As a rule, given the low performance of the alarm MIS and extremely complex functions of the crew decision-making, the fire alarm MIS failures are still associated with emergency and catastrophic situations. This is due to the fact that existing MIS have a number of shortcomings:

– low reliability;
– very high false alarm rate (up to 70%);
– lack of special devices for recognizing dangerous flight situations (DFS) at first occurrence;
– significant uncertainties in the crew decision making.

Certainly, the alarm MIS performance directly affects the flight safety. Therefore, improving the reliability of identifying dangerous operating modes of MIS is an urgent task and can be done by introducing information redundancy and creating new structures for failure isolation.

2. Literature review and problem statement

The analysis of structures of management information systems (MIS) of the aircraft (AC) fire alarm [1, 2] shows that increasing the number of parallel redundant sensors is an effective way to improve probabilistic characteristics of the system. However, the increase in the number of sensors requires certain technical and economic resources. It is, therefore, advisable to apply a serial method of information redundancy of MIS. Information redundancy (IR) is a way to ensure the MIS efficiency due to the introduction of redundancy based on information characteristics of these systems.

Serial information redundancy (SIR) is a way to ensure the MIS efficiency, in which the same information source (IS) is requested n times, and a decision is made on the basis of “m of n”, confirmation of the presence of a controlled phenomenon.

The MIS efficiency depends greatly on the reliability of the information, which is the basis for the crew decision making. Existing alarm MIS, primary information sensors have a fuzzy threshold. As a result, information from the real sensor always comes with a certain level of reliability.
the basis for the relevant decisions. Therefore, it is necessary to take appropriate measures for improving the information reliability in MIS. The solution to the problem of data validation during transmission and handling in process control systems is proposed in [3], which considers the principles and methods of using statistical redundancy data for solving problems related to information reliability control based on the minimum mean-square error criterion under various probability distribution laws of controlled features.

The reliability monitoring scheme for active fault-tolerant control systems using the stochastic simulation method is proposed in [4]. In this paper, the previous fault detection and isolation data are used for updating the transition characteristics in the reliability model.

The research of quantitative error estimates of the lambda method in predicting the mean time to first failure and a failure rate of technical systems depending on the reliability of hardware components, system complexity and operation duration is given in [5].

In [6], the method for determining the distribution of the limiting state of time by exceeding the diagnostic parameter, which determines the accuracy of maintaining the zero state is proposed. The deviation change results from destructive processes during operation. For increase rate deviation assessment, in the probabilistic sense, the Fokker-Planck equation for determining the residual life of onboard devices has been used.

The definitions of failure components, which can lead to degradation of the entire production system are investigated in [7]. In this paper, the probabilistic model is based on the Erlang distribution, and the failure rate of components is subject to an exponential law. The estimation of the safe fire detection time required to start fire suppression, based on the method for estimating the time of fire detection by temperature detectors, temperature rise rate, smoke-type light-scattering detectors is given in [8].

The paper [9] analyzes alarm systems for identifying the causes of false alarms and discusses several solutions to reduce their rate.

The paper [10] proposes the fault diagnosis technique, which is based on the fault detection and diagnosis procedure. This technique is based on the use of adaptive filters, developed by means of a nonlinear geometric approach. This improves the system noise immunity.

The issues of increasing the efficiency of using operational information for MIS reliability monitoring are widely covered in scientific publications. However, in fact, most methods are limited to the use of statistical methods of information processing. The issues of information redundancy are considered as problems of information or software protection in a computing part of MIS, and the choice of a method depends on a large number of factors.

The major factor necessary for decision making in DFS is obtaining reliable information from sensors and IS about flight conditions. Therefore, the problem of obtaining reliable information is proposed to be solved by means of serial information redundancy as an effective way of dealing with such phenomena as fire non-detection and false alarm.

Efficiency and performance of MIS depend significantly on the reliability of the information coming to the MIS calculator input from various meters, which monitor the process status.

Real sensors have the ultimate accuracy of presenting the information they monitor. At the same time, the accuracy and reliability of information are determined by both design features and technical reliability of sensors and, as a rule, do not satisfy or poorly meet the requirements.

It is known that both the accuracy and reliability of the monitored information can be significantly improved by statistical processing. If information is fed to the MIS calculators simultaneously from several parallel-connected sensors, such information input method is called parallel information redundancy [2, 11].

If information is fed to the MIS calculators simultaneously from the same sensor serially at a given rate, such information input method is called serial information redundancy. Both methods basically make it possible to significantly improve the accuracy and reliability of the monitored information, which comes from low-quality and technically unreliable sensors.

As noted in [2, 11], parallel information redundancy of sensors requires certain technical and economic resources. Therefore, it seems promising to use serial information redundancy as an effective way to improve the accuracy and reliability of the information received by a crew for decision making in DFS.

3. The aim and objectives

The research aim is to develop a mathematical model for connecting sensors based on the Bayes’ principle to ensure effective performance of the fire alarm MIS by increasing the accuracy of recognition of a controlled DFS.

To achieve this aim, the problem of development of a mathematical model of serial information redundancy with the possibility of its implementation using microprocessor equipment is solved.

4. Development of a mathematical model of serial information redundancy

If the same system of N sensors is periodically requested at a certain time interval and the given information is stored, then, according to the Bayes’ theorem [2, 12], it is possible to substantially increase the probability of correct detection p1 of such a system for the given number N of sensors or to maintain the same by reducing the number N of sensors.

To apply the Bayes’ formula, we introduce two working hypotheses: H1 (fire) and H2 (no fire). Let the probability of the hypothesis H1 be

\[ p(H_1) = \alpha. \]

and the probability of the hypothesis H2, respectively, be

\[ p(H_2) = 1 - \alpha. \]

Let A be the event when a sensor emits the “Fire” signal. Then the probability p(A/H1) is the probability of correct detection, i.e.

\[ p(A/H_1) = \alpha, \]

and the conditional probability p(A/H2) is the probability of “false” alarm, i.e.
According to the Bayes’ theorem, the probability of fire when a sensor emits the fire-alarm signal is determined by the expression:

$$p(A/H_2) = b.$$  

The expression (1) allows determining the probability of fire at the first stage, and $\alpha$ is substituted with $p(H_1/A)$ determined by the formula (1). After simple conversions at the second control stage, we get the probability of fire $p(H_1/A)$ from the following expression:

$$p(H_1/A) = \frac{\alpha - a^2}{\alpha - a^2 + (1 - \alpha)b^2}.$$  

Using the mathematical induction method, it can be shown that the $k$-th serial sensor request, the probability $p(H_1/A)$ is determined by the expression:

$$p(H_1/A)_k = \frac{\alpha - a^2}{\alpha - a^2 + (1 - \alpha)b^2}.$$  

By introducing the variables $\beta = \frac{1 - \alpha}{\alpha}$ and $\gamma = \frac{b}{a}$, we write the expression (3) in the form:

$$p(H_1/A)_k = p_k = \frac{1}{1 + \beta \gamma^k}.$$  

where $p_k$ is the probability of correct fire detection at the $k$-th stage by the Bayes’ method. Let’s analyze the expression (4), for which we find the increment $\Delta k$ of the probability $p_k$ at two adjacent stages:

$$\Delta k = p_{k-1} - p_k = \frac{1}{1 + \beta \gamma^{k-1}}.$$  

After simple conversions, we get the expression:

$$\Delta k = \frac{\beta \gamma^{k-1}(1 - \gamma)}{(1 + \beta \gamma^k)(1 + \beta \gamma^{k-1})}.$$  

The increment $\Delta k$ will be positive if $\gamma < 1$, $a/b > 1$.

To find the functional dependence of $\Delta k$ on $\beta$, we apply the L’Hospital rule and investigate the function $\Delta k(\beta, \gamma)$. After separate differentiation of the numerator and the denominator of the function (5), we get the expression:

$$\Delta k = \frac{\gamma^{k-1}(1 - \gamma)}{\gamma^k(1 + \beta \gamma^{k+1}) + \gamma^{k-1}(1 + \beta \gamma^k)}.$$  

Thus, we can get the following relations:

$$\lim_{\beta \to \infty} \Delta k = \frac{1 - \gamma}{1 + \gamma}.$$  

If $\beta \to 0$, this means that $a > b$. In this case, the increment $\Delta k$ will depend very little on the a priori probability $\alpha$ and depend only on the quality of sensors, i.e. on the ratio $a/b$, where $\gamma < 1$.

It follows from the expression (7) that the increment $\Delta k$ does not depend on the a priori probabilities $\alpha$ and $(1 - \alpha)$ if they are equal, i.e.

$$\alpha = 1 - \alpha$$

and therefore $\beta = 1$.

If we increase $\beta$, i.e. direct $\beta \to \infty$, we get

$$\lim_{\beta \to \infty} \Delta k = \frac{\gamma^{k-1}(1 - \gamma)}{\gamma^k(1 + \beta \gamma^{k+1}) + \gamma^{k-1}(1 + \beta \gamma^k)} = 0.$$  

If $\beta \to \infty$, this means that $\alpha < 1 - \alpha$, and the increment $\Delta k$ will be very small. It essentially depends on the a priori probabilities $\alpha$ and $(1 - \alpha)$, but the increment $\Delta k$ will be positive if a sensor has a high quality, i.e. $a > b$ or $\gamma < 1$.

Serial redundancy of $N$ sensors by the Bayes’ method can be performed using the microprocessor equipment [1, 10] according to Fig. 1.

Fig. 1. The block diagram of serial information redundancy by the Bayes’ method

Fire or no-fire signals from each sensor ($x_1$, $x_2$, ..., $x_n$) come to counters (Fig. 1) and in parallel to the microcontroller, which takes into account the counter readings at the previous stages. The microcontroller calculates the probability of correct detection for each sensor by the expression (4).

The probability can vary from the value $a$ to the value:

$$\frac{1}{1 + \beta \gamma^k}.$$  

The probability of correct detection will be equal to $a$, when the $i$-th counter reading is zero, and the fire-alarm signal from the $i$-th sensor has come.

The probability of correct detection will be

$$\frac{1}{1 + \beta \gamma^k},$$

when the $i$-th counter readings are equal to $k-1$ and another fire-alarm signal came from the $i$-th sensor. If a signal from the $i$-th sensor did not come, the $i$-th counter readings are ignored and the counter is reset to zero, i.e. there was a “false alarm”. Based on the above determination of the probability of correct fire detection by each sensor, the microcontroller calculates the overall probability of correct detection according to the expression:
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\[ p_{ik} = 1 - \prod_{i=1}^{M} (1 - p_{ia}) \]  

(9)

where the probability \( p_{ia} \) varies within

\[ a_i < p_{ia} < \frac{1}{1 + \beta g} \]

\( M \) is the number of sensors emitting fire-alarm signals. The index may vary within \( 0 < M < N \).

If \( M = 0 \), the formula (9) is invalid. The probability \( p_{ik} \) calculated in the microcontroller is displayed as “Fire probability”. If no sensor emits the fire-alarm signal, “Fire probability 0” is displayed.

The issue of implementing the idea of serial information redundancy on microprocessor systems refers to a specific practical engineering development, in particular, the microcontroller software. In this paper, we are talking about the concept of serial information redundancy and implementation of the method of serial switching of detectors, considering a priori information by the Bayes’ principle.

Summarizing previous arguments, we note that the Bayes’ formula can be applied to the interpretation of the following probabilities:

- correct fire detection – \( p_1 \);
- fire non-detection when it is actually present – \( p_2 \);
- “false” alarm – \( p_3 \);
- correct no-fire detection – \( p_4 \).

The latter fire probability is determined on the basis that a sensor is represented as a symmetrical communication channel, and therefore, the probability of fire detection and the probability of no-fire detection are equal. Using these probabilities, it is possible to estimate the system of “memory” redundant sensors, i.e. the system operating by the Bayes’ principle.

Thus, according to the Bayes’ method, the probability \( p_1 \) can be understood as the probability of the hypothesis \( H_1 \), i.e. the probability of an event, when a true event is the “Fire” signal. The probability \( p_2 \) can be treated as the probability of the hypothesis \( H_2 \), when no fire-alarm signal is emitted. The probability \( p_3 \) is the probability of the hypothesis \( H_3 \), when the fire-alarm signal is emitted, and the probability \( p_4 \) is the probability of the hypothesis \( H_4 \) when the fire-alarm signal is not emitted. These probabilities can be determined by the following Bayes’ relations:

A is “Fire” signal; B is “No fire” signal.

\[ p_1 = p(H_1 / A) = \frac{\alpha - a}{\alpha - a + (1 - \alpha) b} \]

\[ p_2 = p(H_2 / B) = \frac{\alpha - d}{\alpha - d + (1 - \alpha) a} \]

\[ p_3 = p(H_3 / A) = \frac{(1 - \alpha) b}{\alpha - a + (1 - \alpha) b} \]

\[ p_4 = p(H_4 / B) = \frac{(1 - \alpha) a}{\alpha - d + (1 - \alpha) a} \]  

(10)

The above formulas (10) represent probabilistic characteristics of one sensor, the information from which came serially \( k \) times. Such a sensor can be represented as a system of redundant \( k \) sensors. From the expression (10), the probability of correct detection can be represented as:

\[ p_1 = p(H_1 / A) = \frac{1}{1 + \beta g} \]  

(11)

where

\[ \beta = \frac{1 - \alpha}{\alpha} \]

and

\[ \gamma = \frac{b}{d} \]

The factor \( \gamma \) estimates the quality of a sensor: the smaller the factor \( \gamma \), the better the sensor quality.

Using the expression (11), we estimate an increment of the probability \( p_1 \) in two serial stages of verification of events. Such an increment can be easily determined by the ratio of the probabilities \( p_{k-1} \), and \( p_{k-1} \), obtained in the \( k-1 \) and \( k \)-th registration of the fire-alarm signal:

\[ \frac{p_{k-1} \gamma}{p_{k-1}} \geq 1. \]  

(12)

The solution of the inequality (12) is the condition \( \gamma < 1 \), from which it can be concluded that an increase in the Bayes’ probability \( p_1 \sim p(H_1 / A) \) does not depend on the ratio of a priori probabilities \( \beta \), but depends only on the sensor quality \( a>b, (\gamma<1) \).

Thus, we obtain a fundamentally important conclusion that extends the potential of the Bayes’ method, since it was usually assumed that a posteriori probabilities depend on a priori probabilities, and the latter are rather difficult to determine. Below, using the nomogram constructed according to the formula (11), it will be shown that, with an increase in the number of experiments, a posteriori probabilities are less dependent on a priori probabilities. The probabilities \( p_2, p_3, p_4 \) can be written using the following expressions:

\[ p_2 = p(H_2 / B) = \frac{1}{1 + \beta g} \]

\[ p_3 = p(H_3 / A) = \frac{1}{1 + \beta g} \]

\[ p_4 = p(H_4 / B) = \frac{1}{1 + \beta g} \]  

(13)

where \( \eta = \frac{a}{d} \). The factor \( \eta \) also determines the sensor quality.

As an example, we show the dependence of the a priori probability on the sensor quality \( \gamma \) with the given values of a posteriori probability \( p_4 \) and the number of repeated serial requests \( k \). We present the estimated dependences obtained on the basis of the expression (11):

\[ \beta = \frac{1 - p_2}{p_1 \gamma^k} \]  

(14)

Since

\[ \beta = \frac{1 - \alpha}{\alpha} \]

we get the expression for the a priori probability in the form:

\[ \alpha = \frac{1}{1 + \beta} \]  

(15)

By substituting the equation (15) into the equation (14), we get the following formula:
The formula (16) is a polyparametric mathematical model of the dependence of the a priori probability α on the sensor quality γ with the given values of a posteriori probability p_i and the number of repeated serial requests k.

\[ \alpha = \frac{p_i \cdot \gamma^k}{1 - p_i (1 - \gamma^k)} \]  

(16)

The results of the analysis of the mathematical model and graphical nomograms

Let’s analyze the obtained mathematical model using the Mathcad mathematical package (USA). The values of α, calculated by the formula (16), are given in Table 1. For the indicated initial data, graphs of the functional dependence \( \alpha = f(P, \gamma, k) \) are plotted (Fig. 2, 3).

Based on the data presented in Table 1, we get the nomogram (Fig. 1), which reflects the dependence of change in the probability characteristics of fire detection in the aircraft engine on the number of repeated serial requests k. Thus, it is seen that an increase in the number of serial repeated requests k reduces the influence of the a priori probability α on the accuracy of event detection when using a poor-quality sensor.

To compare the performance of an information system for different k, the efficiency factor can be represented in the following form:

\[ E = \frac{k^a}{k_i} \]  

(17)

where \( n > i \).

The dependence of the a priori probability on the sensor quality γ with the given values of α and k.

For example, when γ=0.3, the efficiency factor with k=1, P1=0.99 is \( E = 0.064/0.964 = 0.064 \), when γ=0.5, the efficiency factor for k=1, k=6, P1=0.99 is \( E = 0.59/0.98 = 0.602 \), when γ=0.8 and k=1, k=6, P1=0.99, the efficiency factor is \( E = 0.96/0.99 = 0.97 \).

If k and γ are increased simultaneously, we can see that even with a low-quality sensor there is a high fire detection probability. For comparison, according to the efficiency factor for γ=0.3 and γ=0.8 for the same number k, the probability of correct fire detection increases about 15 times. This analysis gives grounds to confirm the effectiveness and usefulness of the proposed solutions.
To analyze the dependence of a priori and a posteriori information on the number of repeated serial requests \( k \) on the basis of the mathematical model, we get the following nomogram for different probabilities of correct detection of fire on board the aircraft, as shown in Fig. 3.

The analysis of mathematical and graphical dependencies allows drawing the following conclusions:

1. With a high quality of an information source \( \gamma < 1 \), the required high a posteriori probability \( p_1 \) is achieved even with a small a priori probability \( \alpha \).
2. An increase in the number \( k \) of serial favorable messages provides the required a posteriori probability \( p_1 = p(H_1|A) \) even with a small a priori probability \( \alpha \) and with a poor quality of an information source estimated by an increase in the factor \( \gamma \).
3. If \( \gamma = 1 \) (extremely low-quality source of information), a posteriori probability is always equal to the a priori probability, \( p_1 = \alpha \), regardless of the number \( k \), i.e. such a system of serial monitoring gives no information.

6. Discussion of the results of the AC fire alarm reliability research

When applying serial information redundancy, attention should be paid to the following conditions:

1. An IS, requested serially over time shall have high technical reliability, determined by the probability \( P \) of failure-free operation.
2. It is necessary to take into account the impact of the a priori probability \( \alpha \) of a controlled event.
3. In order for controlled random events to be statistically independent in time, an IS shall be requested at intervals that exceed the time of correlation between two random and time-related events.
4. The duration of information retrieval from the same IS is limited by the time of its aging.

The latter two conditions limit the number of repeated serial requests \( k \) of the same IS. The number of IS requests during the permissible information aging time depends also on the permissible request rate, determined by the time of correlation of random noise in an IS in the measurement of one or another controlled parameter. The longer the correlation time, the less frequent possible requests of information sources.

If an IS is requested cyclically at time intervals exceeding the time of correlation of its random noise, the given requests in an IS will be virtually uncorrelated with each other. Thus, the number of serial requests of IS is essentially limited by two factors: the permissible information aging time and the time of correlation of random noise in an IS.

The time of correlation of random noise, transient failures and faults determines the minimum time interval for serial data retrieval.

The information aging time in MIS with serial redundancy limits the number \( k \) of serial requests and is a random value. The aging time can be estimated only in specific conditions. Apparently, in most cases, the aging time has a normal distribution law [12], since various conditions and factors affect the aging time.

Assuming that the aging time has a normal distribution law, it is possible to carry out statistical measurements of its parameters, namely average value and variance and consider them when estimating permissible values of serial requests \( k \).

In alarm MIS, it can be assumed that the aging time is subject to an exponential distribution, since emergencies are rare events and easily described by the Poisson’s law [12]. In this case, time intervals between alarms will be random and subject to an exponential distribution. The final number of such random time intervals, in fact, determines the information aging time. After all, in emergency and dangerous flight conditions, one cannot hesitate, it is necessary to take urgent but correct measures, and this will require several repeated messages. Therefore, it is needed to analyze the influence of the time of correlation of random noise and information aging on the accuracy and reliability for making extremely difficult decisions.

7. Conclusions

The developed mathematical model shows that the information reliability determined by an increase in the probability of correct fire detection \( p_1 \) and, accordingly, a decrease in the probabilities of non-detection \( p_2 \) and false alarm \( p_3 \) increases if:

- to increase the number of serial requests \( k \);
- to improve the sensor quality, i.e., reduce the factor \( \gamma \).

If the a priori probability \( \alpha \) of the controlled phenomenon is low, the probabilities \( p_1 \) and \( p_2 \) change slowly with increasing \( k \), and the probability \( p_3 \) (false alarm) can be quite high, in comparison with the probability of non-detection \( p_2 \).

If the probability \( \alpha \) is sufficiently high, the probability \( p_1 \) effectively increases with increasing \( k \), and the probability of non-detection \( p_2 \) will be greater than the probability of false alarm \( p_3 \).
References


