УДК 621.59

V.V. Trandafilov, M.G. Khmelniuk
orcid.org/0000-0001-9905-9958 vlad.trandafilov@gmail.com
Odessa National Academy of Food Technologies, 112 Kanatnaya str., Odessa, 65039, Ukraine

COMPUTER SIMULATION OF A STIRLING REFRIGERATOR

In present numerical research, the mathematical model to accurately simulate the performance and detailed behavior of Stirling refrigerator was investigated. The mathematical model is for an alpha Stirling refrigerator with helium as the working fluid and will be useful in optimizing the mechanical design of these machines. A complete non-linear mathematical model of the machine, including helium thermodynamics, and heat transfer from the walls, as well as heat transfer and gas resistance in the regenerator, is developed. Non-dimensional groups are derived, and the mathematical model is numerically solved. Important design parameters are varied and their effect on refrigerator performance determined. The simulation results of a Stirling refrigerator are presented and these include heat transfer and coefficient of performance.

Keywords: *Stirling refrigerator – Mathematical model – Non-dimensional parameters – Energy efficiency – Helium*

В.В. Трандафилов, М. Г. Хмельнюк
Одесская национальная академия пищевых технологий, ул. Канатная, 112, Одесса, 65039, Украина

КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ХОЛОДИЛЬНОЙ МАШИНЫ СТИРЛИНГА

В данном численном исследовании рассмотрена математическая модель для точного моделирования производительности и подробного поведения холодильной машины Стирлинга. Математическая модель для альфа холодильной машины Стирлинга с гелием в качестве рабочего вещества будет использоваться в оптимизации механической конструкции этих машин. Разработана полная нелинейная математическая модель машины, в том числе термодинамика гелия и теплопередачи от стенок, а также передача тепла и сопротивление газа в регенераторе. Получены безразмерные группы, и численно решена математическая модель. Изменяются важные конструктивные параметры, и определяется их влияние на производительность холодильной машины. Представлены результаты моделирования холодильной машины Стирлинга, которые включают передачу тепла и коэффициент преобразования.

Ключевые слова: *Холодильная машина Стирлинга – Математическая модель – Безразмерные параметры – Энергоэффективность – Гелий*

I. INTRODUCTION

Fuel and energy resources price surge as well as global worming push us to set problem of searching innovative technologies for energy transformation, to develop new technologies on the base of thermodynamic cycles with high efficiency, to use new working mediums in order to develop ecologically safe energy system, which can satisfy industrial sector alone with housing and utilities for minimum amount used.

This work analyzes a specific application of energy harvesting that converts a mechanical energy input into thermal energy, or thermocompressive energy harvesting. This application is the Stirling refrigerator. Refrigerators use mechanical energy to drive heat from a low temperature source to a high temperature sink, and the Stirling machine is one of these that can be used for this purpose. Though such a device was proposed in the mid-1800's [1], Stirling refrigerators still have limited commercial use for household refrigeration and air-conditioning because of the better performance and convenience of conventional vapor-compression and absorption machines. Smaller Stirling machines are mostly used to cool computer parts. For these reasons, the Stirling machine may be more useful as an energy harvesting device. The mechanical input to the Stirling machine could be an ambient vibration or force, and the resulting temperature difference could be used to cool or heat for a given application. One such application of interest is obtaining mechanical energy from movement such as a walking or rocking motion to produce heat flow that provides a comfortable temperature for the user. Helium-filled Stirling machines have an advantage in this type of application because of their simplicity.

Although the Stirling machine does not operate with ideal cycles, the ideal Stirling cycle demonstrates the principles behind the Stirling machine.

The ideal Stirling cycle begins with an isothermal expansion in the expansion space. The expansion space is kept at a constant temperature by absorbing heat from the low temperature source. The second step is an isochoric heat transfer as the heated working fluid from the expansion space passes through the regenerator, cooling the fluid. The third step is an isothermal compression of the working fluid in the compression space. The compression space is kept at a constant low temperature through the removal of heat to the high temperature sink. The last step is an isochoric heat transfer from the regenerator to the cool fluid from the compression space. This completes the ideal Stirling cycle and returns the cycle to the first step of isothermal expansion of the working fluid.

From the variety current layout schemes and a design of individual nodes of Stirling machines, can distinguish three basic configurations: alpha, beta, and gamma. Each type operates under the same Stirling cycle, but the internal configurations differ slightly. The alpha type Stirling machine is selected for the research because of the rather large power-to-volume thereby reducing the capital cost of proceedings. The machine consists of two chambers separated by a regenerator. The alpha Stirling machine can be seen in Figure 1.

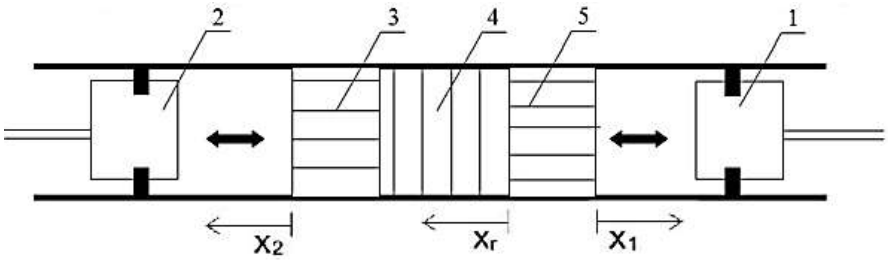

Figure 1 *- Alpha Stirling refrigerator*
1 - the expander, 2 - the compressor, 3 - the warm heat exchanger,
4 - the regenerator, 5 - the cold heat exchanger.
The arrows point in the positive direction for each coordinate.

II. MATHEMATICAL MODEL

A literature review has been carried out covering the relevant literature for mathematical models of Stirling refrigerators. Even though the operation of the Stirling refrigerator differs from that of the ideal Stirling cycle, some researchers rely on the latter for their studies [2]. Wu et al. [3] and Kaushik and Kumar [4] used a finite time thermodynamic analysis of Stirling machines. By assuming the ideal cycle partially or completely, any analysis of the refrigerator deviates from the actual physics occurring during operation. In their review, Chakravarthy et al. [5] classified the Stirling refrigerator as a periodic refrigeration system, which indicates that the pressure and flow rate in the refrigerator fluctuate periodically. Thombare and Verma [6] provided a thorough review of the work done on Stirling cycle - based machines. Although the review focused on engines, the analysis of the departure of Stirling machines from the ideal Stirling cycle remains relevant. Waele [7] provided a comprehensive overview of Stirling cryocoolers and other thermal machines. Tekin and Ataer [8] looked at improving the design of a V-type Stirling cycle refrigerator in a previous model using a thermodynamic approach. Chen and Yan [9] developed a model for a Stirling refrigerator that investigated the effects of non-ideal regenerators. Erbay and Yavuz [10] took a more practical approach to the analysis of Stirling refrigerators by studying the cooling load per unit volume. Omari [11] compared the differences between the ideal and real Stirling cycles that occur in the refrigerator, but lacked analysis of the effect of system parameters on performance. The real cycle includes heat transfer in each section of the refrigerator during any part of the cycle, as well as a non-ideal regenerator. Including these physical processes is important because a more physically sound model provides the designer with a better estimate of how the refrigerator will perform under actual operating conditions.

This work provides a more detailed and comprehensive model for Stirling refrigerators than previously published. By allowing for heat transfer throughout the entire system and duration of the cycle, the present analysis minimizes the simplifications made in previous derivations. It improves upon the model Chen and Yan [9] developed because they include isochoric and isothermal processes in the cycle. It also provides a deeper analysis of the factors that affect refrigerator performance compared to Tekin and Ataer [8], since they assumed the regenerator to be adiabatic. In addition, it improves upon the model of Erbay and Yavuz [10] because they included the isochoric processes of an ideal Stirling cycle. In the following, the governing equations for each section of the refrigerator are developed. The derivations use the basic conservation laws, and also account for heat transfer in the cylinders and regenerator. The model is solved numerically for a number of cycles to remove the effect of the initial transients on the solution. The numerical results are presented to illustrate the performance of the machine, and design parameters that affect performance are derived and discussed.

The model is based on the following considerations: the working fluid is an ideal gas, helium; there is no leakage of mass from the system; the input motion of the pistons is sinusoidal; the walls of the compression and expansion space are at a constant temperature. It is assumed that the heat generated by friction in the regenerator is negligible.

Consider the Stirling refrigerator in Figure 1. The chamber that extracts net heat from the surroundings will be referred to as the expansion space, and the chamber that releases net heat to the surroundings as the compression space. For convenience, variables associated with the former will have a subscript 1, while those associated with the latter will have a subscript 2. The prescribed motions of the pistons are:

$$x_1 = x_{0,1} + \hat{x}_1 \sin(\omega t) \quad (1)$$

$$x_2 = x_{0,2} + \hat{x}_2 \sin(\omega t + \phi) \quad (2)$$

The pistons move with a frequency ω, an amplitude $\hat{x}_i$ and central position $x_{0,i}$ for the chamber i piston. The phase shift ϕ in the motion of the two chamber pistons results in the reciprocating flow between the two chambers; ϕ will turn out to be a critical parameter for the proper operation of the machine.

Energy balance on chamber i gives:

$$\underbrace{c_v M_i \frac{dT_i}{dt}}_{1} = \underbrace{h_i \pi D_i x_i (T_{w,i} - T_i)}_{2} + \underbrace{\left[c_v \dot{m}_{in}(T_{in} - T_i) + \frac{P_{in}}{\rho_{in}} \dot{m}_{in}\right]}_{3} - \underbrace{\left[c_v \dot{m}_{out}(T_{out} - T_i) + \frac{P_{out}}{\rho_{out}} \dot{m}_{out}\right]}_{4} - \underbrace{P_i \frac{dV_i}{dt}}_{5} \quad (3)$$

Term 1 represents the rate of change of the temperature of the air in a chamber with respect to time t, where M_i and T_i are the mass and temperature of the helium, respectively, and c_v is the heat capacity per unit mass of helium at constant volume. Term 2 is the heat transfer from the walls of the chamber to the helium, where h_i is the convective heat transfer coefficient, D_i is the piston diameter, x_i is the distance from the regenerator, and $T_{w,i}$ is the temperature of the chamber wall. Term 3 represents the enthalpy of the incoming flow to the chamber, where $\dot{m}_{in}$ is the mass flow rate into the chamber, T_{in} is the temperature of the incoming flow, P_{in} is the pressure at the inlet, and ρ_{in} is the density of the incoming flow. Term 4 represents the enthalpy of the outgoing flow, where $\dot{m}_{out}$ is the mass flow rate out of the chamber, T_{out} is the temperature of the outgoing flow, P_{out} is the pressure at the outlet, and ρ_{out} is the density of the outgoing flow. Term 5 is the work done by the helium on the piston, where P_i is the pressure on the piston and V_i is the volume of the helium in the chamber.

Eq. (3) can be applied to the two chambers for one-dimensional helium flow in either direction, i.e. from chamber 1 to chamber 2 or vice versa. For incoming flow from the regenerator, Term 4 becomes zero, while for outgoing flow to the regenerator, Term 3 becomes zero. This is a result of the physics of the helium flow.

Assuming ideal gas behavior for the helium:

$$P_i x_i \frac{\pi D_i^2}{4} = M_i R T_i \quad (4)$$

where R is the specific gas constant for helium.

Similar equation can be derived for the regenerator section. Consider the regenerator as a metal tube of length L_r, and diameter D_r. Energy balances for the helium in the regenerator, whose temperature $T_r(x_r, t)$ varies with space and time, as well as the regenerator wall, whose temperature $T_w(t)$ varies only with time, are:

$$M_w c_w \frac{dT_w}{dt} = -\int_0^{L_r} h_r \pi D_r (T_w - T_r) dx_r \quad (5)$$

where M_w is the mass of the regenerator wall is, c_w is the specific heat per unit mass of the metal, and x_r is a location along the regenerator from chamber 1 to chamber 2.

The velocity of the helium in the regenerator depends on the pressure difference between the chambers as:

$$\dot{m} = \begin{cases} \sqrt{\dfrac{\rho \pi D_r^5 (P_1 - P_2)}{32 L_r f}} & \text{for } P_1 \geq P_2 \\ -\sqrt{\dfrac{\rho \pi D_r^5 (P_2 - P_1)}{32 L_r f}} & \text{for } P_2 \geq P_1 \end{cases} \quad (6)$$

where f is the friction factor. The inertia of the helium has been neglected.

To nondimensionalize the equations, define the following non-dimensional groups:

$$t^* = \frac{t\omega}{2\pi}, \; x^* = \frac{x_i}{\hat{x}_1}, \; v_i^* = \frac{v_i}{\omega \, \hat{x}_i}, \quad T_i^* = \frac{T_i - \overline{T}}{\Delta T}, \; P_i^* = \frac{P_i - \overline{P}}{\Delta P}, \; \dot{m}^* = \frac{\dot{m}}{\widehat{M}\omega}, \quad (7)$$

where the starred variables are non-dimensional. Also:

$$\Delta T = \frac{\overline{T}(V^{k-1} - V_{min}^{k-1})}{V_{min}^{k-1}} \quad (8)$$

$$\Delta P = \frac{\overline{P}(V^k - V_{min}^k)}{V_{min}^k} \quad (9)$$

where $k = c_p/c_v$, $\overline{T}$ is the starting helium temperature, and $\overline{P}$ is the starting helium pressure. ΔT and ΔP are characteristic temperature and pressure rises, respectively, if the helium were to be compressed adiabatically to its minimum volume.

The computer simulation of an alpha Stirling refrigerator then requires a solution of 17 non-linear ODE's with appropriate switching conditions, supplemented by a set of nonlinear algebraic equations describing the behavior of elements of the model. Stationary, periodic solutions to the refrigerator operation at a given, constant angular velocity of the refrigerator are presented. The simulation program uses a Matlab language library of procedures and solvers (Euler's first-order method). Time and spatial steps of 0:0001 and 0:001, respectively, were used to ensure numerical convergence to the solution.

The simulation uses the physical parameters of the refrigerator given in Table 1 and describes in [12].

The initial pressures are taken to be 10 atm, and all initial temperatures 303 K. The convective heat transfer coefficient for each section of the refrigerator is assumed to be in the range for gases under forced convection. Because of the low Reynolds number in the regenerator, the friction factor corresponds to that for laminar flow. The temperature of the chamber walls remains constant at 303 K, so any heat transfer to or from the walls is considered to be with a cold reservoir for chamber 1 and with a hot reservoir for chamber 2.

Table 1- Physical parameters

Parameter(s)	Value
Piston diameter D	50 mm
Regenerator pipe diameter D_r	5 mm
Regenerator pipe length L_r	10 mm
Convection coefficients h_r, h_1, h_2	500 W/m^2K
Frequency of piston motion ω	6.28 rad/s
Phase angle ϕ	90°
Piston amplitude $\hat{x}_1, \hat{x}_2$	50 mm
Piston center position $x_{0,1}, x_{0,2}$	80 mm (from regenerator)
Regenerator friction factor f	0.1
Regenerator mass M_w	1

III. RESULTS AND DISCUSSIONS

The model was used to simulate the performance of the Stirling refrigerator over two cycles. The results show the second cycle of operation because the first contains transient effects caused by the initial conditions, which decay by the second.

Figure 2 shows the *P-V* diagram of the expansion space. This plot shows the refrigeration cycle of chamber 1. This curve can be seen to be quite different from that of the ideal cycle.

Because this Stirling refrigerator is not ideal, there is unwanted heat transfer to the cold reservoir (chamber 1 walls) and unwanted heat transfer from the hot reservoir (chamber 2 walls).

At all reliability of machines operating on the Stirling cycle, the drawback is the presence of the regenerator, which leads to complication of machines, a large free volume and the increase of the hydraulic resistance, the combined influence of which results in rounding *P-V* diagram of the refrigerator, i.e. to reduce its specific capacity. Conclusion from this is that to improve the efficiency of the machine requires constant improvement of the design of the regenerator.

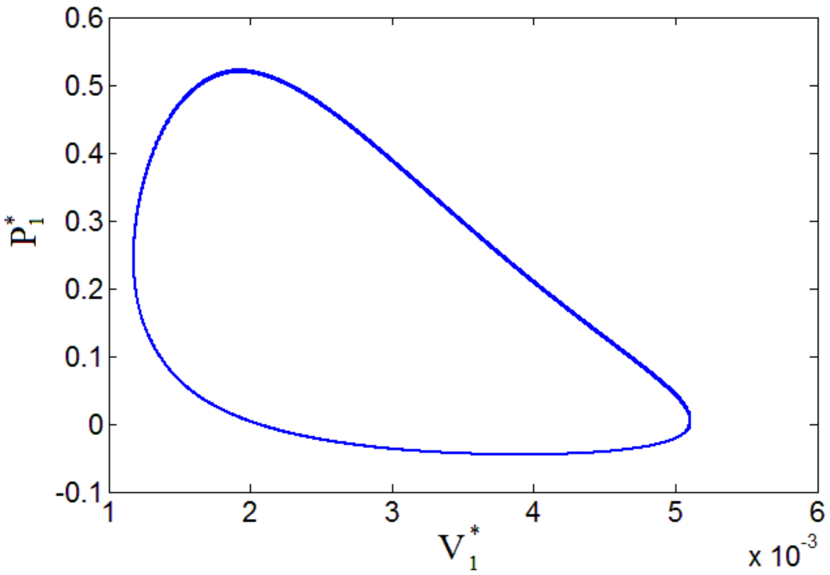

Figure 2 *- Pressure-volume diagram for chamber 1 over a cycle.*

Figure 3 illustrates the temperature, pressure, and mass of the air in chamber 1 along with its piston motion. Figure 4 has the temperature, pressure, and mass of the helium in chamber 2 along with its piston motion. Comparing the two figures, a temperature difference between the two chambers is seen to develop at half of the cycle duration, where the temperature of the chamber 1 helium drops as the temperature of the chamber 2 helium rises. This causes the chamber 1 walls to transfer heat to the helium in that chamber, cooling the walls. Also, heat is transferred from the hotter helium in chamber 2 to the walls, heating them.

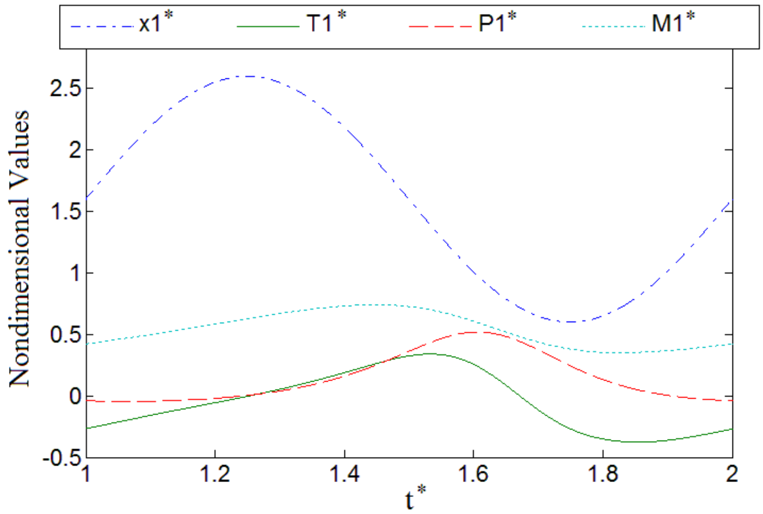

Figure 3 *- Helium temperature T_1^* (solid -), pressure P_1^* (dashed - -) and mass M_1^* (dotted ..) in chamber 1 over a cycle. This also shows the chamber 1 piston motion x_1^* (dash-dot -.).*

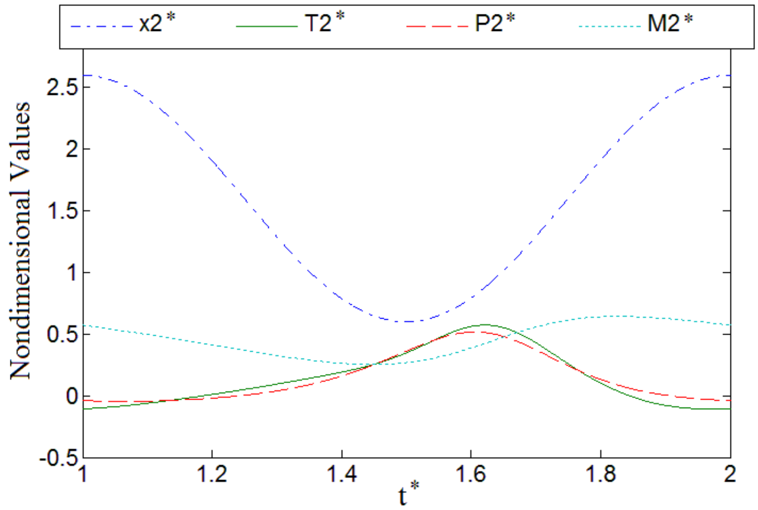

Figure 4 *- Helium temperature T_2^* (solid -), pressure P_2^* (dashed - -) and mass M_2^* (dotted ..) in chamber 2 over a cycle. This also shows the chamber 2 piston motion x_2^* (dash-dot -.).*

The pressure difference between the chambers enables the flow of the helium through the regenerator, which can be seen in Figure 5. The phase shift in the sinusoidal motion of the pistons causes the pressure difference between the two chambers and thus mass fluctuations in each chamber. This mass flow between chambers is an important aspect of the Stirling refrigerator because it couples the two chambers together. The mass difference between the chambers also affects the heat transfer in each chamber.

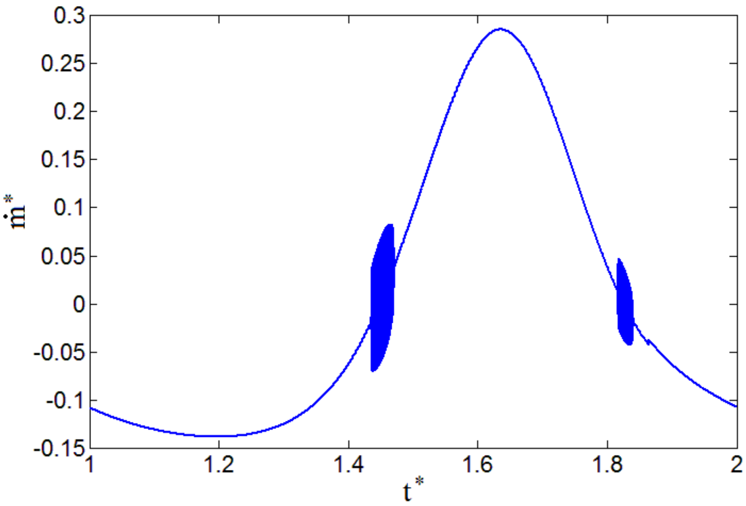

Figure 5 *- Mass flow rate $\dot{m}^*$in the regenerator over the cycle.*
The slight mass flow rate fluctuation around 0 is numerical due to the change of sign of the velocity.

To see how the heat transfer changes, define a non-dimensional heat transfer term Q_i^*, which is:

$$Q_i^* = \int_1^2 \frac{h_i D_i \hat{x}_i}{h_0 D_0 \hat{x}_{0,i}} x_1^* (T_w^* - T_i^*) \frac{2\pi dt^*}{w t_f} \quad (10)$$

$h_0, D_0,\ \hat{x}_{0,i}$ are all of the values used in the simulation described in Table 1. for the convective heat transfer coefficient, piston diameter, and piston amplitude. t_f is the time it takes to run a cycle of that refrigerator. Figure 6 shows the heat transfer at every instant throughout the cycle.

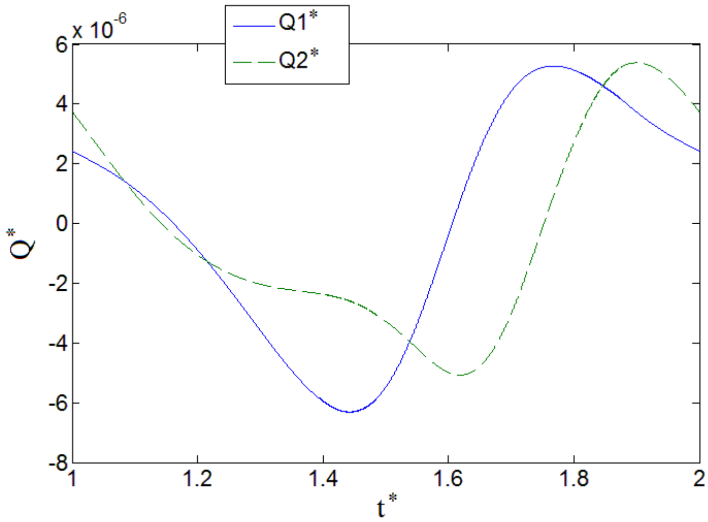

Figure 6 *- Heat transfer Q^*throughout the cycle in each chamber. Heat transfer from the walls to the helium is positive, so chamber 1 (expansion space) has a net cooling effect on the chamber wall, while chamber 2 (compression space) has a net heating effect on the chamber wall.*

To evaluate the refrigerator performance, the heat transfer and work input must be found:

$$Q_1 = \int_0^\tau h_1 x_1 D_1 \pi \left(T_{w,1} - T_1\right) dt \quad (11)$$

$$W = \int_0^\tau P_1 \frac{dV_1}{dt} dt + \int_0^\tau P_2 \frac{dV_2}{dt} dt \quad (12)$$

Over a cycle, the coefficient of performance is the ratio of the heat transfer to the total work input to the refrigerator, so that:

$$COP = \frac{Q_1}{W} \quad (13)$$

where *COP* is the coefficient of performance of a refrigerator. Table 2 shows results relevant to the performance of this Stirling refrigerator.

Table 2- Results from one cycle of a Stirling refrigerator

Variable	Value
Mean chamber 1 helium temperature	300.6048 K
Mean chamber 2 helium temperature	306.6404 K
Heat Transfer to chamber 1 from walls	1.9705 J
COP	1.2616

IV. CONCLUSIONS

In this paper, a Stirling refrigerator has been modeled and simulated numerically. The mathematical model investigated here serves as a comprehensive representation of the operation of a Stirling refrigerator. By minimizing the assumptions and limitations of previous studies, this model can be used and manipulated for a variety of different initial conditions, sizes, and applications. The governing equations are derived for both chambers and the regenerator using the basic conservation and transport laws. The model allows for heat transfer in each section of the refrigerator at any time. The results show a clear temperature difference between the two chambers, a net cooling power of 1.9705 J per cycle, and a COP of 1.2616 for chamber 1. Mathematically, this indicates that a small Stirling refrigerator could be used as an energy harvesting device.

В.В. Трандафілов, М. Г. Хмельнюк

Одеська національна академія харчових технологій, вул. Канатна, 112, Одеса, 65039, Україна

КОМПЬЮТЕРНЕ МОДЕЛЮВАННЯ ХОЛОДИЛЬНОЇ МАШИНИ СТІРЛІНГА

У даному чисельному дослідженні розглянута математична модель для точного моделювання продуктивності та докладної поведінки холодильної машини Стірлінга. Математична модель для альфа холодильної машини Стірлінга з гелієм в якості робочої речовини буде використовуватися в оптимізації механічної конструкції цих машин. Розроблена повна нелінійна математичну модель машини, в тому числі термодинаміка гелію і теплопередача від стінок, а також передача тепла і опір газу в регенераторі. Отримано безрозмірні групи, і чисельно розв'язано математичну модель. Змінюються важливі конструктивні параметри, і визначається їх вплив на продуктивність холодильної машини. Представлені результати моделювання холодильної машини Стірлінга, які включають передачу тепла і коефіцієнт перетворення.

Ключові слова: *Холодильна машина Стірлінга – Математична модель – Безрозмірні параметри – Енергоефективність – Гелій*

REFERENCES

1. **Gosney W. B.** (1982), "*Principles of Refrigeration". Cambridge University Press, U. K.*
2. **McFarlane P., Semperlotti F., Sen M.** (2013), "*Mathematical model of an air-filled alpha Stirling refrigerator*". *Journal of Applied Physics*, No 114.
3. **Wu F., Chen L., Wu C., Sun F. (**1998), "*Optimum performance of irreversible engine with imperfect regeneration*". *Energy Conversion and Management*, No 39(8), pp. 727-732.
4. **Kaushik S.C., Kumar S.** (2000), "*Finite time thermodynamic analysis of endore-versible Stirling heat engine with regenerative losses*". *Energy*, No 25, pp. 989-1003.
5. **V.S. Chakravarthy, R.K. Shah.** (2011), "*A review of refrigeration metods in the temperature range 4-300 k". Journal of Thermal Science and Engineering Applications*, No 3.
6. **Thombare D.G.** (2006), "*Technological development in the Stirling cycle engines*". *Renewable and Sustainable Energy Reviews*, No 12, pp. 1-38.
7. **A.T.A.M. de Waele.** (2011), "*Basic operation of cryocoolers and related thermal machines*". *Journal of Low Temperature Physics*, No 164, p. 179.
8. **Tekin Y., Ataer O.E.** (2010), "*Performance of V-type Stirling-cycle refrigerator for different working fluids*". *International Journal of Refrigeration*, No 33, pp. 12-18.
9. **Chen J., Yan Z.** (1996), "*The general performance characteristics of a Stirling refrigerator with regenerative losses*". *Journal of Physics D: Applied Physics*, No 29, pp. 987-990.
10. **Erbay L.B., Yavuz H.** (1997), "*The maximum cooling density of a realistic Stirling refrigerator*". *Journal of Physics D: Applied Physics*, No 31.
11. **Omari D.** (1996), "*Mathematical Modeling of Thermocompressive and Thermoacoustic Machines"*. PhD thesis, University of Notre Dame, Department of Chemical Engineering. IN 46556.
12. **McFarlane P., (**2014), "*Experiments with a presure-driven Stiling refrigerator with flexible chambers". Journal of Applied Physics*.