) and \( \hat{k}_1, \hat{k}_2 \); the elements of matrix \( K_o \) are determined based on the Euler angles [15, 16]:

\[
\hat{i}_1 = \{ i_x, i_y, i_z \};
\]

\[
\hat{k}_1 = \{ i_x, i_y, i_z \};
\]
forces and moments are due to the rotating motion of ROV. Viscous forces and moments are calculated based on computational or experimental hydromechanics data [18, 19].

External forces and moments include the force of gravity and Archimedes’ buoyant force, as well as force influence of the umbilical cable on ROV. Calculating the first two is not difficult and is based on the known relations [15].

Forces and moments of umbilical cable arise as a result of its interaction with water, which impact the ROV in the point of its fixing. Present work employs a mathematical model of the dynamics of umbilical cable, developed in [20]. It takes into account the forces of gravity and buoyancy, reactive forces that act on the umbilical cable, as well as the curve of current.

Propulsive forces and moments of ROV occur as a result of the operation of propulsion complex:

\[
T_p = \sum_{i=1}^{I} T_i, \quad i = 1, 2, \ldots, I;
\]

\[
T = [F, \hat{M}], \quad \hat{F} = \hat{F}_{d}, \quad \hat{M} = \hat{i} \times \hat{F}_{d},
\]

where \(T_i\) is the matrix of forces and moments of the \(i\)-th propulsion device; \(I\) is the total number of propulsion devices; \(F_i, \hat{M}_i\) are the vectors of force and moment of the \(i\)-th propulsion device, respectively; \(F_{d}\) is the scalar value of force of the \(i\)-th propulsion device, \(\hat{d}\) is the unit vector that describes the direction of the force action of propulsion device, \(\hat{i}\) is the vector that describes the location of propeller relative to the center of ROV masses.

Here we consider the ROV who’s each propulsion device consists of a propeller that is set into motion by a DC electric motor. When describing the model of propulsion device, in order to reduce the notation, symbol \(i\) in the role of the index shall not apply.

A mathematical model of screw propeller consists of non-linear equations of its force \(F_i\) and retarding torque \(Q_i\) [19]:

\[
F_i = f(\omega_i, \nu_i);
\]

\[
Q_i = f(\omega_i, \nu_i),
\]

where \(\omega_i\) is the speed of rotation of screw propeller, \(\nu_i\) is the translational screw propeller speed relative to water.

Parameter \(\nu_i\) is defined as the projection of vector of propulsion device velocity relative to water \(\hat{v}_i\) on the vector of its action direction \(\hat{d}\) with regard to the translational \(\hat{v}\) and rotating \(\hat{\omega}\) speeds of ROV:

\[
\nu_i = \hat{v}_i + \hat{\omega} \times \hat{r};
\]

\[
\nu = \text{proj}_d \hat{v}_i.
\]

Mathematical model of dynamics of the DC electric motor is known and is well explored. That is why we shall reproduce it in a contracted form:

\[
\begin{align*}
\frac{dI}{dt} &= k_u i - k_u \omega_i;
\end{align*}
\]

\[
\frac{d\omega_i}{dt} = k_i i - k_i \omega_i - Q_i,
\]

where \(L, R, i, \omega\) are, respectively, the inductance, resistance and current of the anchor of electric motor; \(k_u\) is the gain factor...
of voltage transformer (driver) of propulsion device; u is the controlling influence; \(k_a\) is the linear parameter that characterizes electromechanical properties of electric motor; \(J\) is the moment of inertia in the system “rotor – reducer – screw propeller” brought to screw propeller; \(k_r\) is the parameter that characterizes resistance moment of the rotor rotation in a liquid dielectric.

The first equation in system (8) describes the dynamics of electrical processes in the propulsion device. The second equation in system (8) describes the dynamics of mechanical processes in the propulsion device.

The model’s input is fed with a set of controlling influences from its propulsion devices \(U\):

\[
U = \{u_1, u_2, \ldots, u_i, \ldots, u_I\}, \quad i = 1, 2, \ldots, I.
\]

At the output, we receive speed \(V\) and positional \(R\) kinematic parameters of ROV.

A generalized structure of the ROV mathematical model is shown in Fig. 2.

\[
\text{Model of gravitational force} \quad \text{Model of buoyancy force} \quad \text{Model of propulsion device} \quad \text{Model of propulsion complex} \quad \text{Basic equation of ROV dynamics} \quad \text{Model of water viscosity} \quad \text{Model of umbilical cable} \quad V, U \quad R, T
\]

Fig. 2. Structure of mathematical model for a remotely operated underwater vehicle

We shall consider ROV, which is equipped with six propulsion devices. Their location and directions of action relative to RCS are given in Table 1.

<table>
<thead>
<tr>
<th>i</th>
<th>Propulsion device</th>
<th>Location: (\hat{x}_i, \hat{m})</th>
<th>Direction of action: (\hat{x}_i, \hat{m})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Left marching</td>
<td>((-0.3, 0, -0.3))</td>
<td>((1, 0, 0))</td>
</tr>
<tr>
<td>2</td>
<td>Right marching</td>
<td>((-0.3, 0, 0.3))</td>
<td>((1, 0, 0))</td>
</tr>
<tr>
<td>3</td>
<td>Front vertical</td>
<td>((0.33, 0.2, 0))</td>
<td>((0, -1, 0))</td>
</tr>
<tr>
<td>4</td>
<td>Stern vertical</td>
<td>((-0.35, 0.2, 0))</td>
<td>((0, -1, 0))</td>
</tr>
<tr>
<td>5</td>
<td>Upper lateral</td>
<td>((0, 0.2, 0))</td>
<td>((0, 0, 1))</td>
</tr>
<tr>
<td>6</td>
<td>Bottom lateral</td>
<td>((0, 0.2, 0))</td>
<td>((0, 0, 1))</td>
</tr>
</tbody>
</table>

Such a configuration of propulsion devices provides for the capability to manage controlled spatial motion by all six degrees of freedom of ROV.

4. 2. Synthesis of automated control system for the ROV spatial motion by the method of inverse dynamics

For the synthesis of SAC, we shall employ a method of inverse dynamics because it is effective for the synthesis of SAC for complex non-linear objects [21]. In addition, applying a mathematical model of the dynamics of an umbilical cable allows us to take full account of the external forces that act on ROV.

A method of inverse dynamics in the classical form requires constructing a mathematical model of ROV in the form of six differential equations. Each equation should match the degree of freedom of ROV and take into account the dynamics of ROV hull and its propulsion devices. Such a problem is extremely difficult due to the substantial non-linearities in the mathematical model of ROV, complicated relationships between its parameters and the existence of elements in the model that are assigned by the means of approximation. These elements include hydrodynamic coefficients of hull and screw propellers of ROV.

In this regard, we shall apply decomposition of the reference model in accordance with [22]. This will allow us to invert a mathematical model of ROV by its components, as well as apply a matrix form of the basic law of ROV dynamics.

Let us form a reference submodel of change in the positional kinematic ROV parameters in the form of a differential equation of first order. Based on it, we shall determine a matrix of the desired values of the ROV motion velocity relative to ground in the projections along RCS axes \(R_{G}^{\parallel}\):

\[
R_{G} = [\text{diag}\{\tau_g\}]^T (R - R_{G}) + R_{G},
\]

\[
\tau_g = [\tau_x, \tau_y, \tau_z, \tau_{\Omega_x}, \tau_{\Omega_y}, \tau_{\Omega_z}],
\]

\[
R_{G} = [\hat{x}_G, \hat{y}_G, \hat{z}_G, \hat{\Omega}_x, \hat{\Omega}_y, \hat{\Omega}_z]^T,
\]

where \(R_{G}\) is the matrix of time constants of reference submodel, \(R_{G}\\) is the matrix of the given positional kinematic ROV parameters; a dot marks time derivative.

Let us invert the second equation of system (3) and receive a matrix of the desired values of the ROV motion velocity relative to water in the projections along the RCS axes \(V_{d}\):

\[
V_{d} = K_{d}^{\parallel} R_{G} - V_{s}.
\]

We shall form a reference submodel of change in speed kinematic ROV parameters in the form of a differential equation of first order. Based on which, we shall obtain a matrix of the desired values of ROV accelerations in the projections along the RCS axes \(V_{d}^{\parallel}\):

\[
\tau_v = [\tau_u, \tau_v, \tau_w, \tau_{\Omega_x}, \tau_{\Omega_y}, \tau_{\Omega_z}],
\]

\[
V_{d} = [\text{diag}\{\tau_v\}]^T (V_{d} - V) + V_{d},
\]

where \(\tau_v\) is the matrix of time constants in the reference submodel, \(V_{d}\) is the matrix of given accelerations of ROV kinematic parameters.

Matrix \(V_{d}\) cannot be obtained by direct differentiation of matrix \(R_{G}\), because their elements are presented relative to different coordinate systems. It is obtained based on equation (10):

\[
V_{s} = \frac{d}{dt} (K_{d}^{\parallel} R_{G} - V_{s}).
\]

where \(K_{d}^{\parallel}\) is the coupling matrix whose elements are calculated in accordance with (4) based on the given values of Euler angles from matrix \(R_{G}\).

Current speed \(V_{s}\) may change over time. But in comparison with the dynamics of ROV, this change will occur slowly. Therefore, we can assume that \(V_{s} = 0\).
Let us invert the first equation of system (3) and obtain a matrix of the desirable resultant forces and moments of ROV in the projections along the RCS axes $T_i$:

$$T_i = I \dot{V}_i + KIV.$$  \hfill (12)

Desired forces and moments of propulsion devices, $T_{pd}$, are determined based on (5):

$$T_{pd} = T_i - T_a - T_b - T_s - T_u - T_v.$$  \hfill (13)

where $T_{pd}$ are the undefined forces and moments that act on ROV. Matrices $T_{pd}$ are calculated based on the appropriate direct models of the ROV components.

Matrix $T_{unc}$ introduces a correction to the inaccuracies of ROV mathematical model and it is an element of the uncertainty compensation contour. The essence of compensation contour is as follows. Based on the matrix of real ROV accelerations $\ddot{V}$ and its speed kinematic parameters $V$ from (12), we determined the matrix of resultant forces and moments $T_i$:

$$T_i = I \dot{V}_i + KIV.$$  \hfill (14)

$T_i$ matrix demonstrates the result of which forces and moments contributed to obtaining acceleration $V_i$. Next, based on (13) and matrices $T_{pd}$, we determine matrix $T_{unc}$:

$$T_{unc} = T_i - T_a - T_b - T_s - T_u - T_v.$$  \hfill (15)

Matrices $T_{pd}$ are taken from the previous step of regulator's operation as a result of calculation in accordance with mathematical models of the ROV elements. In addition, in equation (13), it is recommended to apply the elements of matrix $T_{unc}$ obtained as a result of filtering the matrix $T_{unc}$. This is necessary in order to minimize the impact of inaccuracies of sensors or numerators of SAC. As a filter, it is possible to use aperiodic link of first order with a single coefficient of amplification. Thus, SAC is provided with adaptive properties.

At the next step, it is necessary to allocate the elements of matrix $T_{pd}$ among the propulsion devices. The sum of the forces and moments of each propulsion device must form matrix $T_{pd}$. Such problem for ROV with the accepted configuration of propulsion complex (Table 1) is solved as follows.

Propulsion devices are pairwise oriented along the RCS axes (Table 1). Therefore, to ensure the desired propulsion moment $M$, the forces of propulsion pairs should differ by the following magnitudes:

$$\Delta F_{ir} = M_{ir} / D_{ir} ; D_{ir} = |x_i - z_i|$$

$$\Delta F_{ir} = M_{ir} / D_{ir} ; D_{ir} = |x_i - z_i|$$

where $\Delta F_{ir}$, $D_{ir}$ are, respectively, the difference in forces and distance between the marching propellers along the $z$ axis; $\Delta F_{ir}$, $D_{ir}$ are, respectively, the difference in forces and distance between the lag propellers along the $y$ axis.

Then the desired forces, which have to be produced by the propulsion devices, will be determined from the following equations:

$$F_{sd} = \{F_{s1d}, F_{s2d}, F_{s3d}, F_{s4d}, F_{s5d}, F_{s6d}\};$$

$$F_{sd} = \{F_{sd} + \Delta F_{ir}, F_{sd} + \Delta F_{ir}, F_{sd} + \Delta F_{ir}\};$$

where $F_{sd}$ is the set of desired forces of propulsion devices.

Digital indexes 1, 2...6 correspond to Table 1. Equations (14) and (15) essentially represent an inverse model of propulsion complex consisting of six propulsion devices. They are derived based on equations (6) according to the configuration of propulsion complex (Table 1).

Based on the established forces $F_{sd(i)}$, using a method of inverse dynamics, we shall determine controlling influences. For this purpose, based on (7), we obtain the inverse model of a screw propeller:

$$\omega_{sd(i)} = f(F_{sd(i)}, V_{sd(i)}).$$

where $F_{sd(i)}$ is the desired magnitude of propulsion device force obtained from (15). We shall form a reference submodel for the elimination of errors in the speeds of screw propeller rotation and obtain, based on it, desired acceleration $\omega_{sd(i)}$:

$$\omega_{sd(i)} = \omega_{s(i)} - \omega_{s(i)} \tau_{s(i)} = \tau_{s(i)}.$$

where $\tau_{s(i)}$ is the time constant of reference submodel.

A set of time constants $\tau_{s(i)}$ forms a set of time constants of propulsion complex $\tau_s$:

$$\tau_s = \{\tau_{s1}, \tau_{s2}, \tau_{s3}, \tau_{s4}, \tau_{s5}, \tau_{s6}\}.$$

At the next step, an inverse model of electric motor is constructed based on (8). When describing the inverse model of propulsion device, in order to reduce the notation, symbol $i$ in the role of the index shall not apply.

It is known that electrodynamic transient processes that occur in the propulsion device electric motor are significantly faster than the mechanical transient processes. Based on this, we shall assume that current of electric motor changes instantly depending on the controlling voltage. This allows us to obtain controlling influence from (8):

$$k_{i} = \{i_{o} + k_{i} \omega_{i} + Q_{i} + M_{sd(i)}\}$$

$$u = R_{i} + k_{a} \omega_{i}.$$

where $M_{unc}$ is the moment that characterizes the impact of uncertainties on the operation of propulsion device.

Parameter $M_{unc}$ introduces a correction to the inaccuracies of mathematical model of propulsion device and it is an element of the compensation contour. It is determined from model (18) based on actual angular acceleration $\omega_i$ and angular velocity $\omega_i$ of screw propeller. Similar to the compensation contour of indeterminate forces and moments of ROV, parameter $M_{unc}$ is recommended to filter. In this case,
its filtered value $M_{uncf}$ is also possible to obtain at the output of aperiodic link of first order.

The structure of SAC for the ROV multidimensional motion is shown in Fig. 3.

Fig. 3. Structure of the system of automated control of remotely operated underwater vehicle

In Fig. 3, the following designations are adopted:
- $EM_{R,V,m}$ – reference submodels of SAC in accordance with (9), (11) and (17);
- $IM_{R,V,T,P,p,m}$ – inverse models of ROV elements in accordance with (10), (12), (13), (15), (16) and (18);
- $CU_{T,M(i)}$ – compensation units of indeterminate forces and moments of ROV and propulsion devices, respectively;
- $\Omega_s$ – a set of actual angular velocities of screw propellers:

$$\Omega_s = \{\omega_{s1}, \omega_{s2}, \omega_{s3}, \omega_{s4}, \omega_{s5}, \omega_{s6}\}.$$

A set of reference submodels and inverse models of ROV elements creates a control law for the ROV motion by six degrees of freedom.

4.3. Mathematical modeling and identification of SAC shortcomings when managing a multi-dimensional object

Let us consider the SAC operation when managing ROV whose basic parameters are given in Table 2.

Since the umbilical cable is fixed not in the centre of ROV masses, it will create not only disturbing forces but also disturbing moments.

SAC tuning comes down to adjusting its reference models. Because the reference models are linear, and their amplification coefficients are equal to unity, then tuning is reduced to selecting time constants $\tau_r$, $\omega_r$.

Let us choose time constants such as to ensure the exponential character of SAC transient processes:

$$\tau_r = [110.50.50.5] s;$$

$$\tau_v = 0.25\tau_r; \quad \tau_{\omega} = 0.03 s.$$

We shall perform simulation of SAC under the following conditions. At the start of simulation at $t=0$ s, ROV will have the following positional kinematical parameters:

$$\tilde{r} = \{0, -50, 0\} m; \quad \tilde{q} = \{0, 0, 0\}.$$

The problem of control will be stated as follows: ROV must arrive at the point with the following positional kinematical parameters:

$$\tilde{r}_g = \{5, -55, 5\} m;$$

$$\tilde{q}_g = \{15, -45, -45\}.$$

The speed of current will be assigned by vector:

$$\tilde{v}_s = \{-0.5, 0, 0\} m/s.$$

The given angular orientation of ROV is chosen so that the hull of ROV is directed to point $\tilde{r}_g$.

Results of the simulation are shown in Fig. 4.

Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hull shape:</td>
<td></td>
</tr>
<tr>
<td>- length</td>
<td>Three-axis ellipsoid:</td>
</tr>
<tr>
<td>- height</td>
<td>$-1$ m;</td>
</tr>
<tr>
<td>- width</td>
<td>$-0.35$ m;</td>
</tr>
<tr>
<td>- $0.65$ m;</td>
<td></td>
</tr>
<tr>
<td>Buoyancy</td>
<td>Zero</td>
</tr>
<tr>
<td>Weight</td>
<td>120 kg</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.12 m$^3$</td>
</tr>
<tr>
<td>Resultant of force of weight and buoyancy</td>
<td>120 N</td>
</tr>
<tr>
<td>The point of application of resultant of forces of weight and buoyancy</td>
<td>${0, 0.1, 0}$ m</td>
</tr>
<tr>
<td>Propulsion device power</td>
<td>500 W</td>
</tr>
<tr>
<td>Amplification coefficient of propulsion device's driver</td>
<td>310</td>
</tr>
<tr>
<td>Range of permissible controlling influences of propulsion device</td>
<td>$[-1, 1]$ V</td>
</tr>
<tr>
<td>Umbilical cable length</td>
<td>100 m</td>
</tr>
<tr>
<td>The point of fastening an umbilical cable runner on ROV</td>
<td>${-0.25, 0, 0}$ m</td>
</tr>
</tbody>
</table>

The overall transient process lasted for $\approx 16$ s. In this case, almost over the entire transition process, rotating coordinates of ROV were uncontrolled. This is due to the fact that the desired values of forces $F_d$, computed by regulator, were too large. As a result, SAC operated under saturation mode. Because of this, the corresponding magnitudes $\Delta F_{w,v,rr,lt}$, which had to create the desired moments $M_d$, had no effect.
4. Fundamentals of the method for scaling errors in maintaining manageability at multi-dimensional automated control

Any technical object has certain limitations, due to which it cannot provide working out of controlling influences from the regulator. Simulating SAC for the multidimensional ROV motion demonstrates that the mode of saturation occurs because of too large controlling influences that are received at the output of regulator. In this regard, the main condition to maintain manageability by all degrees of freedom of ROV is to drive SAC out of the saturation mode.

An analysis of simulation results reveals that at low values of errors in the controlled magnitudes, SAC does not enter the mode of saturation. An increase in one or several errors leads to the saturation of one or more SAC contours. We shall use this property of ROV SAC as the basis for maintaining ROV manageability by all degrees of freedom. In this regard, the main idea of the method for maintaining manageability is scaling errors in control by the criterion of saturation of the SAC contours.

We shall introduce designations for the errors in control and their scaled values:

\[ e_j = g_j - r_j, \quad j = 1, 2, \ldots, J; \]
\[ e_j^* = e_j \kappa_j, \quad \kappa_j \in [0, 1], \]

where \( e_j \) is the error of control in the \( j \)-th controlled magnitude; \( g_j \) is the given value of the \( j \)-th controlled magnitude; \( r_j \) is the actual value of the \( j \)-th controlled magnitude; \( \kappa_j \) is the scale factor of the \( j \)-th error in control; \( e_j^* \) is the scaled value of the \( j \)-th error in control; \( J \) is the total number of controlled magnitudes.

It is necessary to find such maximum values of coefficients \( \kappa_j \), at which none of the SAC contours enters the saturation mode. When managing ROV, a mode of saturation occurs due to the limitations in its controlling influences. Then \( \kappa_j \) are chosen so that they satisfy the following expression:

\[ \forall i = 1, 2, \ldots, I: u_{\min(i)} \leq u_i = \varphi_i (E_{\text{sat}}) \leq u_{\max(i)}, \]
\[ E_{\text{sat}} = \{ e_1, e_2, \ldots, e_j, \ldots, e_J \}, \quad j = 1, 2, \ldots, J, \]

where \( u_i \) is the controlling influence; \( I \) is the number of controlling influences; \( u_{\min(i)} \) and \( u_{\max(i)} \) are, respectively, the lower and upper boundary of controlling influence; \( E_{\text{sat}} \) is the set of scaled errors in control; \( \varphi_i \) is the function that implements control law, the enumeration of its arguments, in addition to \( E_{\text{sat}} \), may include other variables necessary for its work.

At each control cycle, parameters \( \kappa_j \) should be refined. In order not to change the structure of regulator, we propose submitting to its inputs, instead of the given values of controlled parameters \( g_j \), their scaled values \( \gamma_j \):

\[ \gamma_j = r_j + e_j. \]

It should be noted that derivatives of errors, as before, are taken not from scaled but from the actual errors \( e_j \). If control contours contain the integral of the error, then it is taken from the scaled errors \( e_j \).
Information and controlling systems

Object of control can be in a state when no combination \( \kappa_i \) satisfies requirement (19). In this case, it is proposed to minimize functional \( G \):

\[
G(K) = \sum_{i} \beta_i \to \min, \quad i = 1, 2, \ldots, I;
\]

\[
K = \{ \kappa_1, \kappa_2, \ldots, \kappa_I, \kappa_J \}, \quad j = 1, 2, \ldots, J;
\]

\[
\beta_i = \begin{cases} u_i - u_{\text{mm}(i)} & \text{if } u_i > u_{\text{mm}(i)}; \\ u_{\text{mm}(i)} - u_i & \text{if } u_i < u_{\text{mm}(i)}; \\ 0, & \text{else}, \end{cases}
\]

where \( K \) is the set of coefficients \( \kappa_i, \beta_i \) is the parameter that describes to what extent the \( i \)-th controlling influence passed beyond permissible limit.

This will allow SAC to maintain “the least saturated” regime.

If the minimum of functional (21) matches several \( K \) variants, then among them we select the one whose sum of elements \( S \) is the largest:

\[
S_{\text{max}}(K) = \sum_{j=1}^{J} \kappa_j.
\]

Determining coefficients of \( \kappa_i \) presents a complicated variational problem. Its simplification is possible in the following way. We shall introduce a scaling parameter \( \delta \) instead of scaling coefficient \( \kappa_i \), which must satisfy the following requirement:

\[
0 \leq \delta_{\text{mm}(j)} \leq \delta_j \leq \delta_{\text{opt}(j)}; \quad j = 1, 2, \ldots, J;
\]

\[
\delta_{\text{opt}(j)} = \begin{cases} \delta_{\text{mm}(j)}, & \text{if } |e_j| \geq \delta_{\text{mm}(j)}; \\ \delta_{\text{mm}(j)}, & \text{if } |e_j| = \delta_{\text{mm}(j)}; \\ |e_j|, & \text{else}, \end{cases}
\]

where \( \delta_{\text{mm}(j)}, \delta_{\text{max}(j)} \) are, respectively, the lower and upper assigned constant limits of absolute value of scaled error, \( \delta_{\text{opt}(j)} \) is the floating upper limit of absolute value of scaled error.

The choice of parameters \( \delta_{\text{min}(j)} \) and \( \delta_{\text{max}(j)} \) is based on the general evaluation of properties of the control object and its regulator. Parameter \( \delta_{\text{max}(j)} \) is chosen large enough so that at \( |e_j| > \delta_{\text{max}(j)} \) at least one of the SAC contours enters the saturation mode. Parameter \( \delta_{\text{min}(j)} \) is chosen sufficiently small so that at \( |e_j| < \delta_{\text{min}(j)} \) none of the contours of SAC is under the saturation mode. This will allow us to narrow the range of search for the parameters of scaling down to range \( [\delta_{\text{min}(j)}, \delta_{\text{opt}(j)}] \).

Scaled control error is determined as follows:

\[
\varepsilon_j = \begin{cases} e_j, & \text{if } |e_j| \leq \delta_j; \\ \text{sign}(e_j) \delta_j, & \text{else}. \end{cases}
\]

The scaled given value of controlled parameter \( \gamma_j \) will be determined, as before, based on (20).

At the next step, we shall introduce coefficient \( k_j \) for scaling in the range \( [\delta_{\text{mm}(j)}, \delta_{\text{opt}(j)}] \):

\[
\delta_j = k_j (\delta_{\text{opt}(j)} - \delta_{\text{mm}(j)}) + \delta_{\text{mm}(j)}, \quad k_j \in [0, 1].
\]

Now, if we accept

\[ k = k_1 = k_2 = \ldots = k_J, \]

then variational problem (21) is simplified to the task of minimizing the functional with one argument \( k \):

\[
G(k) = \sum_{i} \beta_i \to \min.
\]

In this case, SAC will adjust to the weakest degree of freedom. This method of scaling errors somewhat decreases the SAC effectiveness because strong control contours may not depend on the parameters of the weak ones. And reducing the coefficient will equally affect all the contours of SAC.

In this regard, it is proposed to combine controlled parameters into groups. And to form one scaling coefficient \( k_i \) for each group. This technique will level a group's controlled parameters with the weakest one in the group, rather than in the entire system:

\[
G(K_k) = \sum_{i} \beta_i \to \min;
\]

\[
K_k = \{ k_{g_1}, k_{g_2}, \ldots, k_{g(n)} \}, \quad n = 1, 2, \ldots, N; \quad N \leq J.
\]

If the number of groups \( N \) is equal to the number of controlled magnitudes \( J \), then the variational problem (21) is not different from (26), with the exception of constraints (22), taken for (26).

Determining coefficients for \( K_k \) implies the application of iterative methods, which may significantly increase necessary requirements for the computing resources of SAC. Instead of using iterative search algorithms, it is proposed to use a tuning contour for each \( k_{g(n)} \). One can implement it in the form of aperiodic link with a coefficient of amplification equal to 1:

\[
t_{k_{g(n)}} = h - k_{g(n)};
\]

\[
h = \begin{cases} 0, & \text{if } s_n = \text{true}; \\ 1, & \text{else}, \end{cases}
\]

where \( t_0 \) is the time constant of link; \( h \) is the link input, which determines the motion direction of \( k_{g(n)}, S_n \) is the mode saturation flag for the \( n \)-th SAC contour.

The work of such contours will keep SAC on the verge of saturation. Parameter \( S_n \) does not necessarily have to depend on the saturation of elements of the \( n \)-th contour only. It can be received as a result of logical operations over the states of any elements that can enter the mode of saturation.

Configuring contours (27) will be operational if for SAC as whole the increase in errors brings it closer to the saturation mode. And vice versa, if reducing the errors in general brings it out of the mode of saturation.

Application of configuring contour allows us to redistribute the strong and weak degrees of freedom or their group. For this purpose, we shall introduce a dependence of link time constants (27) on the state of flags of saturation \( S_n \):

\[
t_{k_{g(n)}} = \begin{cases} \tau_{\text{down}(n)}, & \text{if } S_n = \text{true}; \\ \tau_{\text{up}(n)}, & \text{else}, \end{cases}
\]
where $\tau_{\text{down}(n)}$ and $\tau_{\text{up}(n)}$ are the constant values that are selected from the conditions of strengthening or weakening of the $n$-th group of controlled parameters.

If a group of controlled parameters $n$ should be strengthened as a whole, the condition must be satisfied:

$$\tau_{\text{down}(n)} > \tau_{\text{up}(n)}.$$  

Conversely, for weakening the group $n$ as a whole, the condition must be satisfied:

$$\tau_{\text{down}(n)} < \tau_{\text{up}(n)}.$$  

If a group of controlled parameters $n=p$ needs to be strengthened relative to the group of controlled parameters $n=q$, then the conditions must be satisfied:

$$\tau_{\text{down}(p)} > \tau_{\text{down}(q)}; \quad \tau_{\text{up}(p)} < \tau_{\text{up}(q)}.$$  

The indicated conditions form the basis for relative selection of time constants of configuring contours.

When choosing absolute values $\tau_{\text{down}(n)}$ and $\tau_{\text{up}(n)}$, it is necessary to take into account the SAC parameters and peculiarities of its actual implementation.

Configuring contours form a unit for maintaining manageability, which is switched in the gap of given influences $g_j$. At the output, magnitudes $\gamma_j$ are formed, which essentially are the scaled values of given influences.

4.5. Designing a unit for maintaining manageability of ROV SAC

The motion of ROV is divided into translational and rotating. That is why we shall divide controlled parameters into the group of translational ones and the group of rotating ones with the respective coefficients $k_r$ and $k_q$. To denote the group of translational controlled parameters, we shall use index $r$, for the rotating ones – $q$.

None of the contours of SAC should enter the mode of saturation, because operation of every particular propulsion device can affect the motion of ROV as a whole. That is why we shall use one flag of saturation $s$ for both configuring contours:

$$s = \begin{cases} \text{true, if } \exists n_i \mid |u_i| > u_{\text{in}}, i = 1, 2, \ldots, I; \\ \text{false, else,} \end{cases}$$

where $u_{\text{in}}=1$ is the boundary value of controlling influence.

Studies of the inverse regulator of ROV demonstrated that the rotating degrees of freedom are weak relative to the translational ones. But the quality of translational motion of ROV depends on its keeping the given angular orientation. Therefore, for quality control, the rotating degrees of freedom should be strengthened relative to the translational ones. In this regard, time constants of configuring contours are selected in the following way:

$$\tau_r = \begin{cases} 0.1 \text{ s, if } s = \text{true}; \\ 0.5 \text{ s, else}; \end{cases} \quad \tau_q = \begin{cases} 0.5 \text{ s, if } s = \text{true}; \\ 0.1 \text{ s, else}; \end{cases}$$

Let us assign the lower and upper bounds for the translational and rotating groups:

$$\delta_{\text{min}(r)} = 0.1 \text{ m} ; \quad \delta_{\text{max}(r)} = 5 \text{ m} ; \quad \delta_{\text{min}(q)} = 1^\circ ; \quad \delta_{\text{max}(q)} = 45^\circ .$$

Next, the system is to calculate the floating upper bounds for the errors in each group and own scaled errors in accordance with (23)–(25). Expressions (23)–(25) can be rewritten in the vector form:

$$\tilde{e}_{\text{up}(r)} = \begin{cases} \delta_{\text{up}(r)}, \text{ if } |\tilde{e}_{\text{r}}| \leq \delta_{\text{up}(r)}, \\ \tilde{e}_{\text{r}}, \text{ else,} \end{cases} \quad \tilde{e}_{\text{up}(q)} = \begin{cases} \delta_{\text{up}(q)}, \text{ if } |\tilde{e}_{\text{q}}| \leq \delta_{\text{up}(q)}, \\ \tilde{e}_{\text{q}}, \text{ else,} \end{cases}$$

$$\delta_{\text{r}} = k_r (\tilde{e}_{\text{r}} - \delta_{\text{min}(r)}) + \delta_{\text{min}(r)}; \quad \delta_{\text{q}} = k_q (\tilde{e}_{\text{q}} - \delta_{\text{min}(q)}) + \delta_{\text{min}(q)}; \quad \tilde{e}_{\text{r}} = \frac{\tilde{e}_{\text{r}}}{\text{ort}(\tilde{e}_{\text{r}})} \delta_{\text{r}}, \text{ else.}$$

Scalable control task $R_\gamma$ will be determined based on (20) in the matrix form:

$$R_\gamma = [x, y, z, \theta, \phi, \psi, \gamma]_T = R + [\tilde{e}, \tilde{e}]_T.$$

The structure of ROV SAC takes the form, shown in Fig. 5.

SCALABLE CONTROL TASK $R_\gamma$ will be determined based on (20) in the matrix form:

$$R_\gamma = [x, y, z, \theta, \phi, \psi, \gamma]_T = R + [\tilde{e}, \tilde{e}]_T.$$
ability. Parameters of ROV, SAC and conditions of simulation coincide with those from chapter 4.3.

Dynamics of transient processes are shown in Fig. 6.

During the first 3 seconds, ROV takes on the given angular position \( \theta \) and its orientation in space coincides with the given direction of motion. In this case, hydrodynamic resistance decreases and total duration of the transition process does not exceed 8 s. In this case, there is no overshooting along both translational and rotating coordinates.

6. Discussion of method of scaling errors for maintaining manageability

The problem of “weak” and “strong” degrees of freedom manifests itself when managing not only ROV, but other multidimensional objects as well. Application of high-precision automated control methods does not guarantee quality in the work of regulators under the modes of saturation. Under multidimensional control and with the impossibility to separate motions, there may occur a loss of manageability by the weak degrees of freedom.

The proposed method of maintaining manageability brings SAC out of the saturation mode by way of scaling errors. It should be noted that the method does not guarantee absolute prevention of SAC entering the mode of saturation. However, if SAC entered saturation mode, it means that the capacity of propulsion devices is not enough to enable correct control when moving under the assigned conditions.

The main disadvantage of the devised method for maintaining manageability is the need for iterative search for such scales of errors at which SAC does not enter the mode of saturation. The larger the dimensionality of the object, the more complex is the search for solution.

Present work proposes a variant to realize the method with one required parameter. In this case, however, all the contours of control will be weakened and the SAC operation may slow down.

A compromise variant is to divide controlled parameters into groups. This will make it possible to reduce the number of required parameters of scaling and not to slow down the work of SAC as a whole. The application of aperiodic link instead of iterative search for scaling coefficients allows lowering demands for computational resources.

The proposed method for maintaining manageability cannot be applied together with the regulators of ROV, synthesized by the control method under sliding modes. This is due to the fact that such regulators operate under saturation regimes by default.

Applying the proposed method is possible in SAC whose controlling influence decreases with a decrease in control error. It is not difficult to demonstrate that such a property is characteristic not only for the inverse regulator, synthesized in present work, but for the PID-like regulators as well.

Stability of ROV SAC is provided by the choice of time constants of reference model. Unit for maintaining manageability, which contains contours for scaling an error, does not affect the stability of SAC as a whole. Rather, the effect of the unit for maintaining manageability on the operation of regulator ultimately comes down to the unsigned modification of control problem, which in this case does not, by definition, affect the stability of SAC. Therefore, we can conclude that SAC remains stable if it was stable before the introduction of the unit for maintaining manageability in its structure.

7. Conclusions

1. We obtained a matrix notation of the basic law of dynamics of marine movable object as a solid body that moves in the flow of liquid. Own masses and moments of inertia of the body and the added masses and moments of inertia of the fluid are brought to a separate matrix. This makes it possible to apply the resulting equation without structural changes to study the dynamics of spatial motion of remotely operated underwater vehicles with different parameters. The equation is used in direct form to simulate the motion of underwater vehicle and in inverse form for the synthesis of automated control system of its spatial motion.

2. An inverse regulator is synthesized for remotely operated underwater vehicle with six degrees of freedom based on the method of inverse dynamics and decomposition of reference model as the basis for the synthesis of automated control system over its spatial motion.

3. Using a method of computer simulation, we studied the work of inverse regulator of remotely operated underwater vehicle and demonstrated a loss of manageability by the weak rotating degrees of freedom. The need to devise
a method for maintaining manageability of a multi-dimensional system of automated control of underwater vehicle is substantiated.

4. We formulated the fundamentals of the method for maintaining manageability at automated control of a multi-dimensional object, the essence of which is the bringing the contours of its automated control system out of the modes of saturation by scaling control errors.

5. A system of automated control over spatial motion of remotely operated underwater vehicle is designed based on the synthesized inverse regulator and unit for maintaining manageability. It provides controlled motion of underwater vehicle by six degrees of freedom without losing manageability.

6. Using a method of computer simulation, we studied the developed system of automated control over spatial motion of remotely operated underwater vehicle. Results of the study demonstrated that the unit for maintaining manageability provides the operation of automated control system on the verge of saturation in its contours. This enables the motion of underwater vehicle by six degrees of freedom without losing manageability.

References


