NUMERICAL INVESTIGATION OF THE PROBLEM OF NONLINEAR THREE-PHASE FILTRATION

1. Introduction

As is known [1], important practical problems of oil and gas production, hydraulic engineering, chemical technology, etc. are associated with the problem of multiphase filtration. This problem is the most urgent for the theory and practice of oil and gas production.

The tasks of multiphase filtering have certain specific features that do not make it possible to always use the finite difference method that has become a classic finite-difference method for their numerical solution. Therefore, there is a need to develop difference schemes on adaptive grids, taking into account the features of the solution [2, 3].

Adaptive grids reduce artificial viscosity and oscillation of a numerical solution. Even if the computational grid has minimal points, they again give satisfactory results in terms of quantity and quality for the entire region (even with the inclusion of zones with solution features).

Therefore, it is relevant to study the processes of joint filtration of oil, gas and water in a porous medium, taking into account:

- real geological field data (elastic properties and formation heterogeneity);
- properties of filtered liquids and gas (compressibility, dependence of viscosities on pressure);
- relative phase permeability and capillary forces.

2. The object of research and its technological audit

The object of research is the numerical simulation of the filtration process of oil, gas and water on adaptive grids, taking into account some properties of liquids during their joint flow.

The tasks of multiphase filtration have specific features that do not make it possible to always use the classical finite difference method in their numerical solution.

One of the most problematic places in the theory of multiphase filtration is that the steps of the spatial variable must be crushed in areas of a sharp change not only in the gradient of water saturation, but also in the gradient of gas saturation. This is explained by the fact that due to the very low viscosity, free gas under the action of the pressure gradient overtakes the other components of the mixture – water and oil. Therefore, there is a need to develop difference schemes on adaptive grids, which makes it possible to take into account the features of the solution.

3. The aim and objectives of research

The aim of research is development of efficient and sufficiently universal numerical methods for solving the problem of one-dimensional three-phase filtration.
To achieve this aim it is necessary to perform the following objectives:
1. Build a mathematical model of the process of one-dimensional three-phase filtration in a porous medium.
2. Build an adaptive grid, taking into account the grinding steps in areas of abrupt change not only the gradient of water saturation, but also the gradient of gas saturation.
3. Develop an algorithm for the difference-iterative method in moving grids.
4. On the basis of numerical experiments, discuss the issues of computational implementation of the proposed method and give practical recommendations for its application.

4. Research of existing solutions of the problem

When conducting hydrodynamic calculations associated with the conduct and operation of oil and gas fields in the water-pressure mode, the Rapoport-Lis two-phase model is mainly used. However, due to the fact that in many fields there is a three-phase flow, it is necessary to consider a model of three-phase flow and evaluate the effect of the gas phase on the displacement process. And this, in turn, leads to the need to develop a numerical method for solving three-phase filtration problems described by a system of nonlinear partial differential equations. To study this process, numerical methods are mainly used [4, 5].

In the works [6, 7], the Buckley – Leverett model is used, and in [4] the effects of capillary forces are also calculated. For the numerical solution of the problems obtained, the idea of the method is used [8]. However, this method does not take into account the characteristics of multiphase filtration, and in some cases gives qualitatively and quantitatively incorrect results [4].

Among the main directions of solving this problem, identified in the resources of the world scientific periodicals, the works [9–11] can be highlighted. However, in [9], the extrusion is assumed to be piston, and approximate analytical solutions are obtained. A numerical method, ideologically different from the finite difference method and finite element method, is proposed in [10]. It should be noted that modifications of these works are widely used by many authors [11].

In this paper, an effective difference-iterative method in moving grids is proposed, which has the property of adaptability to the features of problems and is distinguished by high accuracy [2].

5. Methods of research

5.1. Mathematical formulation of the problem. Let’s suppose a circular horizontal reservoir of radius \( R \) and thickness \( H \) is considered. It is assumed that the reservoir does not let fluid in from below and above, in the center of the reservoir there is a perfect hydrodynamic production well of radius \( r = r_w \). on the contour \( r = R \) there is a battery of wells pumping compressible fluid into the reservoir.

Let’s suppose that in the reservoir in question there is a three-phase compressible fluid, the phases are immiscible and at the initial moment of time \( t = 0 \) are in a state of capillary equilibrium. Based on the Rapoport-Lis model for liquids with reduced conditions, the isothermal planar-radial nonlinear filtering problem is described by a system of partial differential equations in a dimensionless form:

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \rho \phi \psi \right) &= \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right) = \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right), \\
\frac{\partial}{\partial t} \left( \rho \phi \psi \right) &= \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right) = \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right), \\
\frac{\partial}{\partial t} \left( \rho \phi \psi \right) &= \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right) = \frac{\partial}{\partial r} \left( \rho \phi \psi \frac{\partial \psi}{\partial r} \right).
\end{align*}
\]

\((r,t) \in \Omega_t = \{r < r_w, \ R < t \leq T\}.

The following notation is used here: \( \lambda_\alpha \) – phase mobility coefficient:

\[
\lambda_\alpha (r, \rho_a, s_1, s_2, s_3) = k(r) \frac{f_\alpha (s_1, s_2, s_3)}{\mu_\alpha (p_a)};
\]

\(k(r)\) – permeability; \( r \) – polar coordinate; \( t \) – time.

Function:

\[
\psi_1 = \begin{cases} 1 - G_1 |\text{grad} P|, & |\text{grad} P| \geq G_1, \\ 0, & |\text{grad} P| < G_1, \end{cases}
\]

takes into account the viscoplastic property of oil [1, 12]. Here \( G_1 \) – the initial pressure gradient.

To the system of equations (1) it is necessary to add the equations of state of liquids and gas:

\[
p_a = F_a (p_a), \alpha = 1,2,3, \quad \rho_a = \frac{\rho_\alpha}{\rho_a}, \alpha = 1,2,3,
\]

and ratios:

\[
s_1 + s_2 + s_3 = 1.
\]

Let’s note that, in general, in problems of multiphase filtration, the pressure \( p_{a\alpha(1,2,3)} \) is not equal to each other and differs by the value of the corresponding capillary pressure, i. e.:

\[
p_1 - p_2 = p_{h1} (s_1, s_2, s_3), \\
p_3 - p_1 = p_{h2} (s_1, s_2, s_3),
\]

where \( p_{h1} \) – the capillary pressure between the oil and water, and \( p_{h2} \) – the capillary pressure between the gas and the oil.

Knowing \( p_{h1} \) and \( p_{h2} \), it is possible to use them to express the capillary pressure between gas and water:

\[
p_3 - p_2 = p_{h1} + p_{h2}.
\]

It should be noted that functions \( f_\alpha (s_1, s_2, s_3), \alpha = 1,2,3 \) have the following properties in the tasks of three-phase filtration [6, 7]:

\[
f_1 (s_1, s_2, s_3) = 0, s_1 \leq s_1', \\
f_2 (s_1, s_2, s_3) = 0, s_2 \leq s_2', \\
f_3 (s_1, s_2, s_3) = 0, s_3 \leq s_3',
\]

where values:

\( s_\alpha \) \( \alpha = 1,3 \) are called residual saturations.
Let’s suppose that the functions unknown in system (1):

\[ p_n(t, r) \text{, } (\alpha = 1.2, 3). \]

Let’s define them from the initial and boundary conditions.

Let’s suppose that at time \( t = 0 \), i.e. before the start of operation, the values of the functions being defined are known, i.e.:

\[ p_n(r, 0) = \phi(r), \quad r \leq R, \quad \alpha = 1.2, 3. \]  \( \text{(5)} \)

For the problem under consideration, the boundary conditions can be in the following form.

For a production well (i.e., at \( r = r_w \)), it is assumed that only oil is extracted from the formation, and at this point the capillary pressure jump is not taken into account:

\[ \frac{\partial p_n}{\partial r} = 0, \quad r \rightarrow r_w, \quad 0 < t \leq T. \]  \( \text{(6)} \)

On the external contour (i.e., at \( r = R \)), it is assumed that only compressible fluid (water) is pumped into the formation and, accordingly, the first and third phases do not flow:

\[ \frac{\partial p_n}{\partial r} = 0, \quad r = R, \quad 0 < t \leq T. \]  \( \text{(7)} \)

Thus, according to conditions (5)–(7), the task can be set as follows. It is necessary to find such functions:

\[ p_n(t, r) \text{, } (\alpha = 1.2, 3), \]

so that they would satisfy the system of equations (1) and the initial and boundary conditions (5)–(7).

Assuming the existence and uniqueness of the solution to problem (1)–(7), let’s give an approximate method for solving it.

### 5.2. Numerical method for solving the problem.

First of all, let’s note one specific feature of multiphase filtration tasks. It has been established that during the whole study, usually a sharp change in phase pressures occurs around the wells. Therefore, when modeling multiphase nonlinear filtering processes, the boundary conditions must be approximated with high accuracy, i.e., the grid step around wells must be reduced [2, 3].

Thus, let’s cover the area:

\[ \Omega = \{(r, t): r \leq r \leq R, \quad 0 < t \leq T\} \]

uneven grid \( \omega_n = \omega \times \omega \) as follows.

The nodes of the irregular grid \( \omega_n = \omega \times \omega \) by \( r \) are defined as follows:

\[ r_i = r_{i-1} + h, \quad i = 1, M - 1; \quad r_0 = r_w, \quad r_M = R, \]

\[ \tilde{R}_{i+1} = n_i h_i, \quad i = 1, 2, \ldots, i - 2, \ldots, i + \bar{N}_i, \]

\[ \tilde{R}_{i+1} = \left\{ h_{i, r}, \quad \varepsilon_i \leq 2 \tilde{R}_{i+1} \right\}, \]

\[ \tilde{R}_{i-1} = \frac{R - L_i}{n_i}, \quad h_{i, \tau_{i+1}} = h_{i, \tau_{i+2}}, \quad i = 1, 2, \ldots, i + n + 1. \]

Here

\[ n = \left\{ \begin{array}{ll}
1, \quad i = 1, 2, \ldots, i, i + 2, \ldots, i + \bar{N}_i, \\
2, \quad i = \bar{N}_i + 1, \bar{N}_i + 2, \ldots, i - 2.
\end{array} \right. \]

\[ i_0 = \max \left\{ i: \tilde{R}_i + \sum_{i=1}^{i-1} h_{i, n} \leq L_i \right\}. \]

where \( N \) is the number of partitions of the segment \( \left[ L_i, L_i \right] \), 
\( n \) is the number of constant steps; \( n' \) is the number of steps of the segment \( [L_i, R] \).

The task of finding the required values of the functions resulting from solving the system (1)–(7) will be solved at an arbitrary \( t = t_x \) by using the iterative-difference method, it is advisable to carry out calculations with an increment in time. Thus, the calculations begin with a very small step in \( t \) and gradually increase to a given \( T \) [2]. In accordance with this, the grid nodes \( \omega_n \) are defined as follows:

\[ t_0 = t_1 + \tau, \quad n = 1, N - 1, \quad t_n = 0, \quad t_N = T, \]

\[ \tau = \left\{ \begin{array}{ll}
\tau_{n}, \quad 1 \leq n \leq n_{N-1}, \\
\tau_{max}, \quad n = N.
\end{array} \right. \]

Here

\[ n = \left\{ n: \varepsilon, n_{max} \leq \tau_{max} \right\}, \]

where \( n \) is an integer part; \( k \) is a positive integer.

Thus, increasing the step \( t \) occurs every \( k \) time layer. In the calculations, \( k = 100 \) and \( \tau_{max} = 4 \cdot 10^{-8} \).

Using an implicit conservative scheme, let’s approximate the differential problem (1)–(7) on a non-uniform grid \( \omega_n \) with the following discrete problem:

\[ \frac{1}{h} \left( \frac{\Delta p_{i+1, n}}{h_{i+1}} - \frac{\Delta p_{i, n}}{h_{i+1}} \right) = \]

\[ = r_m \left[ \frac{f_{i+1} \Delta p_{i+1, n} + f_{i, n} \Delta p_{i, n}}{h_{i+1}} \right], \]

\[ \frac{1}{h} \left( \frac{\Delta p_{j+1, n}}{h_{j+1}} - \frac{\Delta p_{j, n}}{h_{j+1}} \right) = \]

\[ = r_m \left[ f_{j+1} \Delta p_{j+1, n} + f_{j, n} \Delta p_{j, n} \right], \]

\[ \frac{1}{h} \left( \frac{\Delta p_{i,j+1, n}}{h_{i,j+1}} - \frac{\Delta p_{i,j, n}}{h_{i,j+1}} \right) = \]

\[ = r_m \left[ f_{i,j+1} \Delta p_{i,j+1, n} + f_{i,j} \Delta p_{i,j, n} \right], \]

\[ \frac{1}{h} \left( \frac{\Delta p_{i,j+1, n}}{h_{i,j+1}} - \frac{\Delta p_{i,j, n}}{h_{i,j+1}} \right) = \]

\[ = r_m \left[ f_{i,j+1} \Delta p_{i,j+1, n} + f_{i,j} \Delta p_{i,j, n} \right], \]

\[ \frac{1}{h} \left( \frac{\Delta p_{i,j+1, n}}{h_{i,j+1}} - \frac{\Delta p_{i,j, n}}{h_{i,j+1}} \right) = \]

\[ = r_m \left[ f_{i,j+1} \Delta p_{i,j+1, n} + f_{i,j} \Delta p_{i,j, n} \right]. \]
\[ p_n(r_0) = p^0_n, \quad n = 0, \quad 0 \leq i \leq M, \quad \alpha = 1.23, \quad (9) \]

\[
\begin{align*}
\left[ p^0_i \right] &= \varphi^0, \\
\left[ h^1_i (p^0_i - p^0_{i+1}) \right] &= h^1_i (p^0_i - p^0_{i+1}), \\
\left[ h^1_i (p^0_{i+1} - p^0_{i-1}) \right] &= h^1_i (p^0_{i+1} - p^0_{i-1}).
\end{align*}
\tag{10}
\]

Thus, replacing \( p^{0+}_i \) with \( \hat{Y}_{ai} \), let's introduce the following matrices and vectors:

\[
B_i = \begin{bmatrix}
-1 & 0 & 0 \\
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}, \quad C_i = \begin{bmatrix}
0 & 0 & 0 \\
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}, \quad \hat{F}_0 = \begin{bmatrix}
-\varphi_1 \\
-\varphi_2 \\
-\varphi_3
\end{bmatrix}.
\]

Here

\[
\begin{align*}
h &= \frac{1}{2} (h_i + h_{i+1}), \\
\left( r p \lambda_{nl} \right)_{i+1} &= \left( r p \lambda_{nl} \right)_i + \frac{\left( r p \lambda_{nl} \right)_{i+1} - \left( r p \lambda_{nl} \right)_i}{2},
\end{align*}
\]

\[
f_{ai} = \begin{bmatrix}
p_i (1 - s_i - s_{i+1}) \rho (\lambda_{nl + 1}^1) + n \rho (\lambda_{nl + 1}^2) \\
p_i s_i \rho (\lambda_{nl + 1}^1) + n \rho (\lambda_{nl + 1}^2) \\
p_i = p_i s_i - p_i s_{i+1}, \quad f_{ai} = -p_i s_i
\end{bmatrix},
\]

\[
f_{ai} = \frac{\partial p}{\partial t}, \quad \alpha = 1.23, \quad s_i = s_i (p_i), \quad s_i = s_i (p_{i+1}),
\]

\[
s_i = 1 - s_i (p_i) - s_i (p_{i+1}),
\]

\[
\frac{ds_i}{dt} = \frac{ds_i}{dt} - \frac{ds_i}{dt} - \frac{ds_i}{dt}.
\]

By approximating the derivatives \( \frac{dp}{dt} \) on the right side of system (8) as follows:

\[
\frac{dp}{dt} = \frac{p_{ai} - p_{ai+1}}{1}, \quad \alpha = 1.23,
\]

and having carried out some elementary transformations, let's reduce (8) to the following form:

\[
\begin{align*}
&h^1 f^1_i (r p \lambda_{nl})_{i+1}^{(i)} p_{ai+1}^{(i)} - h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)} p_{ai}^{(i)} + \\
&+ h^1 f^1_i (r p \lambda_{nl})_{i+1}^{(i)} + r m t f^1_i f_{ai+1}^{(i)} p_{ai+1}^{(i)} + \\
&+ h^1 f^1_i (r p \lambda_{nl})_{i+1}^{(i)} - r m t f^1_i f_{ai}^{(i)} p_{ai}^{(i)} + \\
&+ r m t f^1_i f_{ai}^{(i)} p_{ai}^{(i)} - r m t f^1_i f_{ai+1}^{(i)} p_{ai+1}^{(i)} = \\
&= -r m t f^1_i f_{ai+1}^{(i)} p_{ai+1}^{(i)} + f_{ai+1}^{(i)} p_{ai+1}^{(i)} - f_{ai}^{(i)} p_{ai}^{(i)} - f_{ai+1}^{(i)} p_{ai+1}^{(i)}.
\end{align*}
\]

System (12) is non-linear, the defined functions are included in the coefficients of the system. In order to linearize nonlinear coefficients, let's use the simple iteration method, i. e, at each new time point, these coefficients are considered known from the previous time point [13]:

\[
T^{i+1} (r, p) = T^i (r, p).
\]

Taking into account the above, the difference problem (12) can be written in the following three-diagonal canonical form:

\[
A_i \hat{Y}_{ai} + B_i \hat{Y}_{ai} + C_i Y_{ai+1} = F_i + F_{ai}, \quad i = 0, 1, 2, \ldots
\]

The elements of the matrices A, B, and C, as well as the vector \( \hat{F} \), have the following form:

\[
A_i = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)}, \quad B_i = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)}, \quad C_i = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)},
\]

\[
F_{ai} = -r m t f^1_i (f_{ai+1}^{(i)} p_{ai+1}^{(i)} + f_{ai}^{(i)} p_{ai}^{(i)} - f_{ai+1}^{(i)} p_{ai+1}^{(i)});
\]

\[
F_{ai+1} = -r m t f^1_i (f_{ai+1}^{(i)} p_{ai+1}^{(i)} + f_{ai}^{(i)} p_{ai}^{(i)} - f_{ai+1}^{(i)} p_{ai+1}^{(i)});
\]

\[
C_i = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)};
\]

\[
C_{ai+1} = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)};
\]

\[
C_{ai+1} = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)};
\]

\[
h_{ai+1} = h^1 f^1_i (r p \lambda_{nl})_{i}^{(i)} + r m t f^1_i f_{ai}^{(i)}
\]

\[
(12)
\]
Thus, specifying the initial approximations in the form of (9) in order to find the \(i\)-th approximation from the system (13), let’s use the matrix version of the sweep method (Thomas algorithm) \([13, 14]\). At each moment of time, the iterative process is continued until such a value of \(i_0\) that the conditions are:

\[
\begin{align*}
\max_{b_{i0}} Y_{l}^{(i+1)}(t) - Y_{l}^{(i)}(t) & \leq \varepsilon_1, \\
\max_{b_{i0}} Y_{0}^{(i+1)}(t) - Y_{0}^{(i)}(t) & \leq \varepsilon_2, \\
\max_{b_{i0}} Y_{3}^{(i+1)}(t) - Y_{3}^{(i)}(t) & \leq \varepsilon_3,
\end{align*}
\]

would be satisfied at the same time. Here, \(\varepsilon_1, \varepsilon_2, \varepsilon_3\) – the predetermined accuracy for finding the \(i\)-th component of the approximate solution.

### 6. Research results

The iterative-difference method proposed above was applied to the numerical simulation of one three-phase filtration process. The following initial data were used for the calculations:

\(R = 100 \text{ m}, \; H = 10 \text{ m}, \; \tau = 0.1 \text{ m}; \; m = 0.2; \; k = 10^{-12} \text{ m}^2\);

\(\mu_1 = 0.3 \text{ poise}; \; \mu_2 = 0.01 \text{ poise}; \; \mu_3 = 0.0013 \text{ poise};\)

\(\phi_1(t) = 139.5 \text{ atm}; \; \phi_2(t) = 140 \text{ atm}; \; p_g^R(t) = 140 \text{ atm};\)

\(S_1^R(t) = 0.25; \; S_2^R(t) = 0.45; \; S_3^R(t) = 0.3; \; G_1 = 100 \text{ dyne/cm}^2;\)

\(R_s = 100 \text{ m}; \; k_0 = 10^{-12} \text{ m}^2; \; \mu_s = 0.01 \text{ poise}; \; T_0 = 10^4 \text{ s};\)

\(p_0 = 140 \text{ atm}; \; p_0 = 1 \text{ g/cm}^3.\)

Here \(R_s, k_0, \mu_s, T_0, p_0, p_0\) – the characteristic measurement values used to bring the original problem to a dimensionless form. The equations of state of the phases are given in the following form \([15]\):

\[
\begin{align*}
\rho_1(p_1) &= 0.000853 p_1 + 0.82592, \\
\rho_2(p_2) &= 0.01035 p_2 + 0.99989, \\
\rho_3(p_3) &= 0.063 p_3.
\end{align*}
\]

Prior to the beginning of the process, the phase saturation was \(S_1 = 0.45, \; S_2 = 0.25, \; S_3 = 0.30\), respectively. The dependences of relative permeabilities and capillary forces on saturations were taken following \([15]\).

The conducted numerical experiments show that at the beginning of the impact process, an enlarged zone of oil saturation begins to form in the reservoir – an oil shaft, which dimensions increase and which, displacing the gas, moves towards the production well. The displacement of oil occurs almost after the oil shaft. Under the «width» and «increase» of «oil shaft» profile refers to the following values:

\[
l_* = \max \{r : r \in [r_s, R_s]\},
\]

\[
h_* = \max \{r : S_i(r) < S_i^R, \}
\]

\[
r \in [r_s, R_s].
\]

Fig. 1 shows the results of calculations at \(\phi_s - \phi_1 = 0.5 \text{ kgf/cm}^2\) in various times for different values of oil viscosity using an adaptive grid. Here, the solid line shows the graphs reflecting the state of the oil shaft at different points in time, and the dashed lines indicate graphs showing water saturation.

It should be noted the following feature of the process. When the oil shaft approaches the production well, only gas escapes from the reservoir. Fig. 1 shows that as the viscosity of the oil decreases, the time for the oil shaft to approach the production well decreases.

![Fig. 1. The state of the oil shaft at different points in time](image)

Fig. 2 shows the effect of oil viscosity on the size and «growth» of the oil shaft. Here, the solid line shows the graph of oil saturation and the dashed line graph of gas saturation at time \(t = 3\) months \((\phi_2 - \phi_1 = 0.7 \text{ kgf/cm}^2)\).

In Fig. 2, the initial distribution of oil and gas saturation is shown by dots. Curves marked with numbers 1, 2, 3, 4, 5, 6 are the results of the calculations, respectively, at \(\mu_1 = 0.01; \; 0.02; \; 0.03; \; 0.1; \; 0.2; \; 0.4 \text{ poise}\).

As can be seen from Fig. 2, with an increase in oil viscosity, the length \(l_*\) and the «increase» \(h_*\) of the oil shaft decrease, and the decrease in \(l_*\) compared to \(h_*\) occurs at a higher rate. For example, from Table 1 that when increasing the value of \(\mu_1\) by 2, 10 and 40 times, \(l_*\) decreases by about 41%, 73%, 86%, and \(h_*\), respectively, decreases by 5%, 36%, 63%.

Thus, based on the above, in the presence of a free gas phase, the geometrical dimensions of the oil shaft...
characterize the completeness of the oil displacement, so that an increase in the shaft dimensions is the result of the oil being displaced by water.

![Fig. 2. The effect of oil viscosity on the size and growth of the oil shaft](image)

Table 1

<table>
<thead>
<tr>
<th>$\mu_1$, poise</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
</tr>
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<tbody>
<tr>
<td>$h_j$, m</td>
<td>60.01</td>
<td>33.02</td>
<td>25.29</td>
<td>14.49</td>
<td>11.21</td>
<td>9.40</td>
</tr>
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</table>

Let’s consider the effect of the displacement rate on the process of three-phase displacement (Table 2).

Table 2

<table>
<thead>
<tr>
<th>$k$</th>
<th>0.5 kgf/cm²</th>
<th>1 kgf/cm²</th>
</tr>
</thead>
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<tr>
<td>$r$</td>
<td>$S_1$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4500</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4500</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4500</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4525</td>
<td>0.2502</td>
</tr>
<tr>
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<td>0.5199</td>
<td>0.2505</td>
</tr>
<tr>
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</tr>
<tr>
<td>0.7</td>
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</tr>
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<td>0.3617</td>
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<td>0.3139</td>
<td>0.4813</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2726</td>
<td>0.5226</td>
</tr>
</tbody>
</table>

The results of calculations show that with an increase in the filtration rate, the geometric dimensions and the «increase» of the oil shaft increase dramatically. As can be seen from the Table 2, with an increase in the pressure difference to 0.4 kgf/cm², the shaft length increases by 35.4 %, and the increase by 9.6 %.

7. SWOT analysis of research results

**Strengths.** The proposed algorithm can be used to develop the definition of the optimal mode and time of the processes of thermal conductivity in inhomogeneous media.

**Weaknesses.** The weaknesses of research include the complexity of the calculation.

**Opportunities.** Using the obtained research, it is possible to increase the oil recovery of the field.

**Threats.** To implement the research it is necessary to purchase additional equipment, which means cash costs.

8. Conclusions

1. A mathematical model of a one-dimensional three-phase filtration process in a porous medium is constructed, which more adequately describes the process under consideration. A feature of this model is that it takes into account many of the above properties of filtered liquids and gas, for example, compressibility, viscoplasticity, the dependence of viscosity on pressure, relative phase permeability and capillary forces (the Rapoport-Lia model).

2. An algorithm for constructing an adaptive grid is given, which takes into account the refinement of steps in areas of abrupt change in the solution, and which makes it possible to more accurately approximate the derivatives of the desired functions.

3. The algorithm of the difference-iterative method in moving grids is developed, which takes into account the features of the solution and produces automatic thickening in the areas of abrupt solution change.

4. On the basis of numerical experiments, it is established that at the beginning of the impact process, an oil shaft begins to form in the reservoir, the dimensions of which increase and which, displacing the gas, move towards the production well. A displacement of oil occurs almost after the oil shaft.

References


