1. Introduction

A hydraulic balancing device is the automatic device that balances axial force, which acts on the rotor of centrifugal multistage pump. Existing procedures of static calculation [1, 2], as well as the proposed one [3], are based on determining the dimensions of cylindrical and face chokes of balancing the axial force of a pump. In this case, it is necessary to ensure a reliable work of the pump, which prevents contact between working surfaces of the face pair at minimum fluid leaks. Leaks on the hydraulic balancing device cause volumetric energy losses. Friction of the movable surfaces of the unloading unit against the liquid leads to mechanical energy losses. Volumetric and mechanical losses on the unloading unit, in the total balance of energy losses, are of key significance (to 10 % of pump capacity). That is why development of the procedure for calculating the hydraulic balancing device under condition of minimal energy losses at the accepted rigidity of static characteristic reprensents a relevant task. Present article addresses the solution of the given problem.

2. Literature review and problem statement

World market puts forward high requirements to the competitiveness of pumping equipment. In order to satisfy them, it is necessary to improve pumping equipment when both designing new and modernizing the existing analogs. According to the energy balance, losses in pumps are divided into hydraulic, volumetric and mechanical. Hydraulic losses, those in the flowing part of a machine, are reduced to the lowest possible. Volumetric and mechanical losses, in the total energy balance, are of essential significance. Their larger share falls on the devices for balancing the axial force. In most energy multistage centrifugal pumps, a hydraulic balancing device balances the axial force. Hydraulic balancing device balances the axial force under all modes of pump operation. Static calculation of hydraulic balancing devices is based on selecting the geometric parameters of the unloading unit under condition that in the assigned range of change in the axial force, a face clearance and the magnitude of leaks do not exceed the permissible limits. An increase in the working value of a face clearance, accepted in the calculation, leads to the growth of volumetric energy losses.
experience. Using the parameters accepted, they perform a checking calculation of hydraulic balancing device.

The parameters accepted are considered permissible if the leaks do not exceed 5% of feed under nominal mode, while the minimum permissible magnitude of a face clearance is within the limits of $(1+1.2) \cdot 10^{-5} R_2$.

Article [2] reduces the calculation of a hydraulic balancing device to determining a working pressure differential on the disk and the magnitude of leak through the unloading unit and using their subsequent correction.

In paper [3], it is considered that the magnitude of the outside radius of the disk of a hydraulic balancing device and excess pressure do not affect fluid leaks through the unloading unit. The leaks are proportional to the width of a face clearance. The magnitude of a face clearance should be selected as small as it is allowed by the reliability of pump operation. Here, the length of a cylindrical choke is a determining parameter in the calculation of a hydraulic balancing device.

In the static calculation, given in article [6], considerable attention is paid to the temperature of fluid, which increases with the passage of the fluid through a pump and the hydraulic balancing device itself. In order to avoid cavitation in the chamber after the balancing device, it is proposed for the static pressure after the balancing device and in a by-pass pipe to exceed the pressure of the saturated vapor.

The technique is represented most fully in papers [7–9] and includes static calculation and subsequent testing of a hydraulic balancing device by dynamic stability.

The above procedures for the calculation of a hydraulic balancing device are based on the equation of axial equilibrium of a rotor and the equation of fluid flow rate through the unloading unit. The main difference in all calculations is what geometric parameters are accepted originally, and which are calculated based on the condition of equilibrium, as well as what parameters are considered as the most influential for the operation quality of unloading mechanism.

Given the aforementioned, it is proposed to develop procedures for the calculation of a hydraulic balancing device, underlining which is a closed system of equations by the geometric parameters of a hydraulic balancing device, obtained under condition of minimum energy loss on the hydraulic balancing device at the accepted rigidity of static characteristic. In the given procedure, the equation of equilibrium of a rotor and the equation of fluid flow rate is proposed to be considered based on the one-dimensional model of fluid flow in a movable coordinate system [10, 11].

3. The aim and tasks of the study

The aim of present work is to substantiate and develop a procedure for the statistical calculation of a hydraulic balancing device, which operates with minimum energy losses at the assigned value of rigidity of its static characteristic.

To achieve the aim, the following tasks were set:
- to obtain calculation dependences for determining the energy losses on a hydraulic balancing device;
- to substantiate the existence of condition of minimum energy losses on a hydraulic balancing device at the accepted magnitude of static characteristic rigidity;
- to examine a dependence between the static characteristic rigidity of a hydraulic balancing device and the energy losses on it.

4. Materials and methods for examining the development of procedure of calculation of a hydraulic balancing device

4.1. Initial data for the development of calculation procedure

The clearances of cylindrical ($h_1$) and face ($h_2$) chokes (Fig. 1) are considered to be known from the condition of manufacturability. Configuration of a hydraulic balancing device is defined by three geometric parameters: the length of cylindrical choke ($l$), the inner ($R_1$) and outer ($R_2$) radii of the disk of a hydraulic balancing device.

Two equations of equilibrium of a rotor underlie the calculation procedure:

$$T_{hi} + F_{st} = 0$$

at

$$F_{st} = F_{t} \bigg|_{h_1-h_2=0.001R_2};$$

$$mT_{hi} + F_{st} = 0$$

at

$$F_{st} = F_{t} \bigg|_{h_1=0};$$

where $m = \frac{T_{hi}}{T_{st}}$ is the coefficient that establishes the required magnitude of static characteristic rigidity of a hydraulic balancing device.

The equation of energy losses (power) on a hydraulic balancing device is accepted as a closing equation

$$N_{i} = N_{m} + N_{c},$$

representing the function $F(N_{c}, R_1, R_2, l)$ under condition $N_{i} = N_{m}^{\text{min}}$.
The volumetric losses $N_v$ are determined by the commonly accepted approach, according to which the power of volumetric losses on an unloading unit is equal to

$$N_v = \rho g q (H_1 - H_1^\text{in} ),$$

where $q$ are the leaks through a hydraulic balancing device, $H_1$ is the head of the stage of a pump, $H_1^\text{in}$ is the dynamic head of an impeller.

The power of mechanical losses is received as the sum of power losses on the cylindrical and face movable surfaces of an unloading unit of the axial force. The power losses on cylindrical surfaces, determined by different authors, more or less coincide. However, the power losses on face surfaces, calculated employing different procedures, differ significantly. That is why in the present work, using an analogy of determining the energy losses in a cylindrical pipe of round cross section, we obtained calculation formulas for power losses in the cylindrical and front chokes. The formulas received are in good agreement, by mechanical energy losses, with data from the most essential literary sources.

According to the given analogy, the power of mechanical losses in a cylindrical choke is found by formula:

$$N_c^i = \frac{\lambda}{16} \rho \pi \omega R_1^3,$$

where

$$\lambda = 0.11 \left( \frac{\lambda}{D_r} + \frac{68}{\text{Re}} \right)^{0.25},$$

$$\text{Re} = \frac{6 D_r \omega}{v}.$$

The power of mechanical losses in a face choke is determined by dependence

$$N_f^i = \frac{\lambda}{80} \rho \pi \omega \left( R_2^5 - R_1^5 \right),$$

where

$$\lambda = 0.11 \left( \frac{\lambda}{D_r} + \frac{68}{\text{Re}} \right)^{0.25},$$

$$\text{Re} = \frac{6 D_r \omega}{v}.$$

Then the summary energy losses on the unit of central unloading are equal to

$$N_i = N_n + N_v = \sum_{i=1}^{2} N_n^i + \sum_{i=1}^{2} N_v^i + N_v.$$

Taking into account (1)–(3), the previous equation takes the form

$$N_i = \rho \pi \omega \left[ \frac{1}{16} \left( \lambda R_1^3 + \lambda R_1^3 \right) + \frac{1}{80} \left( \lambda R_1^5 - \lambda R_1^5 \right) + \frac{1}{80} \left( R_2^5 - R_1^5 \right) + \frac{1}{80} \left( R_2^5 - R_1^5 \right) \right] +$$

$$+ \rho g q (H_1 - H_1^\text{in} ).$$

Dependence (4) will make it possible to determine the energy losses in an unloading unit of the axial force.

4.2. Sequence of solving the problem on the static calculation of a hydraulic balancing device

The proposed procedure for the static calculation is based on the computation of a closed system of equations by the parameters of a hydraulic balancing device from the condition of minimum energy losses at the accepted static characteristic rigidity. The system of equations is represented in the form:

$$T_{1,i} + F_{1,i} = 0,$$

$$m T_{1,i} + F_{1,i} = 0,$$

$$F \left( N^m_{\text{in}}, l, R_1, R_2 \right) = 0,$$

where

$$T_{1,i} = \pi \left( \tau_{1,i}^2 - \tau_{2,i}^2 \right) \left[ \gamma (H_1 - H_1^\text{in}) - \frac{\rho \phi \omega}{8} \left( \tau_{1,i}^2 - \tau_{2,i}^2 - \tau_{2,i}^2 \right) \right],$$

$$F_{1,i} = F_{1,i} \left( H_1, \theta, 0, 0 \right);$$

$$F_{1,i} = F_{1,i} \left( H_1, \theta, 0 \right);$$

$$F_{1,i} = \frac{-\rho q}{8 \pi h} \times$$

$$\times \left[ 1 - \frac{R_2^2}{R_1^2} - 2 \ln \left( \frac{R_2}{R_1} \right) + \frac{\lambda}{2 h_1} \left( \frac{R_2^3 - R_1^3}{R_2^3} \right) + \zeta_2 \left( R_2^3 - R_1^3 \right) \right],$$

where

$$q = g_c \sqrt{\gamma (H_1 - H_1^\text{in}) - \frac{\rho \phi \omega}{8} (R_2^3 - R_1^3)}$$

are the leaks through a hydraulic balancing device, here

$$g_c = \frac{g_1 g_2}{g_1 + g_2}$$

is the equivalent conductivity of sequentially connected cylindrical and face chokes, where

$$g_1 = 2 \pi R_1 h_1 \left( 0, 5 \phi \zeta \right)^{0.5}$$

is the conductivity of a cylindrical choke;

$$\zeta_1 = 1, 5 + \lambda_2 \frac{1}{2 h_1}$$

is the resistance coefficient of a cylindrical choke;

$$g_2 = 2 \pi R_1 h_1 \left( 0, 5 \phi \zeta \right)^{0.5}$$

is the conductivity of a face choke;

$$\zeta_2 = 0, 2 + \frac{R_1^3}{R_2^3} + \lambda \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \frac{1}{1},$$

is the resistance coefficient of a face choke.
As a result, we obtained a closed system of equations for determining the geometric parameters of a hydraulic balancing device.

4.3 Sequence of realization of the calculation procedure of a hydraulic balancing device

1. According to the conditions of pump operation, one accepts the values of coefficient \( m \), which determine the static characteristic rigidity.

2. A series of values \( R_i \) are assigned from \( R_0 < R_i < R_2 \) and, according to them, for the accepted \( m \), from the first and second equations of system (5), one finds appropriate values of \( l \) and \( R_1 \) and, from (9), the magnitude of leak \( q \).

3. By the values of \( R_i, R_0, l \) and \( q \), from equation (4), one finds the values of energy losses of energy \( N \) on a hydraulic balancing device.

4. A dependence graph \( N = f(R_i) \) is constructed.

5. By dependence \( N = f(R_i) \), one finds \( R_{opt} \) corresponding to the value of \( \min N \), based on which, from the first and second equations of system (5), one obtains parameters \( R_i = R \), \( R_0, l \) and \( l \). These parameters define the geometry of a hydraulic balancing device under condition of minimum energy losses on a hydraulic balancing device for the accepted value of static characteristic rigidity.

5. Results of examining the procedure for the calculation of a hydraulic balancing device

The proposed procedure for the calculation of a hydraulic balancing device was verified based on the commercially available feed pump PE 600-300 (AO “Sumy Plant – NasosEnergoMash”, Ukraine); design schemes and drawings are given in [12].

The basis of the proposed procedure for the statistical calculation of a hydraulic balancing device is a closed system of equations (9) for determining the basic dimensions of a hydraulic balancing device \( R, R_0, l \). These equations are obtained from the condition of minimum power losses on a hydraulic balancing device at its accepted static characteristic rigidity. The radial clearance of a cylindrical choke for the pump PE 600-300 is \( h_1 = 0.25 \times 10^{-3} \) m, and that of the face one is selected from the condition of manufacturability and in the absence of contact between the working surfaces of a face pair is taken equal to \( h_2 = 0.001 \times R_2 \) [2].

The axial force, which acts on the rotor, depends on the pump-operating mode. Its largest value is at zero feed of the pump. For the pump PE 600-300, the axial force at zero feed of the pump is 1.2 times larger than the axial force under nominal mode. Furthermore, according to the research results, axial force over the period of pump operation, because of the wear of the front seal of pump’s impeller, can increase by 2-2.5 times. Given this, for the wide class of pumps, which are not superimposed with special conditions on the reliability of a hydraulic balancing device, we recommend accepting during calculation the coefficient that establishes the required magnitude of static characteristic rigidity equal to three (\( m = 3 \)).

The essence of the calculation procedure proposed is in defining the geometric parameters of a hydraulic balancing device, which satisfy the accepted static characteristic rigidity under condition of minimum energy losses in a hydraulic balancing device. These parameters are found from the dependence of power losses on a hydraulic balancing device \( N_{mm} = f(R_i) \), as well from the dependences of power losses on the rigidity coefficient of a hydraulic balancing device \( N_m = f(m) \).

Results of the calculations according to dependences \( N_v = f(R_i), N_m = f(R_i) \) and \( N_f = f(R_i) \), for the accepted values of rigidity \( m = 2, 3, 4, 5, 6 \) are given in Fig. 2–6. According to the results of calculations, given in Fig. 2–6, a generalized characteristic of power losses on a hydraulic balancing device \( N = f(R_i) \) for the accepted values of static characteristic rigidity takes the form represented in Fig. 7.

![Figure 2](image1.png)

**Fig. 2.** Dependence of power losses on the radius of disk of a hydraulic balancing device \( R_i \) for \( m = 2 \): \( a \) – volumetric \( N_v \), mechanical \( N_m \); \( b \) – total losses \( N_f \)

![Figure 3](image2.png)

**Fig. 3.** Dependence of power losses on the radius of disk of a hydraulic balancing device \( R_i \) for \( m = 3 \): \( a \) – volumetric \( N_v \), mechanical \( N_m \); \( b \) – total losses \( N_f \)
Fig. 7. Generalized characteristic of power losses on a hydraulic balancing device at the change in radius of the disk R₂.

Fig. 7 shows that for the pump PE 600-300, dependence Nₖ=f(R₂) has a distinctly pronounced optimum on the energy losses on a hydraulic balancing device. The minimum value of function Nₖ=f(R₂) is taken in the range R₀<R₂<r₂. An increase in coefficient m and, therefore, an increase in the static characteristic rigidity, lead to an increase in energy losses Nₖ=min on a hydraulic balancing device. As shown by Fig. 7, the dependence of change in the minimum power losses on rigidity coefficient Nₖ=min=f(m) takes the form (Fig. 8).

The use of characteristics Nₖ=f(R₂) and Nₖ=min=f(m) makes it possible to determine the desired dimensions of a hydraulic balancing device. In this case, the condition of minimum energy losses at different values of m is satisfied.

6. Discussion of results of the static calculation of a hydraulic balancing device employing the procedure proposed

In the present work, we received a closed system of equations for determining the optimal geometric dimensions of a hydraulic balancing device. The given system of equations makes it possible, for the assigned parameters of a pump, to obtain unique solution and to determine the geometric parameters of a hydraulic balancing device. Closing the system of equations was achieved by receiving reliable calculation formulas for determining the energy losses on the node of a hydraulic balancing device.

The research conducted in order to devise a procedure for the calculation of a hydraulic balancing device has a scientific and practical value. They might be used in the calculation of devices for unloading the axial force, which act on the rotor of a centrifugal multistage pump.

At the same time, however, present work did not solve a problem on the calculation of optimal geometric dimensions of a hydraulic balancing device from the condition of minimal energy losses at the maximal static characteristic rigidity. Its solution for the model of one-dimensional flow of fluid is the object of further studies.

7. Conclusions

Employing an analogy with determining the losses in a cylindrical pipe of round cross section, we obtained cal-
calculation dependences for finding the energy losses in the cylindrical and face chokes of a hydraulic balancing device that are a function of energy losses from the geometric parameters of a choke, rotation frequency of the surfaces and the mode of fluid flow.

It is demonstrated that with an increase in the diameter of disk of a hydraulic balancing device, mechanical energy losses increase while the volumetric energy losses decrease. Thus, the dependence of total energy losses on a hydraulic balancing device has the optimum value for each of the presented values of static characteristic rigidity of a hydraulic balancing device.

The condition of minimum energy losses on a hydraulic balancing device, at its accepted static characteristic rigidity, made it possible to close the system of equations regarding the geometric parameters of a hydraulic balancing device and to obtain a unique solution of the system of equations and determining the geometric dimensions of a hydraulic balancing device.

It is shown that each value of static characteristic rigidity of a hydraulic balancing device matches specific value of minimal energy losses on a hydraulic balancing device. With an increase in the rigidity of static characteristic, the optimum by minimal losses increases.

References