Introduction

Radial-axial turbines are widely used in power and technological units: turboexpanders of various purpose, cogeneration plants working with low boiling media, actuators of pumps, etc. Compared with axial designs they have a higher efficiency for machines with a relatively small mass flow of the working medium.

Until recently, blades with the «plate» profile have mainly been used in the rotors of the radial-axial turbines. Despite of that they provided a fairly high level of internal efficiency in steady design conditions. In unsteady modes of off-designs conditions, due to leakage and off-design flow angles in the flow channels, substantial separated flows occurred, leading to a significant reduction in the efficiency. In order to improve the aerodynamic characteristics of the radial-axial type rotor both for the steady conditions and especially at variable operating conditions, blades of complex spatial forms with the corporeal profiles are used [1–4].

Stator blade profiling in radial and radial-axial turbines received much less attention than profiling rotor blades. This is due to the fact that usually the impact of the stator on overall losses of kinetic energy is much smaller than that of the rotor. But with the implementation of new technologies in the production of turbomachines, including the use of magnetic bearings, there is an opportunity to create the high-speed turbines with the rotational speed up to 10–20 thousand rev/min., and in some cases up to 300 thousand rev/min [5, 6]. This allows us to create turbines with high pressure drops in one stage. Increasing the pressure drop in the stage, usually leads to a reduction of outlet flow angle from the stator domain, thus increasing the profile and trailing edge losses. Moreover the flow in the stator domain exceeds the supersonic speed, which is why there are additional supersonic and shock wave losses [7, 8]. As a result, the kinetic energy losses in the stator increase significantly and become similar or higher than the kinetic energy loses in the rotor.

The article presents the method of analytical profiling of radial stator blades, using of which in combination with numerical modelling of viscous flows allows us to increase the efficiency of the flow parts of high-loaded centripetal turbines.

Analytical method of profiling of radial blades

The basis of the method of development of radial blade profile geometry is an approach of parametrization and analytical profiling of axial blades [9]. Unlike in the method described in [9], in the present case the profile is given not in a Cartesian but in a curvilinear coordinate system $\hat{\Phi}\hat{r}$ (fig. 1).

Curvilinear coordinates $\hat{\Phi}\hat{r}$ are associated with the cylindrical coordinates $\varrho r$ by the relations:

$$r = r_0 - (\hat{r} - \hat{r}_0), \quad \varrho = \frac{\hat{\Phi} - \hat{\Phi}_0}{t} \frac{2\pi}{N},$$

where \( \phi_o, \hat{r}_o \) – coordinates of point \( o \) in the coordinate system \( \phi \hat{r}, r_\sigma \) – radial coordinate of point \( \sigma \) in the cylindrical coordinate system, \( N \) – number of blades in the cascade, \( t = \frac{2\pi r_\sigma}{N} \) – cascade pitch at point \( \sigma \).

The profile is described by the leading and trailing edge, as well as by the pressure and suction side curves. The trailing and leading edges are given by circles, the suction side – by a polynomial of the 5th order, pressure side – by a polynomial of the 4th order:

\[
\hat{\phi}(\hat{r}) = \sum_{i=0}^{5} a_i \hat{r}^i, \quad a_i = \text{const,} \quad (1)
\]

\[
\hat{\phi}(\hat{r}) = \sum_{i=0}^{4} a_i \hat{r}^i, \quad a_i = \text{const.} \quad (2)
\]

The following data are used to describe the profile cascade: \( b_s \) – profile width; \( \alpha_1 \) – skeletal angle at the inlet; \( r_1 \) – the leading edge radius; \( \alpha_2 \) – the effective exit flow angle; \( r_2 \) – the trailing edge radius; \( t \) – cascade pitch; \( \Delta \alpha_1, \Delta \alpha_2 \) – the angles of “sharpening” of the leading and trailing edges; \( \alpha_{2bev} \) – the “bevel” angle of suction side, \( \alpha_{co} = \alpha_{2s} + \alpha_{2bev} \); \( 1_s, 2_s, 1_p, 2_p \) – tangency points of the leading and trailing edges with the pressure and suction side curves (fig. 1).

The coefficients of the curve (1) that describe the suction side are iteratively calculated from the relations:

\[
\begin{align*}
&\hat{\phi}_s(\hat{r}_{1s}) = \tan(\alpha_1 + \Delta \alpha_1) \\
&\hat{\phi}_s(\hat{r}_{1s}) = \{ \hat{\phi}_s(0) \} \\
&\hat{\phi}_s(\hat{r}_O) = \hat{\phi}_O \\
&\hat{\phi}_s(\hat{r}_{2s}) = \tan(\alpha_{co}) \\
&\hat{\phi}_s(\hat{r}_{2s}) = \hat{\phi}_{2s} \\
&\hat{\phi}_s(\hat{r}_{2s}) = \tan\{ \alpha_{2s} \}. \quad (3)
\end{align*}
\]

The variable parameters for the relation (3) are \( \alpha_{2s} \) and \( \hat{\phi}_0^s \), whose selection should lead to the determination of the required cascade throat \( O \) and the minimum curvature of the curve (1) [8]. The value of throat is determined by the given values of cascade pitch and effective angle:

\[
O = t \cos \alpha_{2ef}.
\]

After determination of the suction side curve and inscribing of trailing and leading edges, the coefficients of the curve (2) for the pressure side are iteratively calculated with the use of the following relations:
where \( \hat{r}_{1c}, \hat{\Phi}_{1c}, \hat{r}_{2c}, \hat{\Phi}_{2c} \) – coordinates of tangency of the suction side curve with the circles of the leading and trailing edges, which are determined by the given angle \( \alpha_i = \Delta \alpha_i \) at the leading edge and variable angle \( \alpha_2 \) at the trailing edge. The angle \( \alpha_2 \) is determined from the range \( \alpha_{c_1} \leq \alpha_2 \leq \alpha_{c_2} \) so as to obtain the minimum curvature of the pressure curve [10].

**Approbation of the analytical method of profiling of radial blades. The stator of a high-loaded radial-axial ORC turbine stage**

As an object of research a radial-axial 100 kWe ORC turbine stage was selected. Its view is shown in fig. 2 and the results of its investigation are shown in the paper [11]. Numerical modelling of spatial viscous flows was provided using the software package IPMflow, which is an extension of the software packages FlowER and FlowER-U [12, 13]. The computational grid of the flow domain consisted of more than 1 million cells.

The turbine (stage) works at the following thermodynamic parameters: inlet parameters – pressure 1200 kPa, temperature 553 K; outlet parameters – pressure 17 kPa; rotational speed – 14000 rev/min, working fluid – silica oil (MDM) in the vapor state. Thus, the stage operates at a very high pressure drop - with the pressure ratio more than 70.
The stator blade profile for this turbine (variant 1) was developed using the standard method, which is used for developing of the axial blades [9]. Two variants of turbines with different types of stator blade profile were also investigated: stator blade with a “half-drop” profile (variant 2) and the profile, which was developed using the proposed method (variant 3). A view of all investigated variants of profile cascades denoted by appropriate numbers is shown in fig. 3.

In the table, the main integral characteristics of the stage for stator variants 1–3 are collected to be discussed consecutively later.
The main integral characteristics

<table>
<thead>
<tr>
<th>Variant of the stator</th>
<th>Number of stator blades</th>
<th>Mass flow, kg/s</th>
<th>Kinetic energy loses in the stator related to the enthalpy drop in stage, %</th>
<th>Kinetic energy loses in the stage, %</th>
<th>Stage efficiency with considering the outlet speed losses, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57</td>
<td>18.26</td>
<td>20.01</td>
<td>7.95</td>
<td>9.9</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>18.61</td>
<td>17.4</td>
<td>6.89</td>
<td>8.3</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
<td>18.51</td>
<td>15.05</td>
<td>5.75</td>
<td>7.96</td>
</tr>
</tbody>
</table>

Fig. 4 shows a flow visualization in the blade-to-blade channel and a distribution of adiabatic Mach number on the blade surface of variant 1 of the stator. The presented results show that the maximum value of Mach number is reached downstream of the throat and its value is not more than 2. This suggests that over-expansion of the flow downstream of the throat is relatively small for such flows, whereby the level of supersonic and shock wave losses should be relatively low. However, due to the high number of blades (57) the level of profile and trailing edge losses is increased. The kinetic energy losses in the stator related to the enthalpy drop in the stator are 20.01%, and related to the enthalpy drop in the stage – 7.95%. The overall kinetic energy losses in the stage are 9.9%, so the stator contribution to this value is a few times bigger than the rotor contribution. It is important to note that we can’t obtain a cascade with «classical» stator blades and a smaller number of blades to provide a given mass flow rate. Also a stagger angle of the profile determines the profile position in which the profile tails are “curled up” to the flow, which leads to a substantial increase of kinetic energy loses.

At the second variant of the stator, “half-drop” profiles are applied (fig. 3), which helps to reduces the number of blades to 29, and as the result, to reduce the profile and trailing edge losses (see table). The overall efficiency of the stator and stage as a whole tends increase. Fig. 5 shows a visualization of the flow in the blade-to-blade view and a distribution of adiabatic Mach number on the blade surface of variant 2 of the stator.

It seen from the results that in variant 2 of the stator, there are high Mach numbers behind the throat, which leads to an increase of supersonic and shock wave losses.

Fig. 6 shows a flow visualization of the blade-to-blade channel and a distribution graph of adiabatic Mach number on the blade surface of variant 3 of the stator. In this case the number of blades was reduced to 23, but the value of maximum Mach number remained at the level of the first variant of the stator cascade.
For this proposed variant of the stator, the kinetic energy losses related to the enthalpy drop in the stage are reduced by 2,2 and 1,1%, compared the first and the second variant of the cascade, respectively.

Conclusions

An analytical method of profiling of radial turbine stage blades is proposed. In this method the profile is defined in a curvilinear coordinate system. It consists of the leading and trailing edge, the pressure and suction side, which are determined by the polynomial curves of the 5th and 4th order, respectively. The method makes it possible to create highly efficient radial stator cascades of the high-loaded radial-axial stages. A special form of the blade-to-blade channel allows us to get a relatively low level of profile and trailing edge
losses, as well as low supersonic and shock wave losses of kinetic energy. An example of stage with the new stator profile is shown. The new profile allows us to reduce the kinetic energy losses by 2.2% compared to the “classical” profile (used in axial turbines) and by 1.1% compared to the “half-drop” profile.

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