ANALYSIS OF MODERN APPROACHES TO THE FORMATION OF THE PORTFOLIO INVESTOR SHARES STOCK

1. Introduction

Improving the quality of the formation of investment portfolios is significantly associated with the use of modern mathematical methods. Such a task is not easy and requires the development of mathematical and information modeling tools. Therefore, the creation and development of elements of modern information technologies to solve this problem is urgent.

The functional orientation of the production activities of enterprises that are not institutional investors, defines as a priority form of real investment. However, during individual stages of enterprise development, it is advisable to make financial investments. The choice of this form of investment may be due to:

- need for effective use of investment resources, which are formed by the beginning of real investment for selected investment projects;
- in the event that market conditions make it possible to achieve a higher level of return on invested capital than operating activities in «fading» commodity markets;
- if necessary, the use of temporarily free monetary assets as a result of the seasonal activities of enterprises;
- in cases of possible «seizure» of other enterprises for sectoral, commodity or regional diversification of their activities by investing capital in their statutory funds (or acquiring a decisive block of shares), etc.

At the present stage of economic development, investment activity of individual investors and legal entities provides for the investment of excess (temporarily free) funds not in one, but in a large number of investment objects, thereby generating a certain diversified aggregate of them. This method is called «portfolio investment».

Portfolio structure is the ratio of specific types of securities in a portfolio, which is the ratio of the investment portfolio with a specified characteristic of risk and return. The portfolio structure is a ratio of ideal investment objects, thereby generating a diversified aggregate of them.

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2. The object of research and its technological audit

The object of research is an investment portfolio consisting of a set of investment instruments (securities, assets, projects, etc.) in which the investor’s finances are distributed. The question of finding the optimal ratio of assets in the investment portfolio has been and remains promising so far.

The papers [1, 2] address the problems of portfolio investment in the stock exchange market of Ukraine.

The study of the problems of financing innovation activities of business entities and current trends in financial activities in Ukraine are highlighted in [3, 4].
By the middle of the XIX century, risk was determined only qualitatively, that is, most managers used a generalized classification of shares, dividing them into conservative, cheap, growth, profitable and speculative. Such imperfections regarding the evaluation of investments created favorable conditions for scientists – they try to apply the formal analytical technique to the practical problems associated with the choice of investments [5]. As a result, a significant amount of new ideas emerged regarding the investment process, and finally formed the modern theory of investment valuation, or portfolio theory.

One of the most problematic places is the difficulty of managing the parameters of risk and portfolio returns due to the constant change in the financial performance of investment assets in the face of economic uncertainty.

3. The aim and objectives of research

The aim of research is a mathematical and information modeling of a stock portfolio, characterized by an optimal structure and minimal risk. For this it is necessary to solve the following tasks:

1. To analyze the main possible situations in the formation of the investment portfolio and possible solutions.
2. To carry out the analysis and selection of mathematical methods and models.
3. To perform selection of the optimal portfolio structure at the request of the investor, taking into account various rates of return, risk and stock correlation coefficients.

4. Research of existing solutions of the problem

Scientists have proposed a set of basic mathematical models for the formation of a portfolio of stocks, which make it possible to describe both the individual investment quality of stocks, and their combination in the portfolio Markowitz model, Sharpe model, Tobin model and others.

The Markowitz model is based on the fact that the return indicators of various securities are interrelated: with the growth of the return of some securities, a simultaneous growth is observed in some others, while the return in other securities remains unchanged or vice versa, decreases [6].

The Sharpe model considers the relationship between the profitability of each security and the profitability of the market as a whole [7]. This model is a simplified version of the Markowitz model, in which the earnings per share are strictly correlated with the overall market index, which greatly simplifies the procedure for finding an effective portfolio. The use of the Sharpe model requires much less computation, and therefore it is more suitable for practical use.

The Tobin model is more related to the market structure than to the structure of acceptable portfolios [8]. The proposed model is an improved Markowitz model, which may additionally contain risk-free assets, in which it is proposed to use cash.

The authors of [9, 10] conduct an empirical study of the Markowitz model based on mathematical and statistical methods for normalizing the profit of investment assets. But these studies are conducted only for the stock markets of the United States and Japan with a high level of business activity. And in [11], the authors estimate the stability of portfolios and the probability of portfolio turnover using the example of the Indian Stock Exchange. However, the above studies are not acceptable for Ukraine due to the unstable economic situation.

In [12, 13], the authors propose to reformulate the problem of the investment portfolio as a cybernetic system, where the investor is a control system, and the portfolio is a controlled system. However, in this work, the controls of such a system are not fully disclosed, since control can be exercised only under condition of complete certainty of all features of the stock market.

In [14, 15], approaches to the formation of an investment portfolio based on the methods of delta normalization, variation-covariance, and Monte Carlo are considered. The use of the latter is justified, since it is the most suitable and flexible in measuring the value of risk. However, this method is very time consuming and unstable for analyzing a large number of assets.

The work [16] is devoted to the analysis and optimization of the investment portfolio and the calculation of its basic correlation and covariance coefficients for collective investments (investment companies and pension funds). But the question of portfolio formation for private investors remains open.

Summarizing the above analysis of literary sources, it should be noted the lack of funds for the formation of the investment portfolio in the modern stock market. This determines the prospects of studies of the effectiveness of the investment strategy for the formation of a stock of shares.

5. Methods of research

In the formation of the investment portfolio, some standard or non-standard situations may arise. Standard situations are situations when a formed investment portfolio fully complies with the requirements that are put at the previous stage of its formation [17]. These requirements may include: the objectives of the formation of the investment portfolio; value of return or risk of the portfolio and the like. In a standard situation, the formation of a portfolio takes place clearly in the stages that are defined in the first chapter. Such a portfolio is predictable.

Non-standard situations arise when an investment portfolio is formed does not meet (or does not fully comply) with the requirements that were set at the beginning. There are two main non-standard situations:

- when a non-standard situation arises from a standard situation;
- when a non-standard situation leads to a non-standard situation.

The first kind of non-standard situation is possible when the customer sets specific initial conditions:

- it is a specific goal, which must meet the investment portfolio;
- the customer establishes the value of the return and risk of the portfolio, and wants to receive a portfolio, the value of the return and risk of which will be equal to what it has established.

But when forming a portfolio, a situation arises when it is impossible to fulfill the goals set, or the value of the return and risk of the formed portfolio does not correspond to what the customer has set [18]. In this case, there are several solutions to the situation. Firstly, if obtained values of the portfolio indicators are not much different from those set by the customer, it can accept these new conditions. Another way out of this situation is changing
the composition of the portfolio. In this case, it is possible to try to get the set values of the portfolio by changing the share of assets in the portfolio.

The second type of non-standard situation arises when an investor investing funds in an investment portfolio does not define specific goals with which the portfolio must correspond. There may be another situation where, at the previous stage of portfolio formation, an investor does not indicate specific values of portfolio indicators, but after forming a portfolio, it can accept the obtained values, or suggest changing them by changing the structure of the portfolio. In addition, a situation may arise when the investor itself chooses joint stock companies in which it wants to invest his money. In this case, the formation of a portfolio is reduced to the choice of the optimal portfolio structure from the selected assets. Investment portfolios in a non-standard situation and uncertain conditions are formed for a short time and for investors who have the purpose only to increase their income from the formation of the portfolio.

As a rule, the solution of non-standard situations in the formation of an investment portfolio is solved in two ways: it is possible to correct the stages of portfolio formation or change the structure of the portfolio by changing the ratio of assets of joint-stock companies in the portfolio.

The situations described above characterize only a part of the basic situations that may arise during the portfolio formation. However, there may be other situations. The solution of these situations depends on the requirements for the portfolio formation.

Modern portfolio theory treats risk in quantitative terms. At the same time, based on a thorough analysis and evaluation of individual securities, gives a quantitative definition of the purpose of the portfolio. Depending on the set parameters of the ratio of income and risk of the portfolio, the composition of the portfolio and the models with which it will be calculated are determined.

Comparison of the main methods of forming the investment portfolio is given in Table 1.

The main indicators of the portfolio in each model are the risk and return on the portfolio. But for each of these methods, the calculation of these parameters is performed by its formulas. In addition, it can be noted that the overall formulation of the problem of forming investment portfolios is the same: it is necessary to create an investment portfolio for which the maximum value of return will be achieved with a minimum risk of the portfolio. However, each author presents a solution to this problem in his own way.

Today, the Markowitz model is used mainly in the first stage of forming a portfolio of assets in the distribution of invested capital for various types of assets: stocks, bonds, real estate, etc. Sharpe model is used in the second stage, when capital invested in a certain segment of the asset market, is distributed between individual specific assets that make up the selected segment (that is, for specific stocks, bonds, etc.).

6. Research results

As a result of the research, it is found that to solve the problem of forming the optimal investment portfolio taking into account the criteria of minimizing risks and maximizing profits, it is advisable to use a combination of the above methods.

For example, let’s build the optimal portfolio for the Ukrainian stock market, contains data on four Ukrainian enterprises for the period 2015–2016. On the basis of data on stock quotes of Ukrainian enterprises [19], calculate the profitability and risk of each stock (Table 2) and the correlation coefficients between the stock returns (Table 3).

After the calculated return and risk of each stock, it is possible to proceed to the formation of a portfolio of these shares. Let’s assume that the portfolio will include all the shares, but with different shares, and the formation of the portfolio of shares will be carried out in two directions. The first (direct task) is when the profit maximization portfolio is formed, the second (inverse problem) the risk minimization portfolio is formed. In the first case, it is required to set the acceptable value of the risk of the portfolio, in the second – the acceptable value of the portfolio return.

The Lagrange function for the problem of minimizing the risk (inverse problem) for a fixed level of income is written as:

\[ L = \sum_{i} w_i \sigma_i + \lambda_1 \left( \sum_{i} w_i P_i - P \right) + \lambda_2 \left( \sum_{i} w_i - 1 \right), \]  

(1)

where \( \lambda_1, \lambda_2 \) – Lagrange multipliers; \( w_i, w_j \) – vectors, equal parts of each i-th and j-th share in the portfolio; \( \sigma_j \) – the value of the coefficient

### Table 1

<table>
<thead>
<tr>
<th>Comparison factors</th>
<th>Markowitz model</th>
<th>Sharpe model</th>
<th>Tobin model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Possibility of forming an investment portfolio</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Ability to use the calculation of portfolio returns</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Ability to apply portfolio risk calculation</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Accounting for the number of issues in one portfolio</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Admissibility of various types of securities in the portfolio</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Algorithmic accounting of various types of securities in the portfolio</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Accounting for the distribution of invested capital among specific portfolio assets</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Taking into account the possibility of monitoring and maintaining the established securities portfolio</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Stock</th>
<th>Returns, UAH</th>
<th>Risk, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEEN Centrenergo</td>
<td>12.003</td>
<td>0.41</td>
</tr>
<tr>
<td>DON Donbasenergo</td>
<td>15.03</td>
<td>0.75</td>
</tr>
<tr>
<td>ZAEN Zakhidenergo</td>
<td>15.22</td>
<td>0.93</td>
</tr>
<tr>
<td>BAVL Bank Aval</td>
<td>13.71</td>
<td>1.19</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Share</th>
<th>CEEN</th>
<th>DON</th>
<th>ZAEN</th>
<th>BAVL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEEN</td>
<td>1</td>
<td>0.32</td>
<td>0.199</td>
<td>–0.06</td>
</tr>
<tr>
<td>DON</td>
<td>–</td>
<td>1</td>
<td>0.47</td>
<td>0.07</td>
</tr>
<tr>
<td>ZAEN</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>–0.61</td>
</tr>
<tr>
<td>BAVL</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1</td>
</tr>
</tbody>
</table>
of covariance of the stock; \( \sigma_{n} \) – the value of the risk coefficient of each \( i \)-th stock; \( \sigma_{n} \) – the value of the risk ratio of the portfolio.

The minimum risk portfolio is for all assets \( i = 1 \ldots n \) and \( j = 1, 2 \) by the condition:

\[
\frac{\partial L}{\partial w_{i}} = 0, \quad \frac{\partial L}{\partial \lambda_{j}} = 0. \tag{2}
\]

This first-order condition determines the linear system of equations, and therefore allows the use of matrix methods.

Let’s consider the Lagrange function for a future portfolio of twelve assets:

\[
L = \sum_{i=1}^{n} \left( \sum_{j=1}^{2} w_{ij} \sigma_{ij} + \frac{1}{2} \lambda_{j} \sigma_{ii} \right) + \sum_{i=1}^{n} \left( \sum_{j=1}^{2} \lambda_{j} \sigma_{ij} \lambda_{j} \right) + \lambda_{3}(w_{3} + w_{2} + w_{3} + w_{4} - 1). \tag{3}
\]

First order conditions for this task:

\[
\frac{\partial L}{\partial w_{1}} = 2w_{1} \sigma_{11} + 2w_{2} \sigma_{12} + 2w_{1} \sigma_{13} + 2w_{2} \sigma_{14} + \lambda_{1} \sigma_{11} + \lambda_{2} = 0, \\
\frac{\partial L}{\partial w_{2}} = 2w_{2} \sigma_{21} + 2w_{1} \sigma_{22} + 2w_{2} \sigma_{23} + 2w_{1} \sigma_{24} + \lambda_{1} \sigma_{22} + \lambda_{2} = 0, \\
\frac{\partial L}{\partial w_{3}} = 2w_{3} \sigma_{31} + 2w_{1} \sigma_{32} + 2w_{2} \sigma_{33} + 2w_{1} \sigma_{34} + \lambda_{1} \sigma_{33} + \lambda_{2} = 0, \\
\frac{\partial L}{\partial w_{4}} = 2w_{4} \sigma_{41} + 2w_{1} \sigma_{42} + 2w_{2} \sigma_{43} + 2w_{1} \sigma_{44} + \lambda_{1} \sigma_{44} + \lambda_{2} = 0, \\
\frac{\partial L}{\partial \lambda_{1}} = w_{1} \sigma_{11} + w_{2} \sigma_{12} + w_{3} \sigma_{13} + w_{4} \sigma_{14} - \sigma_{11} = 0, \\
\frac{\partial L}{\partial \lambda_{2}} = w_{1} \sigma_{22} + w_{2} \sigma_{23} + w_{3} \sigma_{24} + w_{4} \sigma_{24} - \sigma_{22} = 0, \\
\frac{\partial L}{\partial \lambda_{3}} = w_{3} + w_{2} + w_{3} + w_{4} - 1 = 0.
\]

In the matrix form, the system of equations:

\[
\begin{bmatrix}
2\sigma_{11} & 2\sigma_{12} & 2\sigma_{13} & 2\sigma_{14} & \sigma_{11} & 1 & \vdots & w_{1} \\
2\sigma_{21} & 2\sigma_{22} & 2\sigma_{23} & 2\sigma_{24} & \sigma_{22} & 1 & \vdots & w_{2} \\
2\sigma_{31} & 2\sigma_{32} & 2\sigma_{33} & 2\sigma_{34} & \sigma_{33} & 1 & \vdots & w_{3} \\
2\sigma_{41} & 2\sigma_{42} & 2\sigma_{43} & 2\sigma_{44} & \sigma_{44} & 1 & \vdots & w_{4} \\
\sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & 0 & 0 & \vdots & \lambda_{1} \\
1 & 1 & 1 & 1 & 1 & 1 & \vdots & \lambda_{2}
\end{bmatrix}
\begin{bmatrix}
w_{1} \n w_{2} \n w_{3} \n w_{4} \n \lambda_{1} \n \lambda_{2}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix}.	ag{5}
\]

Denoting the risk-return matrix as \( V \), vector \( (w, \lambda) \) as \( W \), and the vector on the right side as \( K \), let’s write the resulting system of equations as the product \( V \cdot W = K \), the solution of which with respect to \( W \) is:

\[
W = K \cdot V^{-1}. \tag{6}
\]

This solution determines the optimal portfolio of four assets that provides the desired level of return at a fixed level of risk. The vectors \( w_{1}, w_{2}, w_{3} \) and \( w_{4} \) are respectively equal to the share of each stock in the portfolio. The \( d_{p} \) parameter is the allowable return on a portfolio that the user sets.

The values of \( \sigma_{11}, \sigma_{12}, \sigma_{13}, \ldots, \sigma_{44} \) correspond to the values of the covariance coefficient (Table 4).

For comparison, the composition of the portfolios is calculated three variants of the portfolio with different acceptable values of portfolio returns.

For the first case, the value of the permissible return \( d_{p} \) is assumed not less than 40 %. Then we have the following value of the matrix \( K \):

\[
K_{1} = \begin{bmatrix}
0 \\
0 \\
0.6 \\
1
\end{bmatrix}
\]

According to the obtained values of the matrices \( K \) and \( V \) by the formula (6), it is possible to obtain the value of the matrix \( W \):

\[
W_{1} = \begin{bmatrix}
0.487021 \\
0.123290 \\
-0.042951 \\
0.402640 \\
-0.000237 \\
0.000091
\end{bmatrix}
\]

In the second case, let’s take the value of the allowable return \( d_{p} \) not lower than 40 %. Substituting the calculated values of profitability and risk into formula (5), let’s obtain the following matrix values:

\[
K_{2} = \begin{bmatrix}
0 \\
0 \\
0.4 \\
1
\end{bmatrix}
\]

Substituting the calculated values of profitability and risk into formula (5), let’s obtain the following matrix values:

\[
K_{3} = \begin{bmatrix}
0 \\
0 \\
0.6 \\
1
\end{bmatrix}
\]
For the second variant of the value of the matrix \( W \) is as follows:

\[
W_2 = \begin{pmatrix}
0.145898 \\
0.122079 \\
0.024615 \\
0.723408 \\
0.000050 \\
-0.000034
\end{pmatrix}.
\]

In the third case, let’s take the value of the permissible return \( d_p \) not lower than 10 %. Then the matrix \( K \) has the following form:

\[
K_3 = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0.1 \\
1
\end{pmatrix}.
\]

For this case, the matrix \( W \) has the following form:

\[
W_3 = \begin{pmatrix}
-0.365786 \\
0.120263 \\
0.081964 \\
0.803559 \\
0.000481 \\
-0.000221
\end{pmatrix}.
\]

After analyzing the obtained values of the matrices \( K \) for each case, it is possible to determine the composition of the portfolio. The calculated portfolio results are shown in Table 5.

<table>
<thead>
<tr>
<th>Share</th>
<th>Requirements</th>
<th>Portfolio structure</th>
<th>Return 60 %</th>
<th>Return 40 %</th>
<th>Return 10 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEEN</td>
<td>48 %</td>
<td></td>
<td>14 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DOEN</td>
<td>12 %</td>
<td></td>
<td>12 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZAEN</td>
<td>0</td>
<td></td>
<td>2 %</td>
<td>8 %</td>
<td></td>
</tr>
<tr>
<td>BAVL</td>
<td>40 %</td>
<td></td>
<td>72 %</td>
<td></td>
<td>80 %</td>
</tr>
<tr>
<td>Portfolio risk</td>
<td>32 %</td>
<td></td>
<td>28 %</td>
<td>27 %</td>
<td></td>
</tr>
</tbody>
</table>

After analyzing the Table 5, it is possible to conclude that the first portfolio has the highest return, but it also has the maximum risk from all portfolios. However, this portfolio is the most attractive, since its risk is much greater than in the last two options, and the return of such a portfolio is quite high. Therefore, a portfolio consisting of a 48 % stake in CEEN, a 12 % stake in DOEN and a 40 % stake in BAVL is the best option for an investor who has a set to maximize profits with a small percentage of risk.

7. SWOT analysis of research results

Strengths. Compared with peers, the integrated use of portfolio investment methods makes it possible to increase the efficiency of decision-making in the formation of an investment portfolio taking into account the criteria for minimizing risks and maximizing profits in the current political and economic situation of the Ukrainian stock market.

Weaknesses. The use of an integrated approach to the formation of an investment portfolio leads to the complexity of calculations due to the lack of information support, which would allow to automatically monitor and inform the investor about all changes in the status of stocks in the portfolio.

Opportunities. The proposed approach contributes to the efficiency of the formation of the investment portfolio, taking into account the instability of the Ukrainian stock market, which makes it possible to invest capital in the shares of domestic companies with a large percentage of reliability. However, for the most effective use of the proposed approach, it is necessary to stabilize the economic situation of the country and to provide regulatory support for the Ukrainian stock market.

Threats. The weaknesses of the proposed approach include the possibility of taking into account the imperfect regulatory framework of the securities market, which does not ensure the stability of changes in quotations of the considered assets in the market. That is, the results of the proposed approach can be applied only in a stable stock market, when the profitability of assets really depends on past values.

8. Conclusions

1. As a result of the research, the main possible standard and non-standard situations in the formation of the investment portfolio are analyzed. The main ways of their decision, based on the adoption of new conditions for the formation of the portfolio, or the creation of a new portfolio, if the indicators of the previous one do not correspond to certain goals, are defined.

2. The main models and methods used for the formation of the investment portfolio in world practice are analyzed. The main reasons are identified, the main ones being: lack of legislative and regulatory framework for regulating market relations, «shadowing» of the economy, stock market instability, according to which these models are not desirable to thrust into the current political and economic situation in the country. It is proposed to use a set of methods with the help of which one can as accurately as possible calculate the possible risks in the formation of an investment portfolio.

3. An example of the formation of an optimal stock portfolio is given; it meets two main criteria: minimizing risks and maximizing profits. It is shown how due to a change in indicators a part of shares in a portfolio changes.

References


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