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OPTIMAL PLANNING OF TRIP AND ROUND TRIP CYCLE TIME ON AN URBAN ROUTE

Об'єктом дослідження є маршрут міського пасажирського транспорту загального користування. Одним з найбільш проблемних місць при організації перевезень на фіксованому міському маршруті є встановлення планової тривалості виконання рейсу та (або) зворотного рейсу. Складнощі виникають через те, що тривалість виконання рейсу на міському маршруті зазвичай є випадковою величиною, що необхідно враховувати при встановленні її планових значень, використовуваних надалі при складанні розкладів руху. Це, з одного боку, дозволяє покращити ефективність використання маршрутних транспортних засобів за рахунок зменшення їх непродуктивних простоїв, а з іншого – підвищити якість обслуговування пасажирів за рахунок зменшення тривалості очікування останніми транспорту на зупинках.

В ході дослідження використано метод стохастичної оптимізації планової тривалості рейсу. Це дало можливість знайти компроміс у вартісному виразі між ефективністю використання маршрутних транспортних засобів та якістю обслуговування пасажирів. Особливістю запропонованого оптимізаційного методу є врахування в узагальнених витратах непродуктивних простоїв маршрутних транспортних засобів, недоотриманого прибутку транспортного оператора та вартості транспортного часу пасажирів.

Застосування розробленого методу для умов тролейбусного маршруту № 14 міста Запоріжжя (Україна) дозволяє у порівнянні з існуючими плановими показниками зменшити узагальнені витрати на 12 %.

Наразі значно розширилися технічні можливості збирання, накопичення та обробки емпіричної інформації про умови виконання перевезень на міських маршрутах з використанням супутникових систем глобального позиціонування GPS. У таких умовах з використанням розробленого методу забезпечується можливість оперативного врахування при плануванні пасажирських перевезень як експлуатаційних, так і соціально-економічних чинників, у яких ці перевезення виконуються.

Ключові слова: *міський громадський транспорт, тривалість очікування, тривалість рейсу, узагальнені витрати.*

1. Introduction

The length of the round trip cycle time on the city public transport route is one of its most important technical and operational indicators. This value is used to determine the required number of route vehicles (RV), frequency and headway, the distribution of transit vehicles between routes, scheduling and timetabling, and the organization of combined modes of communication on routes [1].

The difficulty of rationing the duration of the round trip cycle time on the route is that it is a random variable, depending on which varies under the influence of a number of both controlled and uncontrolled factors. So, the duration of trip in real operating conditions is affected by [2]:

– traction-dynamic properties of the vehicles used on the route;
– design features of entrance and exit devices;
– the number of doors of the vehicle and the distribution of passengers between them;
– the intensity of passenger traffic;
– the intensity of the traffic flow on the route;
– road and climatic conditions (season of the year and time of the day, condition of the road surface, number of lanes, plan and longitudinal profile of the road, availability and frequency of intersections, technical means for organizing traffic);
– experience and psychophysiological condition of the driver.

Under such conditions, the duration of the layover time at the terminal stops of the route is established for several reasons. This is provision, on the one hand, a short-term rest of the driver, and on the other – the reliability of the timetable due to random fluctuations in the trip duration.

Reduction of the normative duration of a round trip relative to what is actually needed is the reason for the decline in the quality of passenger transportation, which, in particular, is expressed in the following:

– the time spent by passengers on waiting for transport at stops increases [3];
– there are refusals for passengers to board the RVs for overcrowding;
– the unevenness of the utilization degree of the RV passenger capacity is growing. Excessive increase in the normative duration of the round trip leads to an increase in the RV unproductive downtime and, as a result, an increase in the costs of transport enterprises for the maintenance of routes and the cost of travel for passengers.

Thus, on the one hand, the sufficient carrying capacity of the route depends on correct determination of the duration of the round trip, and on the other hand, the efficiency of the RV use and the economic activity of transport enterprises. Taking into account that this directly affects the level of service of passenger transportation on both sides, the problem of reliable planning of the duration of a round trip on the routes of public transport is actual. The urgency of the solution of the problem is increasing in modern conditions, the growth of the level of motorization in the cities, since high-quality transport services for residents by public transport make it possible to attract car owners before using it. This, in turn, makes it possible to reduce the severity of the problems associated with its harmful consequences [4].

2. The object of research and its technological audit

The object of research is a fixed route for public urban passenger transport.

The most technology of transport services for passengers on mass public transport in cities is the route technology, which is most effective at a settlement density of 3,750 people/km^2 [5]. The route technology provides for the RV movement organization along permanent routes in the form of a sequence of transportation cycles (*trips*). During the trip a passenger (entrance and exit of passengers) is carried out at specially equipped areas (*stops*) along the route [6]. Stops divide the route into *links*. Under such conditions, the operation of the transit vehicle is a sequence of individual elements of the transport process cyclically performed during a day (or working shift) (Fig. 1).

RV daily operation begins with its delivery from the parking lot (depot) to one of the terminal stops of the route. Such run is called a dead-head run, and its duration t_0 refers to the diurnal or variable operating cycle of the route vehicles. Next, RVs performs one or several cycles of transportation (*round trips*) on the route, each of which consists of the following consecutive elements:

– movement along the route from the terminus A to the terminus B (*trip in the forward direction*) duration t_{AB};
– layover (recovery) time at the terminus B duration t'_B;
– movement along the route from the terminus B to the terminus A (*trip in the backward direction*) duration t_{BA};
– layover (recovery) time at the terminus A before the next return trip t'_A.

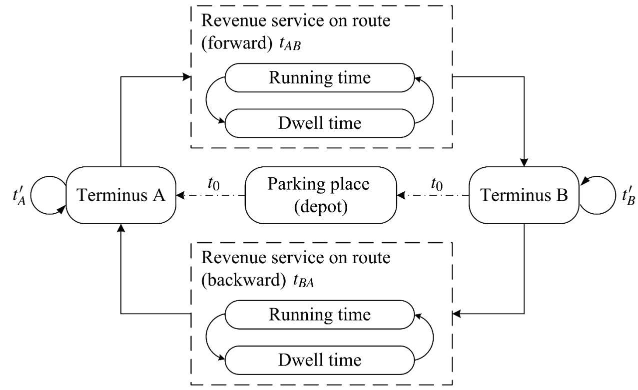

Fig. 1. Structure of the components of the transport process cycle on public urban transport

Throughout each of the trips, RVs moves with the links of the route, spending time directly on traffic (including delays associated with traffic management) and on dwell time at the stops for loading and/or unloading of the passengers. After the last planned round trip (or part of it), RVs returns from the final stop of the dead-head run to the depot.

Thus, the duration of the round trip on the route T_{rt} is determined by the sum of the durations of its components:

$$T_{rt} = t_{AB} + t'_B + t_{BA} + t'_A. \quad (1)$$

In practice, routes can exist, they have only one terminus (circular route) [7], in which case $t_{BA} = 0$ and $t'_B = 0$.

Two methods are used for scheduling the trip duration – *timing* and *calculation* [2].

The timing method is based on measuring the actual time spent on trips and their individual elements directly on the route with the fulfillment of certain requirements and conditions. Based on the results of these measurements, a statistical sample (usually a small volume) is obtained, based on which the calculation time for the trip is calculated by the formula:

$$t_c = \frac{3t_{\min} + 2t_{\max}}{5}, \quad (2)$$

where $t_{\min}$, $t_{\max}$ – respectively, the minimum and maximum value of the actual trip duration, determined from the sampling of the timekeeping data, min.

The calculation method consists in dividing the route into segments within which an approximate equality of the RV traffic conditions is ensured. After that, for each segment, the time spent on running and dwell RV are calculated taking into account all the operating factors in this section using analytical or graphical dependencies [8].

One of the most problematic places in the transportation organization on the route is the establishment of the planned trip and (or) the round trip duration. Difficulties arise because the trip duration on a city route is usually a random variable, which must be taken into account when establishing its planned values, used later when scheduling. This, on the one hand, makes it possible to increase the RV efficiency use by reducing its unproductive idle time, and on the other hand, to improve the quality of passenger service by reducing the waiting time for the transport at stops.

3. The aim and objectives of research

The aim of research is development of a method for optimizing the planned duration of a round trip on a city bus route, which, unlike existing ones, takes into account the perspectives of both transport operators and passengers.

To achieve this aim, it is necessary to solve the following tasks:

1. To develop and substantiate the economic-mathematical model of optimal planning of the round trip duration on a city route, taking into account the probabilistic nature of the transport process of passenger transportation.

2. Based on the empirical data obtained directly on the urban route network, to perform a factorial analysis of the developed model and give recommendations on the establishment of its parameters in practical terms planning the transport process of passenger traffic.

4. Research of existing solutions of the problem

The tasks of reliable determination of the round trip duration and its components both in the whole on the route and on its separate sections are given great attention to scientists and specialists in the field of urban passenger transportation. The main aim of research is mainly aimed at ensuring the reliability of the established timetable on the routes. It is clear that both the early arrival time at the stops and the late arrival of buses from them relative to the established timetable significantly impair the quality of transport services [9].

In [10], a calculation method for determining the trip duration on a bus route is proposed, which consists of the main and additional expenditures of time. In this case, the main time consuming is the running time, dwell time at the stops and the associated maneuvering. Additional time costs include traffic conditions and traffic flow intensity, delays at signalized and uncontrolled crossings. Each component of time expenditure is calculated by analytical, empirical formulas or presented as tabulated values and tables. The obtained value of the trip duration is adjusted in accordance with the mode of movement on the bus route and taking into account the probability of queues of buses before stops and the associated additional delays in traffic.

However, according to most researchers, the calculated trip duration thus obtained appears to be overestimated in most cases. Therefore, in order to improve the efficiency of the transit fleet, transport operators use the planned trip duration, somewhat less than or equal to the calculated one, when designing the timetables [11].

In contrast to the calculated ones, probability-statistical methods for rationing the trip duration and its individual components, which use the historical time data as source data, are much more widespread. The development of this direction contributes to the wide distribution for the control and dispatching of public transport of satellite systems of global positioning (Global Positioning System, GPS) and built on their basis automatic vehicle location registration (AVL) systems. At the same time, technical capabilities have significantly expanded as a collection of empirical information on the duration and speed of movement of vehicles, and its accumulation for subsequent analysis and forecasting [12].

Within the framework of probability-statistical methods for rationing the trip duration, two methodological approaches are formed in scientific research. The first one involves the construction of regression models (usually multifactorial linear models) in which various factors affecting the trip duration are included as independent variables [13–15]. Such factors are: route length, the number of stops, time of the day and season of the year, location of the route on the city's plan, passenger turnover, degree of passenger capacity utilization, road conditions, age and driver experience, RV specifications. The obtained regression curves have a coefficient of multiple determination from $R^2=0.59$ [13] to $R^2=0.69$ [14, 15], which indicates a satisfactory agreement of the obtained regression models and the possibility of obtaining a significant error in their practical application.

The second approach is normalization the trip duration based on the establishment of patterns of its statistical distribution as a random variable. Later, using the modeling methods (statistical modeling [16], modeling by structural equations [17]), the planned trip duration and (or) its components is determined.

The greatest distribution for describing the trip duration on a city route, due to their good correspondence with empirical data and relative simplicity, was the laws of normal and log-normal distribution [18]. However, in [19], based on a statistical analysis of the length of traffic on sections of six bus routes using GPS data, it is established that the hypothesis of its log-normal distribution is not confirmed in about 40 % of the observations. Proceeding from this, it is proposed to describe the duration of the motion with the help of a mixed probability distribution, which is the sum of the random values of the bus traffic duration on individual sections of the route, additionally calculating the correlation moments between the standard deviations of these durations. Based on the developed approach to establish the level of transport quality – the mean of this distribution for the evaluation of the efficiency of transit vehicle use and its standard deviation for assessing the reliability of passenger transport services.

An analysis of the statistical data of the TransLink Transit Autority transport company in Queensland, Australia, is carried out in [20]. On its basis, it is proposed to use the normal distribution law to describe the random value of the actual duration of the actual bus traffic. To describe the duration of the bus idle time at stops, it is proposed to use a discrete distribution, presented in the form of a systematized table. As a result, it is established that the random value of the trip duration, obtained as the sum of the durations of its individual components, is well described by the Gaussian mixed model, the use of which allows the reliability of the bus schedule to be fulfilled by 15 %. Similar studies in Brisbane, Australia, show that, compared to a series of continuous distributions of a random variable, the mixed Gaussian distribution model is better in terms of the accuracy of fit and the stability of the obtained results. In the study for comparison, the normal, log-normal, Weibull, logistic distribution laws and the law of gamma distribution are used.

The paper [16] is devoted to optimization of the trip duration, taking into account its scholasticity by the criterion of minimizing the weighted total expected time of deviations of the actual moments of arrival of buses to stops from the planned and the time of excessive work of the transit vehicles. Using the Monte Carlo method, the optimization model is transformed and presented as a linear programming problem with constraints in the form of inequalities. Based on the optimization results, the sensitivity analysis of the model is performed with the change of the initial data.

Recommended duration of layover (recovery) time at the terminus, according to most studies, is assumed to be 10...15 % of the trip duration [21], which in natural terms is 5...15 min. [22].

Summarizing the analysis of previous studies, it should be noted that they are considering the task of planning the trip duration on the route or only from the perspective of users of transport services (passengers) or only from the perspective of transport operators. It should also pay attention to the fact that an excessive increase in the reliability of the timetable makes, as a rule, an increase in the operating costs of transport operators. This, in turn, increases the cost of transportation services, which negatively affects the attitude of passengers to public transport. Thus, there is reason to state that the problem of determining the optimal planning of the trip duration on a bus route, taking into account the interests of both passengers and transport operators, remains unresolved.

5. Methods of research

As was shown above (Fig. 1), the RV round trip duration on the city route generally consists of the trip duration in each direction and the layover time at each of the two terminuses. Next, let's consider, as an optimized parameter, the trip duration in one of the directions with the addition of the layover time at the terminus at the end of this trip. It is clear that the remainder of the round trip duration (in the opposite direction) can be considered in a similar way.

In practical conditions, as a rule, the trip duration using probability-statistical methods is determined from a statistical sample of a limited volume. Let as a result of N observations on the city route a statistical sample of the actual trip duration $t_1, t_2, \ldots, t_N$ be obtained. Let the planned duration of the voyage be t_s ($t_{\min} \le t_s \le t_{\max}$, where $t_{\min}$ and $t_{\max}$ – respectively, the minimum and maximum value of the trip duration from the set of observations). Then a certain part of the trips is carried out with a duration that is shorter than planned ($t_i \in [t_{\min}, t_s)$), and a part – with the planned duration, or more ($t_i \in [t_s, t_{\max}]$).

If the actual trip duration is less than the planned one, an excessive idle time at the terminus occurs. The total value of this time, accumulated over the entire observation period, is:

$$T_i = \sum_{\substack{i=1 \\ t_i < t_s}}^{N} (t_s - t_i). \quad (3)$$

Such cases are undesirable for the transport operator perspective, since they lead to the loss of available transport resources. These losses are manifested in two aspects. First, an unproductive idle time of vehicle and driver lead to additional operational losses C_1, which in the cost amount are:

$$C_1 = c_i \cdot T_i, \quad (4)$$

where c_i – the cost of the RV idle time per unit time.

Secondly, for lost time could perform an additional number of trips T_i/t_s and, accordingly, give the transport operator the lost additional profit in the amount of:

$$C_2 = \frac{T_i \cdot Q \cdot \delta}{t_s}, \quad (5)$$

where Q – the average number of passengers on the route for the trip, pas.; δ – profit that the transport operator receives from transportation of one passenger.

Thus, the total costs of the transport operator consist of the sum of expenses from unproductive idle time of vehicles and the spent profit, on the average for one performed trip are equal:

$$C' = \frac{C_1 + C_2}{N} = \frac{T_i}{N}\left[c_i + \frac{Q \cdot \delta}{t_s}\right] = \\ = \frac{1}{N}\sum_{\substack{i=1 \\ t_i < t_s}}^{N} (t_s - t_i) \cdot \left[c_i + \frac{Q \cdot \delta}{t_s}\right]. \quad (6)$$

Similarly, if the actual trip duration exceeds the planned one, there is a delay in the RV departure on a return trip, the total amount accumulated over the entire observation period is:

$$T_d = \sum_{\substack{i=1 \\ t_i \ge t_s}}^{N} (t_i - t_s). \quad (7)$$

The delay in the RV departure on a return trip is undesirable for passengers, as it leads to additional time spent by them in waiting for the RV arrival at the stops. Based on the assumption that the total value of this additional waiting time is equal to the total delays in the RV departure to the trip in the opposite direction, let's obtain an expression for calculating losses reflecting the passenger point of view in the form:

$$C_3 = c_{pas} \cdot Q \cdot T_d, \quad (8)$$

where c_{pas} – the cost of passengers waiting for the RVs at the stops per unit time in value terms.

The total cost of passengers on average for one trip performed is:

$$C = \frac{C_3}{N} = \frac{c_{pas} \cdot Q \cdot T_d}{N} = \frac{c_{pas} \cdot Q}{N}\sum_{\substack{i=1 \\ t_i \ge t_s}}^{N} (t_i - t_s). \quad (9)$$

Thus, the problem of optimal planning of the trip duration on the route consists in determining the t_s value that minimizes the total costs of the transport operator and passengers (generalized costs) per one performed trip, that is:

$$C_\Sigma = C' + C'' = \frac{1}{N}\sum_{\substack{i=1 \\ t_i < t_s}}^{N} (t_s - t_i) \cdot \left[c_i + \frac{Q \cdot \delta}{t_s}\right] + \\ + \frac{c_{pas} \cdot Q}{N}\sum_{\substack{i=1 \\ t_i \ge t_s}}^{N} (t_i - t_s) \Rightarrow \min. \quad (10)$$

It is clear that the values of T_i (3) and T_d (7) in the general case depend on the law of distribution of the random value of the trip duration t, therefore let's give the formulation of the problem of optimal planning of the trip duration in a generalized form.

Let's suppose that the random value of the trip duration is given by the probability density function $f(t)$ defined on the interval $[t_{\min}, t_{\max}]$.

Then the mean of the trip duration from those trips which magnitude does not exceed the scheduled one t_s by the mean value theorem [23] is equal to:

$$M[t]_{t<t_s} = \frac{\int_{t_{\min}}^{t_s} t\cdot f(t)\mathrm{d}t}{\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t}. \quad (11)$$

Then the average deviation of the trip duration to the lower side for all trips, the duration of which does not exceed the scheduled one, is:

$$t_s - M[t]_{t<t_s} = t_s - \frac{\int_{t_{\min}}^{t_s} t\cdot f(t)\mathrm{d}t}{\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t}. \quad (12)$$

Since the proportion of trips which duration does not exceed the planned value, in their total amount $\int_{t_{\min}}^{t_s} f(t)dt$, let's finally obtain:

$$\frac{T_i}{N} = \left[t_s - \frac{\int_{t_{\min}}^{t_s} t\cdot f(t)\mathrm{d}t}{\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t}\right]\cdot\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t =$$

$$= t_s\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t - \int_{t_{\min}}^{t_s} t\cdot f(t)\mathrm{d}t. \quad (13)$$

For similar reasons, for a set of trips which duration is greater than or equal to the scheduled one, let's obtain:

$$\frac{T_d}{N} = \left[\frac{\int_{t_s}^{t_{\max}} t\cdot f(t)\mathrm{d}t}{\int_{t_s}^{t_{\max}} f(t)\mathrm{d}t} - t_s\right]\cdot\int_{t_s}^{t_{\max}} f(t)\mathrm{d}t =$$

$$= \int_{t_s}^{t_{\max}} t\cdot f(t)\mathrm{d}t - t_s\int_{t_s}^{t_{\max}} f(t)\mathrm{d}t. \quad (14)$$

Given the duration of layover time at the final stop t', let's obtain the expression for the objective function:

$$C_\Sigma = C' + C'' \Rightarrow \min, \quad (15)$$

$$C_\Sigma = \left[t_s\int_{t_{\min}}^{t_s} f(t)\mathrm{d}t - \int_{t_{\min}}^{t_s} t\cdot f(t)\mathrm{d}t\right]\cdot\left[c_i + \frac{Q\cdot\delta}{t_s+t'}\right] +$$

$$+ c_{pas}\cdot Q\cdot\left[\int_{t_s}^{t_{\max}} t\cdot f(t)\mathrm{d}t - t_s\int_{t_s}^{t_{\max}} f(t)\mathrm{d}t\right] \Rightarrow \min. \quad (16)$$

Let's consider, for example, the most particular case, when the random value of the trip duration is distributed according to the uniform distribution law. The density function of the uniform distribution has the form [23]:

$$f(t) = \begin{cases} \dfrac{1}{t_{\max}-t_{\min}}, & \text{if } t\in[t_{\min}, t_{\max}]; \\ 0, & \text{if } t\notin[t_{\min}, t_{\max}]. \end{cases} \quad (17)$$

Then, substituting (17) into (16) and performing elementary transformations, let's obtain:

$$C_\Sigma = \left[\frac{(t_s - t_{\min})^2}{2(t_{\max}-t_{\min})}\right]\cdot\left[c_i + \frac{Q\cdot\delta}{t_s+t'}\right] +$$

$$+ c_{pas}\cdot Q\cdot\left[\frac{(t_{\max}-t_s)^2}{2(t_{\max}-t_{\min})}\right] = \frac{1}{2(t_{\max}-t_{\min})}\times$$

$$\times\left[(t_s-t_{\min})^2\cdot\left(c_i + \frac{Q\cdot\delta}{t_s+t'}\right) + (t_{\max}-t_s)^2\cdot c_{pas}\cdot Q\right] \Rightarrow \min. \quad (18)$$

Differentiating expression (18) for t_s:

$$\frac{dC_\Sigma}{dt_s} = \frac{1}{2(t_{\max}-t_{\min})}\times$$

$$\times\left[\begin{array}{l} 2(t_s-t_{\min})\cdot\left(c_i + \dfrac{Q\cdot\delta}{t_s+t'}\right) - \\ -2\cdot Q\cdot c_{pas}\cdot(t_{\max}-t_s) - \dfrac{Q\cdot\delta\cdot(t_s-t_{\min})^2}{(t_s+t')^2} \end{array}\right]. \quad (19)$$

Equating the resulting expression to zero, let's obtain a fraction, in the numerator of which there is a cubic equation of the form:

$$Ax^3 + Bx^2 + Cx - D = 0, \quad (20)$$

where

$$A = 2(c_i + Q\cdot c_{pas});$$

$$B = Q\cdot\delta + 2\cdot c_i\cdot(2t' - t_{\min}) + 2\cdot Q\cdot c_{pas}(2t' - t_{\max});$$

$$C = t'\left[t'(c_i + Q\cdot c_{pas}) + Q(\delta - 2\cdot c_{pas}\cdot t_{\max}) - 2\cdot c_i\cdot t_{\min}\right];$$

$$D = Q(\delta\cdot t_{\min}^2 + 2\cdot c_{pas}\cdot t'^2\cdot t_{\max} + 2\delta\cdot t'\cdot t_{\min}),$$

and in the denominator the expression $(t_s + t')^2$ never turns to zero with positive values of t_s and t'. Solving the cubic equation (20), one can find the optimal value of the trip duration t_s^*, as one of its real roots.

If $t' \ll t_s$, then the calculations can be simplified by putting in (18) $t_s + t' \approx t_s$. Then, as a result of differentiation and further equating the resulting derivative to zero, let's obtain a cubic equation for finding the quantity t_s^* of the form:

$$Ax^3 + Bx^2 - C = 0, \quad (21)$$

where

$$A = 2(c_i + Q\cdot c_{pas});$$

$$B = Q\cdot(\delta - 2\cdot c_{pas}\cdot t_{\max}) - 2\cdot c_i\cdot t_{\min};$$

$$C = Q\cdot\delta\cdot t_{\min}^2.$$

If also assume that additional trips do not bring profit ($\delta=0$), which can take place, for example, on an isolated route, the optimal trip time from (21) is determined by a simple formula:

$$t_s^* = \frac{Q \cdot c_{pas} \cdot t_{\max} + c_i \cdot t_{\min}}{Q \cdot c_{pas} + c_i} = t_{\max} \cdot \frac{Q \cdot c_{pas}}{Q \cdot c_{pas} + c_i} + t_{\min} \cdot \frac{c_i}{Q \cdot c_{pas} + c_i}. \qquad (22)$$

In the case of a normal distribution of the trip duration with the probability density function:

$$f(t) = \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(t-\bar{t})^2}{2\sigma_t^2}}, \qquad (23)$$

where $\bar{t}$ – the mean of the trip duration, min.; σ_t – standard deviation of the trip duration, min.

The objective function for determining the optimal trip duration (16) will look like:

$$C_\Sigma = \left[t_s - \frac{\int\limits_0^{t_s} t \cdot \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{(t-\bar{t})^2}{2\sigma_t^2}} \mathrm{d}t}{\int\limits_0^{t_s} \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{(t-\bar{t})^2}{2\sigma_t^2}} \mathrm{d}t} \right] \cdot \left[c_i + \frac{Q \cdot \delta}{t_s + t'} \right] + c_{pas} \cdot Q \cdot \left[\frac{\int\limits_{t_s}^{+\infty} t \cdot \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{(t-\bar{t})^2}{2\sigma_t^2}} \mathrm{d}t}{\int\limits_{t_s}^{+\infty} \frac{1}{\sigma_t\sqrt{2\pi}} e^{-\frac{(t-\bar{t})^2}{2\sigma_t^2}} \mathrm{d}t} - t_s \right] \Rightarrow \min. \qquad (24)$$

Passing to the standardized normal distribution by replacing the variable in the integrands:

$$z = \frac{t - \bar{t}}{\sigma_t}; \; t = z \cdot \sigma_t + \bar{t}; \; dt = \sigma_t dz,$$

after integrating by parts, elementary transformations and substitution of integration boundaries, the objective function (24) will have the form:

$$C_\Sigma = \left[t_s - \bar{t} + \frac{\sigma_t \cdot e^{-\frac{(t_s-\bar{t})^2}{2\sigma_t^2}}}{\sqrt{2\pi} \cdot \Phi\left(\frac{t_s - \bar{t}}{\sigma_t}\right)} \right] \cdot \left[c_i + \frac{Q \cdot \delta}{t_s + t'} \right] + c_{pas} \cdot Q \cdot \left[\bar{t} - t_s + \frac{\sigma_t \cdot e^{-\frac{(t_s-\bar{t})^2}{2\sigma_t^2}}}{\sqrt{2\pi}\left[0{,}5 - \Phi\left(\frac{t_s - \bar{t}}{\sigma_t}\right)\right]} \right] \Rightarrow \min, \qquad (25)$$

where $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int\limits_0^z e^{-\frac{z^2}{2}} \mathrm{d}z$ – integral Laplace function.

Since the derivative of the function (25) with respect to the independent variable t_s contains in its expression the non-elementary Laplace function, the search for its minimum can be performed only by numerical methods. Expression (25) can also be used to find the optimal value for the trip duration, if the latter has a log-normal distribution, for which it is sufficient to apply the logarithm operation to the initial statistical sample of the time length of the trips.

6. Research results

Let's consider a practical example of the application of the optimization model for the trip duration and turnover of the rolling stock on the city trolleybus route No. 14 «Simferopol Highway – Waterfront» in the city of Zaporizhzhia (Ukraine) with a pendulum traffic scheme. Based on the results of a comprehensive survey of passenger flows conducted in April 2017, the sample data is obtained on the duration of $N=20$ trips in each of the directions of the route (Table 1), the main statistics of which are given in Table 2.

Table 1

Observation data on the trip duration on the city trolleybus route No. 14 of the city of Zaporizhzhia (Ukraine)

t_{AB}, min.	60	64	62	62	67	62	63	65	65	65
	55	72	60	68	63	68	60	63	62	65
t_{AB}, min.	60	64	62	66	62	64	55	54	67	63
	61	56	58	60	63	64	68	62	64	54

Table 2

The basic statistics of the trip duration on the city trolleybus route No. 14 of the city of Zaporizhzhia (Ukraine)

Statistics	The meaning of statistics in the direction of movement	
	Forward	Backward
1. Minimal value $t_{\max}$, min.	55	54
2. Maximal value $t_{\max}$, min.	72	68
3. Sample mean $\bar{t}$, min.	63.55	61.35
4. Standard deviation σ_t, min.	3.65	4.13

First, let's establish the character of the distribution of the random variable in each sample. Since we are dealing with small samples, let's apply nonparametric methods. Note that in the case of large $(N>30)$ samples, for this it is possible to apply, for example, the Pearson criterion χ^2.

First, let's test the hypothesis of the normal distribution of the trip duration by the criterion of the mean absolute deviation [24], for which let's calculate the statistics of this criterion using the formula:

$$d = \frac{1}{N \cdot \sigma_t} \sum_{i=1}^{N} \left| t_i - \bar{t} \right|. \qquad (26)$$

Let for the trip duration in the forward direction

$$\sum_{i=1}^{N} \left| t_i - \bar{t} \right| = 54.1, \quad d_{AB} = 0.741,$$

for the trip duration in the opposite direction

$$\sum_{i=1}^{N} \left| t_i - \bar{t} \right| = 65.6, \quad d_{BA} = 0.794 .$$

The hypothesis of the distribution normality is not ruled out if the condition is satisfied $d_1 \le d \le d_2$, where

d_1 та d_2 – the critical values of the criterion, which are equal $d_1 = 0.7304$ and $d_2 = 0.8768$ at the significance level of 0.05 and at the sample size $N = 20$ [24]. Obviously, the value d_{AB} and d_{BA} fall into the area of hypothesis acceptance, therefore the hypothesis of the normal distribution of the trip duration in the forward and opposite directions of the route isn't rejected.

In the future, let's calculate by the expression of the objective function (25) with the argument t_s, which changes with the step Δ_t=1 min., and the values of the constant parameters: $c_i = 0.1$ USD/min.; $c_{pas} = 0.002$ USD/min.; $Q = 158$ pas.; $\delta = 0.021$ USD/pas.; $t' = 10$ min.

The results of the calculation in the form of graphs of changes in generalized costs, depending on the change in the planned trip duration, are shown in Fig. 2. It also provides, for comparison, the graphs of the change in generalized costs for the distribution of the trip duration according to a uniform law.

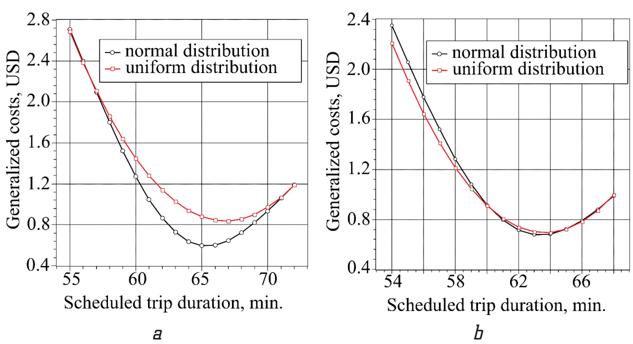

Fig. 2. The dependence of generalized costs on the value of the planned trip duration on trolleybus route No. 14 of the city of Zaporizhzhia: *a* – in the forward direction; *b* – in the opposite direction

Thus, the minimum generalized costs are achieved in the case of the planned trip duration: in the forward direction $t_{AB} = 65$ min. ($C_{\Sigma}^{\min} = 0.597$ USD), in the opposite direction $t_{BA} = 63$ min. ($C_{\Sigma}^{\min} = 0.678$ USD). Taking into account the duration of the inter-trip idle time at each of the final stops $t'_A = t'_B = 10$ min., the optimal planned duration of the round trip on the route according to (1) will be:

$$T_{rt}^{*} = 65 + 63 + 10 + 10 = 148 \text{ min.}$$

In comparison with the existing planned indicators $t_{AB} = 64$ min. and $t_{BA} = 61$ min., the total duration of the trip duration after optimization is increased by 3 min., which gives a reduction in generalized costs per one round trip from $C_{\Sigma}^{\min} = 1.43$ USD to $C_{\Sigma}^{\min} = 1.28$ USD that is, by almost 12 %.

Let's note that in the case of the distribution of random values of the trip duration in each of the directions of the route according to the uniform law, the optimal planned duration of the round trip is $T_{rt}^{*} = 151$ min.

As can be seen from the expression of the objective function of generalized costs (16), the following parameters affect the optimal trip duration on the city route t_s^{*}:
– the nature of the distribution of the random value of the trip duration and its numerical characteristics;
– cost of unproductive RV idle time per unit of time c_i, USD;
– average number of passengers on the route for the trip Q, pas.;
– profit that the transport operator receives from transportation of one passenger δ, USD;
– the passenger's expenses for waiting at a stop in a unit of time c_{pas}, USD;
– duration of layover time at the terminus of the route t', min.

Let's analyze the nature and degree of influence of the model parameters on the resultant indicator t_s^{*} by studying the dependence of the change in its value on the change in each of these parameters.

Fig. 3 shows the characteristic graph of the change in the optimum trip duration, depending on the change in the model parameters for the forward direction of traffic on the investigated route. The upper limits of the range of variation of parameters are chosen in such way as to illustrate their effect on the effective indicator within the limits of its actual values obtained as a result of the timing.

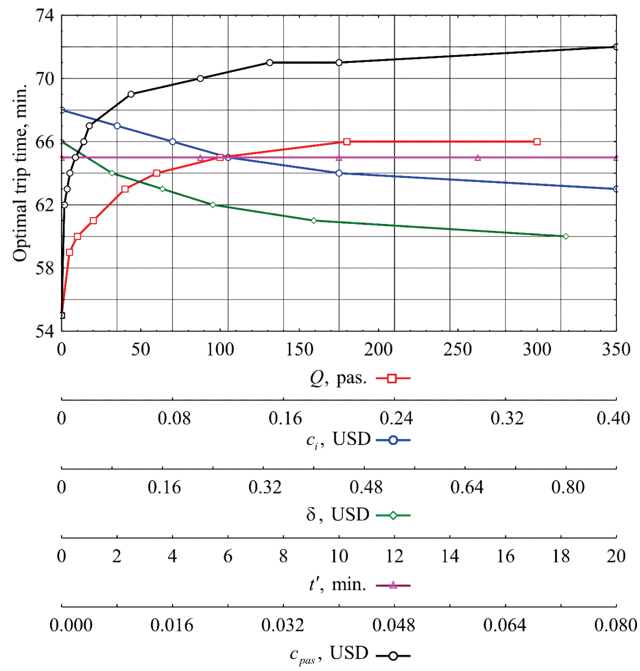

Fig. 3. Characteristic graph of the dependence of the optimum trip duration of the model parameters

The impact of the cost of RV unproductive idle time per unit of time. C_i expresses the so-called fixed costs of the transport operator, which is a component of the transportation cost [2]. These costs, which are attributed to one hour of operation, include driver's wages with accruals, depreciation of transport fleet and general production costs. When it increases c_i, the value of the optimum trip duration on the route decreases nonlinearly in a limited range of 63...68 minutes, which is approximately 30 % of the span of the random value of the duration of the trip.

Influence of the average number of passengers on the route for per one trip. The optimal trip time along the route t_s^{*} increases nonlinearly with increasing Q, while it is limited by the above value, in the example given it is approximately 65 % of the span of the random value of the trip duration. The value of Q is a characteristic of a particular route and reflects the ratio of demand for transport and passenger capacity of the transit vehicles used on this route. According to the comprehensive survey of passenger traffic in the city of Zaporizhzhia in 2017, the actual values of this parameter are in the range:

– for the bus $Q_a = 3.6\ldots113.8$ pas. (an average value is $\bar{Q}_a = 30.05$ pas. and a coefficient of variation $\nu_a = 0.48$);

– for the trolleybus $\bar{Q}_{tr} = 46.9\ldots210.0$ ($\bar{Q}_{tr} = 127.3$ pas., $\nu_{tr} = 0.52$);

– for the tram $\bar{Q}_t = 59.3\ldots104.7$ pas. ($\bar{Q}_t = 76.7$ pas., $\nu_t = 0.57$).

Influence of profit that the transport operator receives from transportation of one passenger. The profit received by the transport operator from the carriage of one passenger is the ratio of the difference between the income from the providing of passenger transportation services and the operating costs to the volume of transport of passengers on the route. As the value of δ increases, the optimal trip time along the route decreases nonlinearly. In practical terms, the value of δ can be estimated using the known values of the tariff for transportation T and the planned profitability of transport R by the formula:

$$\delta = \frac{T \cdot R}{1+R}. \quad (27)$$

If, for example, the size of the tariff (fare) is $T=0.16$ USD, and the planned profitability of traffic $R=0.15$ (15 %), then $\delta=(0.16\cdot0.15)/(1+0.15)=0.021$ USD.

Influence of the passenger's expenses on the RV waiting time at a stop in a unit of time. This parameter is determined by the cost of the so-called one passenger-hour or more general concept of the cost of transport time (VTTI, Value of Travel Time Savings) and is rather subjective. It is an indicator characterizing the passenger's potential expenses due to passive waiting for transport at a stop. In this case, the c_{pas} magnitude can be estimated both from the point of view of the passenger, and from the point of view of the transport operator or the state as a whole [25]. On urban passenger transport it is recommended to take within 35...60 % of per capita income of a citizen per unit of time [26].

According to the main department of statistics in the Zaporizhzhia region [27], the average per capita income in the region in 2016 was 1 thousand 686 USD, which is 0.842 USD/hour with an annual fund of working hours of 2003 hours. Thus, the range of the change in the c_{pas} value saved can be estimated in the range of 0.29...0.50 USD/h or 0.005...0.008 USD/min. As the value c_{pas} increases, the optimal trip duration increases nonlinearly (Fig. 3), propagating practically over the entire range of its timing, with the largest growth rates falling precisely on the interval $c_{pas} \in [0;0.008]$. Therefore, the responsible choice of the c_{pas} value in the proposed optimization model is decisive.

Influence of the duration of the layover time at the route terminus. As can be seen from Fig. 3, the duration of the layover time t' has almost no effect on the optimal trip length on the route, so in practical calculations it can be neglected.

7. SWOT analysis of research results

Strengths. Among the strengths of research, it should be noted the universality of the developed method in relation to the characteristics of the random magnitude of the trip duration on the route. This makes it possible to determine the optimal trip duration for an arbitrary law of distribution of its random variable, presented in analytical or tabular form, or directly from the data of a sample of actual timekeeping observations.

Weaknesses. To the weak sides of the conducted research it is necessary to carry the assumption that the average waiting time of the passenger at the stop when the delay in the RV departure on the trip is delayed by the amount of this delay. Thus, the additional time costs associated with the irregularity of the RV movement are taken into account. In addition, as shown in [28], the appointment of the planned trip duration is less than its mathematical expectation, leads in the limiting case to the actual failure of the timetable for the route. Given this, the obtained value of the optimal trip duration on the route may be subject to adjustment.

Opportunities. Additional opportunities that arise when applying the developed method consist in the possibility of promptly adjusting the planned trip duration. This is made possible by automated collection of time-related data on the duration of elements of the transport process of passenger traffic in cities through the use of GPS monitoring systems for the RV movement. In this case, the accumulated statistical data can be used both for analysis of the implementation of the established schedule and regularity of traffic, and for forecasting changes in planned targets under the influence of external factors. Among such factors, in particular, include: changes in the level of loading of city highways by traffic, traffic signal regulation at intersections, the type of transit vehicles on routes, the opening of new or closing existing routes, and the like.

Threats. Threats in using the developed method in practical conditions consist in the subjectivity of the approach to determining the cost of waiting time for a passenger, which, however, significantly affects the calculation results. The value of transport time is determined by the general state of the country's economy and today it is much smaller in Ukraine than in the economically developed countries of the world. As the decrease in this value reduces the planned trip duration and, thus, the reliability of the timetable along the route, the use of the developed method for routes with a relatively low frequency of movement (with intervals of more than 10 minutes) is limited.

8. Conclusions

1. An optimization model has been developed for planning the trip duration and the round trip on the city public transport route, which takes into account the probabilistic nature of the transport process of passenger transportation, the perspectives of the transport operator and passengers. The analytical dependencies for determination of the optimal value of the trip duration for cases of its normal and uniform distribution are obtained.

On the example of the city trolleybus route No. 14 of the city of Zaporizhzhia, the optimal trip duration in each direction is determined, are random variables distributed according to the normal distribution law. Compared with the existing planned indicators on the route, the total trip duration in the directions after optimization is increased by 3 minutes. That gives a reduction in the generalized costs per one performed a return trip from $C_\Sigma^{\min} = 1.43$ USD to $C_\Sigma^{\min} = 1.28$ USD, which is almost 12 %.

2. Based on the empirical data obtained during the comprehensive survey of passenger flows in the city of

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Zaporizhzhia (Ukraine) in 2017, a factorial study of the developed model is carried out. This makes it possible to obtain the character and degree of influence of its parameters on the value of the optimum trip duration on the route. It has been established that the layover duration in the range of $t'=0...20$ min. practically does not affect the amount of planned trip duration. Recommendations are given on establishing the values of model parameters in practical calculations.

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